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Electron capture in fp shell at presupernova stage*

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The electron capture of neutron-rich nuclei in *fp* shell in massive stars at presupernova stage is discussed based on the nuclear shell model. The Gamow-Teller resonance transition strength is modified by introducing a Gaussian function. As a result, the electron capture rate in high density is evidently larger than the previous results given by some authors.

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1. Introduction

The structure of presupernova and its core plays a key role in supernova (SN) explosion and determines its evolution mode subsequently. Numerical simulation shows that the condition and result of supernova explosion depend on the presupernova model and the evolution mode that is especially sensitive to input physical parameters. The parameters are closely related to the change rate of electron fraction at the presupernova stage. Electron capture rate is the most important physical factor in studying supernova explosion mechanism. It is a key physical process not only for nucleosynthesis and neutrino production in supernova but also for core collapse supernova (including type SNII, SNI_b and SNI_c). When the mass density reaches the critical density at which some nuclei vastly capture electrons, the presupernova core begins to collapse. The electron fraction Y_e in presupernova core is one of the key physical parameters in theoretic research of type II supernova. Though the main cause of core collapse, for type I_a supernova, is the general relativistic effect, electron capture will accelerate the core collapse process. Electron capture not only affects the neutron excess at presupernova evolution stage, but also produces a great deal of neutrinos. Hence the iron core is cooled by this electron capture reaction.

Electron capture, as an important kind of weak interactions in the late stage of stellar evolution, has been widely studied for many years. Bethe *et al*[1] recognized that Gamow–Teller (GT) resonance transition greatly improves electron capture rate and may affect the formation of iron core and the collapse of the core region. Based on the simple shell model, Fuller, Fowler and Newman (FFN)[2−5] did many pioneering works, in which they discussed the location and the strength of GT resonance transition and made a detailed calculation of 226 nuclei with mass number (A) from 21 to 60. Aufderheide (AFWH) *et al*[6] pointed out that the nuclei with $A > 60$ also must be taken into account in theoretical analysis because the core region produced by presupernova evolution is mainly composed of some neutron-rich nuclei. Consequently, they extended FFN's work and discussed GT resonance transitions of some nuclei with $A > 60$, and provided the electron capture rate of 150 most abundant nuclei at the important temperatures and densities where the transitions are comparatively strong. Recently, Langanke and Martinez-Pinedo[7] calculated

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the capture rates of a great many of nuclei based on the shell model. This greatly enriches the data of weak interaction in presupernova stage.

It is easy to calculate the electron capture rate of nucleus if the distribution of excited-state nuclei is known. Unfortunately, most of them are difficult to be known in advance, because many excited states of nuclei are unstable at the presupernova stage. As a rule, the high-excited-state distributions are almost continuous, especially in heavy nuclei. So, it is infeasible to obtain the exact distribution for each nucleus. The excited levels of nuclei are usually analysed in terms of shell model. Based on this model Langanke and Martinez-Pinedo [7] have given good simulation results of electron capture rate for many different nuclei. However, this method is very time-consuming. AFWH[6] made an approximate treatment in which the GT excited states are divided into low-energy region and high-energy region. This treatment is relatively easy, but the precision of the calculated capture rates is limited.

It is very important to provide an accurate weak interaction rate of all comparatively important nuclei in the study on supernova explosion and collapse in the core region. In this paper, by introducing a Gaussian function to represent the distribution of GT resonance transition, we improve AFWH's work. This technique is similar to the method using by Kar *et al*[8] in the analysis of GT resonance for β decay.

2. Calculation of electron capture rate

In the calculation of electron capture rate at the presupernova stage, AFWH divided the excited states into two regions, the low-energy region near the ground state and the high-energy region near the resonance state. The low-energy region, which is represented by the ground state, is easy to study because the spin of the ground state can be obtained from experimental data or estimated by using the shell model. For the high-energy region, one feasible method is to seek a function to describe the distributions of the thick energy levels, but it is very difficult to find an accurate function for each nucleus.

