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# Effect of electron–phonon interactions on the dynamical localization of semiconductor superlattices*

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Effect of the electron–phonon interactions on the property of dynamical localization of semiconductor superlattices in a single-band model under the actions of the dc and ac electric field, respectively, is investigated. The results show that the electron–phonon interactions can destroy the dynamical localization of the electron. For the same phonon frequency, when the strength of the electron–phonon interactions becomes larger, the electron is more quickly scattered. When the phonon frequency equals to the Bloch frequency $\Omega_{\rm B}$ in dc field or $\omega$ in ac field or one of them in dc-ac field, the electron initially localized at the 0th site can be rapidly scattered to the other sites, where $\omega$ is the frequency of the ac field. The property of the dynamical localization of the electron can be seriously changed by the photon–phonon resonant field.

**Keywords:** electron–phonon interactions, dynamical localization
**PACC:** 7335, 7340L

## 1. Introduction

Recently, the availability of high-quality semiconductor superlattices has facilitated some interesting experimental studies, and there is a growing interest in the response of semiconductor superlattices to external electric field. Especially for the response to high-frequency ac electric fields, a thorough study has been conducted and many useful results have been obtained.[1−15] In the investigation of electron evolution in such a system, it is found that an initially localized particle will remain localized if the ratio of the field magnitude to the frequency is a root of the zeroth-order Bessel function.[9−11] This phenomenon was later found to be the effect of band collapse of the quasi-energy miniband,[12] which has been verified experimentally in optical lattices.[13]

In fact the crystal lattice sites are vibrational in a real system. i.e. there are all kinds of phonons existing in a real system. These phonons will interact with electrons. From both the theoretical and experimental points of view, the effect of the electron–phonon interactions must be considered in the investigation of the electron dynamics. For example, Dmitriev and Suris[14] have studied the transport characteristics of electrons under the interactions between electrons and acoustic phonons.

In this paper, we study the effect of electron–phonon interactions on the property of dynamical localization of semiconductor superlattices in a single-band model under the action of an external field. We find that the electron–phonon interactions can change the property of the dynamical localization of the electrons. When the phonon frequency equals the Bloch frequency $\Omega_{\rm B}$ in dc field or $\omega$ in ac field or one of them in a dc–ac field, the electron can be rapidly scattered. The dynamical localization of the electron is seriously destroyed by this electron–phonon resonant field. Therefore, in order to establish dynamical localization of electrons in experiment one should have control over the resonance between the phonon and external electric fields.

## 2. Model

We use the standard tight-binding model of a

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single-band system under the action of an external field $E(t)$. The model Hamiltonian can be written as

$$H(t) = H_0(t) + H_{\rm ep}, \tag{1}$$

where

$$H_0(t) = V\sum_m(|m\rangle\langle m+1| + |m+1\rangle\langle m|) - edE(t)\sum_m m|m\rangle\langle m| \tag{2}$$

is the term without the electron–phonon interactions. $V$ is the nearest-neighbour intersite interactions, $e$ is the charge of the carrier, $d$ is the lattice constant, and $|m\rangle$ is a Wannier state. The external field $E(t)$ can be written as

$$E(t) = E_0 + E_1\cos\omega t, \tag{3}$$

where $E_0$ is the amplitude of the dc field, $E_1$ and $\omega$ are, respectively, the amplitude and the frequency of the ac field. The Bloch frequency is defined as $\Omega_{\rm B} = edE_0$.

To simplify the question, we only consider the interactions between the electrons and the acoustical phonons. The Hamiltonian can be written as[16]

$$H_{\rm ep} = {\rm i}\sum_{k,q} D_q(a_q{\rm e}^{-{\rm i}\omega_q t} + a_{-q}^{\dagger}{\rm e}^{{\rm i}\omega_q t})C_{k+q}^{\dagger}C_k, \tag{4}$$

where $q$ is the wave vector of the phonons. $D_q$ describes the strength of the electron–phonon interactions and $D_q = -D_{-q}$. $a_q(a_q^{\dagger})$ and $C_k(C_k^{\dagger})$ are, respectively, the lowering (raising) operators of the phonon and electron. $\omega_q$ is the frequency of phonon $q$, and $\omega_q = \omega_{-q}$. $k$ is in the Brillouin zone. When $k$ is out of the Brillouin zone, we can use $k - lK_n$ to replace $k$, where $k - lK_n$ is in the Brillouin zone. $K_n$ is the period of the Brillouin zone, and $l$ is an integer. Taking the Fourier transform, we can get the form of the electron–phonon interactions in the Wannier representation,

$$H_{\rm ep} = {\rm i}\sum_{m,q} D_q(a_q{\rm e}^{{\rm i}mq-{\rm i}\omega_q t} + a_{-q}^{\dagger}{\rm e}^{{\rm i}mq+i\omega_q t})|m\rangle\langle m|. \tag{5}$$