The GT operator for a parent nucleus in initial state i will produce a corresponding strength distribution in possible final states, the strongest location is called GT resonance location. Different initial states produce different resonance locations corresponding to the excited states of the parent nucleus. AFWH *et al* put forward a hypothesis that the GT transition distribution of all final states f with energy $E_{\rm f}$ from any initial excited states i with energy $E_{\rm i}$ is similar to that from the ground state except that the resonance location is aboveE_i. With this hypothesis, the electron capture rate of a nucleus can be expressed as[6]

$$\lambda = \ln 2 \sum_f B_{0\rm f} f(Q_{0\rm f}), \tag{1}$$

where $B_{0\rm f} = 10^{-3.596} M_{0\rm f}^2$, $M_{0\rm f}$ is the matrix element of GT transition from ground state to final state f, $Q_{0\rm f}$ is the threshold energy corresponding to the electron capture. $Q_{0\rm f} = Q_{00} + E_0 - E_{\rm f}$, Q_{00} is the transition threshold energy between two ground states in the parent nucleus and daughter nucleus. E_0 and $E_{\rm f}$ are the energies of parent nucleus and daughter nucleus respectively. $f(Q_{0\rm f})$ is the electron capture phase space integral,[2,6] which is determined by mass density ρ, electron fraction $Y_{\rm e}$ and temperature T.

$$f(Q_{0\rm f}) = \int_{\varepsilon_Q}^{\infty} \varepsilon^2 (Q_{0\rm f} + \varepsilon)^2 G(z, \varepsilon) f_{\rm e}(\varepsilon) {\rm d}\varepsilon, \tag{2}$$

where ε is electron energy, $G\ (z, \varepsilon)$ is Coulomb distortion factor, which is approximately given in Ref.[2]. $f_{\rm e}\ (\varepsilon)$ is the electron Fermi-Dirac distribution function; ε_Q, the lowest interaction energy of electrons that take part in the electron capture reaction, is defined as

$$\varepsilon_Q = \begin{cases} |Q_{\rm if}|, & (Q_{\rm if} < -1); \\ 1, & (Q_{\rm if} \geq -1). \end{cases} \tag{3}$$

In order to have the sum in Eq.(1), we also divide the final states into low-energy region near ground state and high energy region near resonance location. In low-energy region, $B_{0\rm f}$ can be substituted with an effective value $B_{\rm eff}$=1.65.$\times 10^{-5}$,[2] in high-energy region, it is completely determined by GT resonance transition. Then the electron capture rate can be written as

$$\lambda = \lambda_0 + \lambda_{\rm GT}, \tag{4}$$

where λ_0 and $\lambda_{\rm GT}$ are electron capture rates of low-energy region and of GT resonance region respectively. $\lambda_0 = \ln 2 B_{\rm eff} f(Q_{00})$ is dominant when the temperature and density are low, while $\lambda_{\rm GT}$ is dominant when they are high. We use $\lambda_{\rm AFWH}$ to represent electron capture rate calculated by using the AFWH's method. So,

$$\lambda_{\rm AFWH} = \lambda_0 + \lambda_{\rm GT}^{\rm AFWH}, \tag{5}$$

where $\lambda_{\rm GT}^{\rm AFWH} = \ln 2 \cdot B_{\rm GT} f(Q_{00} - E_{\rm GT})$, $B_{\rm GT} = 10^{-3.596} M_{\rm GT}^2$, $E_{\rm GT}$ and $M_{\rm GT}$ are the location and

the matrix element of GT resonance transition respectively. $E_{\rm GT}$ can be expressed as $E_{\rm GT} = \Delta E_1 + \Delta E_2 + \Delta E_3$. ΔE_1, estimated by using single particle energies that can be found from relevant reference papers, is the energy difference between the orbits that the new neutron occupies in GT resonance and the ground state, $\Delta E_2$is the particle-hole repulsion energy, ΔE_3 is the energy for breaking a neutron pair if there is an even number of neutrons in the daughter nucleus. Generally, ΔE_2=2MeV, $\Delta E_3 = 12A^{-1/2}$. $M^2_{\rm GT} = \frac{1}{2}\cdot\frac{n^{\rm p}_{\rm i} n^{\rm h}_{\rm f}}{2J_{\rm f}+1}|M^{\rm sp}_{\rm GT}|^2_{\rm if}$, $n^{\rm p}_{\rm i}$ and $n^{\rm h}_{\rm f}$ are the number of proton particles in the initial orbital and that of neutron holes in the final orbit respectively; $J_{\rm f}$ is the angular momentum of the final state. The fraction $\frac{1}{2}$ is a typical quenching factor which fits the situation that GT resonance transition is dominant; $|M^{\rm sp}_{\rm GT}|^2_{\rm if}$ is the single-particle GT resonance transition matrix element which is given in Ref.[6].

3. A modification of GT resonance transition

In AFWH's work, the electron capture rate of GT resonance transition $\lambda^{\rm AFWH}_{\rm GT}$ was estimated by taking into account the contribution only from resonance location but not all excited states, and the precision is thus limited. We develop a reasonable and feasible method, which is similar to that in Ref.[8], to deal with the thick energy states distribution. The GT resonance strength distribution is presented by normalized Gaussian function whose centre is at the resonance location, namely

$$B^{\rm LP}_{\rm GT} = \frac{1}{\sqrt{\pi}\Delta}\exp\left(-\frac{(\varepsilon' - E_{\rm GT})^2}{\Delta^2}\right)\cdot B_{\rm GT}, \qquad (6)$$

where $B^{\rm LP}_{\rm GT}$ is resonance strength , ε' the excited energy, Δ the half-width of Gaussian function that should be obtained through experiments. However, because of the lack of these experimental data, it is infeasible to obtain the half-width for each nucleus. In this paper, it can be discussed through a relevant modification. The electron capture rate should be rewritten as

$$\lambda_{\rm LP} = \lambda_0 + \lambda^{\rm LP}_{\rm GT}, \qquad (7)$$

$$\lambda^{\rm LP}_{\rm GT} = \int_{\varepsilon_{\rm LP}}^{\infty} B^{\rm LP}_{\rm GT}{\rm d}\varepsilon' \int_{Q_{\rm LP}}^{\infty} f(Q_{00},\varepsilon,\varepsilon'){\rm d}\varepsilon, \qquad (8)$$

$$f(Q_{00},\varepsilon,\varepsilon') = \varepsilon^2(Q_{00}-\varepsilon'+\varepsilon)^2 G(z,\varepsilon) f_{\rm e}, \qquad (9)$$

where $\lambda^{\rm LP}_{\rm GT}$ is the GT modified resonance transition rate, $f(Q_{00},\varepsilon,\varepsilon')$ is the modified phase space factor. $\varepsilon_{\rm LP}$, whose exact location depends on the half-width of Gaussian distribution, is the bottom energy of the resonance region. $Q_{\rm LP}$, the lowest energy in electron capture reaction, is given by

$$Q_{\rm LP} = \begin{cases} |Q_{00}-\varepsilon'|, & (Q_{00}-\varepsilon' < -1); \\ 1, & (Q_{\rm LP}-\varepsilon' \geq -1). \end{cases} \qquad (10)$$

For the convenience of discussing the modification, we define an adjustable factor for the half-width as

$$C = \Delta/E_{\rm GT}. \qquad (11)$$

This reflects the modification of Gaussian representation. If $C \to 0$, namely $\Delta \to 0$, Gaussian function changes into a δ function, the results of electron capture will return to that given by AFWH. The half-width of some nuclei, about 6.3MeV, is given by Kar *et al* in discussing β decay through experimental data and theoretic analysis. Due to the experimental difficulty, in this paper the modification of electron capture rate will be discussed by using a series of possible half-width values.

4. Numerical analysis and conclusion

In order to investigate the modification effect on the electron capture rate, we have calculated the capture rate and the resonance transition fraction of a few electron capture reactions in the typical neutron-rich nuclei at the presupernova stage at certain important temperature and density we consider. Table 1 lists the modifications of electron capture rate, $\lambda_{\rm LP}$ and $\lambda_{\rm LP}/\lambda_{\rm AFWH}$ for ^{56}Fe and ^{56}Co with different C at $\rho = 5.86E+07{\rm g/cm}^3$, $T = 3.4\times10^9$K, $Y_{\rm e}$=0.47. Table 2 gives the modified results for ^{56}Fe and ^{56}Mn with different C at $\rho = 1.45E+08{\rm g/cm}^3$, $T = 3.8\times10^9$K, $Y_{\rm e}$=0.45.