Here we give some explanation to this model. We know one superlattice site includes many atom lattice sites. In a real system, these atom lattice sites are vibrational. Since we have taken the effective mass approximation for the electron in the system considered, the interactions of the electrons and the vibrations of these atom lattice sites may be included in the effective mass. Thus, in this model we only consider the effect of the vibration of the superlattice sites.

In order to investigate the dynamics of the system, the particle state can be expressed as $|\Psi(t)\rangle$ as a linear combination of $|m\rangle|n_{q_1}\rangle|n_{q_2}\rangle|n_{q_3}\rangle...$, where $|m\rangle$ is the Wannier state, $|n_{q_i}\rangle$ is the phonon number state which has the frequency $q_i$. In this paper, we only consider the effect of the phonon with a monofrequency on the dynamical localization of the electron. So, we can write $|\Psi(t)\rangle = \sum_{mn_q} C_{mn_q}|m, n_q\rangle$. The following evolution equation for the amplitudes $C_{mn_q}$ can be obtained from Eqs.(1), (2) and (5) (throughout this paper, we set $d = \hbar = 1$),

$$\begin{aligned}{\rm i}\frac{{\rm d}C_{mn_q}}{{\rm d}t} =& V(C_{m+1,n_q} + C_{m-1,n_q}) - eE(t)mC_{mn_q} \\ &+ {\rm i}D_q(C_{m,n_q+1}{\rm e}^{{\rm i}mq-{\rm i}\omega_q t}\sqrt{n_q+1} \\ &- C_{m,n_q-1}{\rm e}^{-{\rm i}mq+{\rm i}\omega_q t}\sqrt{n_q}).\end{aligned} \tag{6}$$

To solve Eq.(6), we must specify the integers $m$ and $n_q$. In this paper we select $m$ to be integers from $-10$ to 10 and $n_q$ from 40 to 60. In fact, the number of the phonons can change from 0 to infinity, but in numerical calculations, we must cut the number of the phonons. We arbitrarily make the number of phonons $n_q$ change from 40 to 60. Of course, the same conclusions will be reached in other phonon number ranges. To more clearly understand the effect of the phonons on the dynamical localization of the electrons, we choose the initial conditions as

$$C_{m,n_q}(0) = \delta_{m,0}\delta_{n_q,50}, \tag{7}$$

i.e., when $t$=0, $n_q = 50$. In the next section we discuss the effect of the electron–phonon interactions on the dynamical localization in the external electric field with Eqs.(6) and (7).

## 3. Numerical results and discussions

Notice that Eq.(6) is uncoupling for phonons, and we can only consider the effect of one kind of phonon with the wave vector $q$ on the property of the dynamical localization of the electron.

### 3.1. The case in a dc field

In the following, we set the Bloch frequency $\Omega_{\rm B}$ as the energy unit and $V/\Omega_{\rm B} = 1/2$. The strength of the electron–phonon interactions can be chosen as $D_q/\Omega_{\rm B} = 0.025$. Figures 1 and 2 show the probability of the electron at the 0th lattice site $P_0$, which is defined as

$$P_0 = \sum_{n_q}|C_{0,nq}|^2, \tag{8}$$

versus the dimensionless time $t^* = \Omega_B t$. The phonon wave vectors $q = \pi$ and $q = \pi/3$ are, respectively, studied in Fig.1 and Fig.2. In both figures, the phonon frequencies are $\omega_q/\Omega_B = 0.5$, 1.0, and 1.5 which correspond to the solid, dashed, and dotted line, respectively.

From Figs.1 and 2, one can see for all phonons with the frequency $\omega_q$, the probability of the electron at the 0th lattice site decays with the time and does not return to 1 again. This means that the electron initially localized at the 0th site will be scattered to the other sites under the electron–phonon interactions. From these figures, one can also find though $P_0$ decays, the decay rates are not the same. For $\omega_q/\Omega_B = 0.5$ and 1.5, the resonant peaks slowly decay which one can clearly observe in the first 40 time steps. But for $\omega_q/\Omega_B = 2.0$, the resonant peak rapidly decays with the time. After a period of time, the amplitude of the resonant peaks decays to values below 0.2. The reason is that when $\omega_q/\Omega_B = 1.0$, the phonon frequency is just the same as the Bloch frequency $\omega_q = \Omega_B$, i.e. there exists resonance between the electric dc field (or the Bloch oscillator) and the phonon field. The electron initially located at the 0th site will be rapidly scattered to the other sites under this resonant field. The property of the dynamical localization of the electron is seriously destroyed by this electron–phonon field. Therefore, in order to observe the dynamical localization in experiment one should have control over the resonance between the phonon and external electric fields.

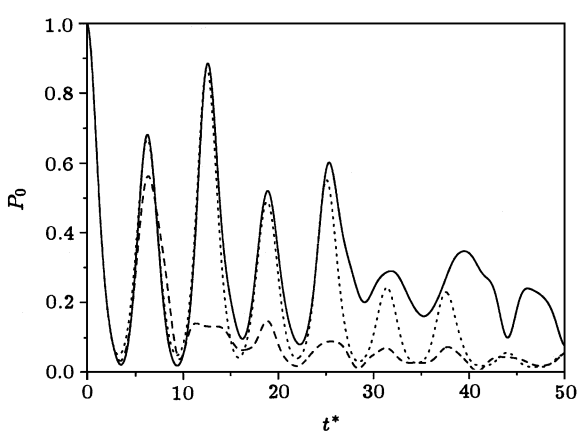



**Fig.1.** Probability of the electron at the 0th lattice $P_0$ versus the dimensionless time $t^* = \Omega_B t$. The superlattice and electric field parameters are chosen to be $V/\Omega_B = 0.5$, $D_q/\Omega_B = 0.025$, and the phonon vector $q = \pi$. The phonon frequencies $\omega_q/\Omega_B = 0.5$, 1.0, and 1.5 which correspond to the solid, dashed, and dotted line, respectively.

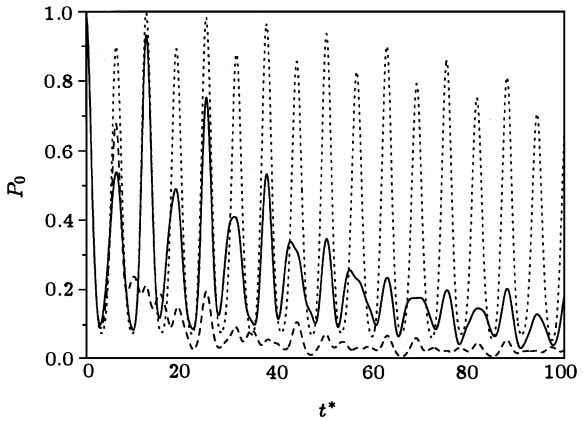



**Fig.2.** Probability of the electron at the 0th lattice $P_0$ versus the dimensionless time $t^* = \Omega_B t$. The superlattice and electric field parameters are chosen as in Fig.1, only the phonon wave vector $q = \pi/3$. The phonon frequencies $\omega_q/\omega = 0.5$, 1.0, and 1.5 which correspond to the solid, dashed, and dotted line, respectively.

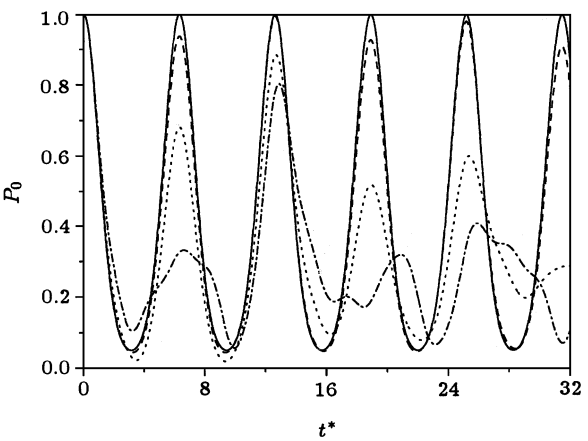



**Fig.3.** Probability of the electron at the 0th lattice $P_0$ versus the dimensionless time $t^* = \Omega_B t$. The superlattice and electric field parameters are chosen to be $V/\Omega_B = 0.5$, $\omega_q/\Omega_B = 0.5$, and the phonon wave vector $q = \pi$. The strength of the electron–phonon interactions $D_q/\omega = 0.00$, 0.01, 0.025, and 0.05 which correspond to the solid, dashed, dotted, dashed-dotted line, respectively.