For ^{56}Fe $(e^-,\nu_{\rm e})^{56}$Mn and ^{56}Mn $(e^-,\nu_{\rm e})^{56}$Cr, as shown in the below two tables, $\lambda_{\rm LP}$ and $\lambda_{\rm AFWH}$ show little difference from each other when $C < 0.15$. This result can be explained by the fact that the modification aims at the GT resonance transition. In fact, the fraction of GT, i.e. $F_{\rm GT}$, is very small for the above two nuclei at the temperature and density shown in the tables. However, for ^{56}Co $(e^-,\nu_{\rm e})^{56}$Fe, the fraction of GT is up to 40% at some points shown in Table 1, this results in an evident modification of the electron capture rates. Therefore, the electron capture

rate increases greatly through the Gaussian modification. For example, $\lambda_{\rm LP}/\lambda_{\rm AFWH} > 4$ when C=0.25. It should be noted that the rates always increase after the Gaussian modification. The reason can be understood as follows. The transition strength distribution is Gaussian, which is symmetrical about the resonance location; the electron distribution is Fermi-Dirac, but the electron energy in the resonance reaction is not symmetrical and the number of electrons taking part in the capture reaction increases evidently. So the total electron capture rate increases accordingly, especially in the nuclei whose resonance transition is dominant. The modified results are also related to the half-width, that is, to the excited-level strength distribution of GT resonance transition.

Table 1. The electron capture rates for ^{56}Fe and ^{56}Co with different C ($\rho = 5.86E+07$g/cm^3, $T = 3.4\times10^9$K, $Y_{\rm e}$=0.47).

	^{56}Fe $(e^-,\nu_e)^{56}$Mn		^{56}Co $(e^-,\nu_e)^{56}$Fe	
	$\lambda_{\rm AFWH} = 6.9529E-8(s^{-1})$, $F_{\rm GT} = 5.936E-4$		$\lambda_{\rm AFWH} = 5.1000E-2(s^{-1})$, $F_{\rm GT} = 4.410E-1$	
C	$\lambda_{\rm LP}$	$\lambda_{\rm LP}/\lambda_{AFWH}$	$\lambda_{\rm LP}$	$\lambda_{\rm LP}/\lambda_{AFWH}$
0.01	6.9529×10^{-8}	1.0000	0.0511	1.0021
0.05	0.6954×10^{-7}	1.0002	0.0583	1.1439
0.10	0.6965×10^{-7}	1.0018	0.0795	1.5601
0.15	0.7041×10^{-7}	1.0126	0.1135	2.2277
0.20	0.7799×10^{-7}	1.1217	0.1598	3.3136
0.25	0.1774×10^{-6}	2.5517	0.2183	4.2839

Table 2. The electron capture rates for ^{56}Fe and ^{56}Mn with different C ($\rho = 1.45E+08$g/cm^3, $T = 3.8\times10^9$K, $Y_{\rm e}$=0.45).

	^{56}Fe $(e^-,\nu_e)^{56}$Mn		^{56}Mn $(e^-,\nu_e)^{56}$Cr	
	$\lambda_{\rm AFWH} = 1.2946E-6(s^{-1})$, $F_{\rm GT} = 2.261E-3$		$\lambda_{\rm AFWH} = 2.3782E-4(s^{-1})$, $F_{\rm GT} = 3.560E-5$	
C	$\lambda_{\rm LP}$	$\lambda_{\rm LP}/\lambda_{AFWH}$	$\lambda_{\rm LP}$	$\lambda_{\rm LP}/\lambda_{AFWH}$
0.01	1.2946×10^{-6}	1.0000	2.3782×10^{-4}	1.0000
0.05	0.1295×10^{-5}	1.0007	0.2378×10^{-3}	1.0000
0.10	0.1300×10^{-5}	1.0045	0.2379×10^{-3}	1.0003
0.15	0.1326×10^{-5}	1.0244	0.2388×10^{-3}	1.0041
0.20	0.1500×10^{-5}	1.1588	0.2565×10^{-3}	1.0785
0.25	0.2982×10^{-5}	2.3037	0.4601×10^{-3}	1.9344

From the above calculation and analysis, we can find that Gaussian modification of resonance-excited-level makes the electron capture rate increase slightly. For the electron capture reaction dominated by low-energy transition, the modification effect is limited, but this effect is very remarkable for the one dominated by resonance transition, because the Gaussian spectrum distribution enlarges the amount of the captured electrons. We thus conclude that the modification of the excited-state is necessary, especially for the situation that the distribution of the excited-level in GT resonance transition is not very concentrative. It is because the density and Fermi energy of the electrons are high enough at the presupernova stage and the high-energy resonance transition is dominant in the electron capture reaction.

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