When $V/\Omega_B = 0.5$, the phonon wave vector $q = \pi$ and the phonon frequency $\omega_q/\Omega_B = 1.0$ are fixed, one can plot the probability of the electron at the 0th site $P_0$ versus the dimensionless time $t^* = \Omega_B t$ under the different strengths of the electron–phonon interactions $D_q$, as shown in Fig.3. In this figure, the strength of the electron–phonon interactions $D_q/\Omega_B = 0.0$, 0.01, 0.025, and 0.05 which correspond to the solid, dashed, dotted, dash-dotted line, respectively. One can notice that when there are no electron–phonon interactions, i.e. $D_q/\Omega_B = 0.0$, $P_0$ oscillates between 1 and

a positive number. When there are electron–phonon interactions, the property of the dynamical localization of the electron will be destroyed, and $P_0$ decays with the time. The strength of the electron–phonon interactions being stronger, i.e. $D_q/\Omega_B$ larger, the dynamical localization is more quickly destroyed. When $D_q/\Omega_B$ increases from 0.0 to 0.05, the amplitude of the resonant peaks decays faster and faster, which can be seen in Fig.3. This shows that the electron–phonon interactions will destroy the dynamical localization. The larger the strength, the faster the dynamical localization is destroyed.

### 3.2. The case in an ac field

In this section, we investigate the dynamics of the electron in an ac field with the electron–phonon interactions. We set $\omega$ as the energy unit and $V/\omega = 1$. The ac field is set as $eE_1/\omega = 2.405$. The reason why we select this parameter is that when there are no electron–phonon interactions, under this parameter, the electron initially localized will remain localized. The strength of the electron–phonon interactions can be chosen as $D_q/\omega = 0.02$. Figure 4 shows the probability of the electron at the 0th site versus the dimensionless time $t^* = \omega t$. The phonon wave vector $q = \pi$ is studied in Fig.4. In this figure, the phonon frequencies are $\omega_q/\omega = 0.8$, 1.0, and 1.2 which correspond to the solid, dashed, and dotted line, respectively.

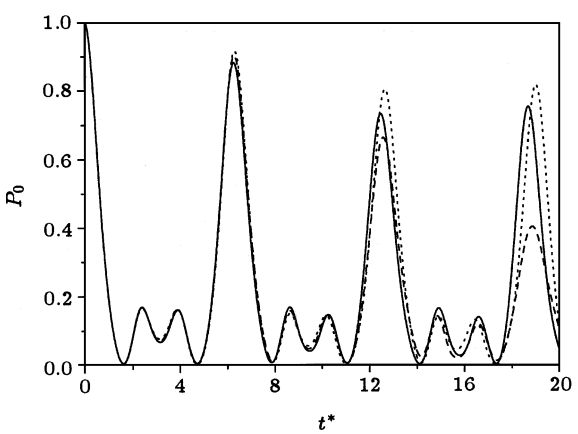



**Fig.4.** Probability of the electron at the 0th lattice $P_0$ versus the dimensionless time $t^* = \omega t$. The superlattice and electric field parameters are chosen to be $V/\omega = 1.0$, $eE_1/\omega = 2.405$, $D_q/\omega = 0.02$, and $q = \pi$. The phonon frequencies $\omega_q/\omega = 0.8$, 1.0, and 1.2 which correspond to the solid, dashed, and dotted line, respectively.

From Fig.4, one can see for all phonon frequencies $\omega_q$, the probability of the electron at the 0th lattice site will decay with the time and does not return to 1 again. That means that the electron initially localized at the 0th site will be scattered to the other lattices under the electron–phonon interactions. From the first 20 time steps, one can find $P_0$ will decays, but the decay rates are not the same. The electron will be scattered more quickly to other sites under the interactions with the phonon $\omega_q/\omega = 1.0$ than with the other phonons. The reason is the same as in a dc field. When $\omega_q/\omega = 1.0$, the phonon frequency just equals the external electric ac field frequency $\omega$. The resonance takes place between the ac electric (or the photon) field and the phonon, i.e. there exists photon–phonon resonance. The electron initially located at the 0th site will be rapidly scattered to the other sites under the effect of this resonant photon–phonon field.

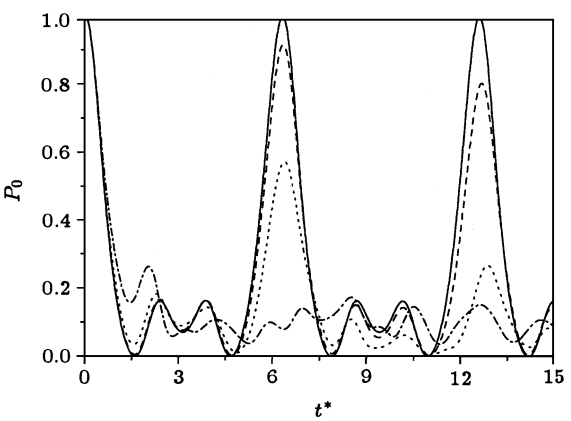



**Fig.5.** Probability of the electron at the 0th lattice $P_0$ versus the dimensionless time $t^* = \omega t$. The superlattice and electric field parameters are chosen to be $V/\omega = 1.0$, $eE_1/\omega = 2.405$, $\omega_q/\omega = 1.2$, and $q = \pi$. The strength of the electron–phonon interactions $D_q/\omega = 0.00$, 0.02, 0.05, and 0.10 which correspond to the solid, dashed, dotted, dashed-dotted line, respectively.

When the other parameters are fixed, one can plot the probability of the electron at the 0th site $P_0$ versus the dimensionless time $t^* = \omega t$ under the different strengths of the electron–phonon interactions $D_q$, as shown in Fig.5. In this figure, $V/\omega = 1.0$, $eE_1/\omega = 2.405$, the phonon wave vector $q = \pi$ and the phonon frequency $\omega_q/\omega = 1.2$. The strength of the electron–phonon interactions $D_q/\omega = 0.0$, 0.02, 0.05, and 0.10 which correspond to the solid, dashed, dotted, dash-dotted line, respectively. One can notice that when there are no electron–phonon interactions, i.e. $D_q/\omega = 0.0$, $P_0$ oscillates between 1 and 0. When there are electron–phonon interactions, the property of the dynamical localization of the electron is destroyed. $P_0$ decays with the time. The strength

of the electron–phonon interactions being stronger, i.e., $D_q/\omega$ larger, the dynamical localization is more quickly destroyed. When $D_q/\omega$ changes from 0.0 to 0.10, the amplitude of the resonant peaks decays faster and faster, which can be seen in Fig.5. It shows that the electron–phonon interactions will destroy the dynamical localization. The larger the strength, the faster the dynamical localization is destroyed.

### 3.3. The case in a dc–ac field

In this section, we investigate the dynamics of the electron in a dc–ac field with the electron–phonon interactions. We also set $\omega$ as the energy unit and $V/\omega = 1$, the Bloch frequency $\Omega_{\rm B}/\omega = 0.5$, and the strength of the electric ac field $eE_1/\omega = 3.832$. With these parameters, the electron initially localized will remain localized without the effect of the phonons (i.e. $D_q = 0.0$). In fact, when the effect of the phonons is considered, the initially localized electrons will be scattered by the electron–phonon interactions, which can be seen in Fig.6. In this figure, we set $D_q/\omega = 0.05$, the phonon wave vector $q = \pi$ and the phonon frequencies are $\omega_q/\omega = 0.5$, 0.7, and 1.0 which correspond to the solid, dashed, and dotted line, respectively.

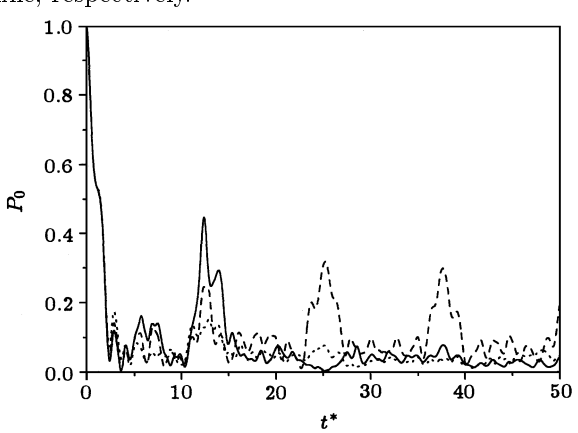



**Fig.6.** Probability of the electron at the 0th lattice $P_0$ versus the dimensionless time $t^* = \omega t$. The superlattice and electric field parameters are chosen to be $V/\omega = 1.0$, $\Omega_{\rm B}/\omega = 0.5$, $eE_1/\omega = 3.832$, $D_q/\omega = 0.05$, and the wave vector $q = \pi$. The phonon frequencies $\omega_q/\omega = 0.5$, 0.70, and 1.0 which correspond to the solid, dashed, and dotted line, respectively.

From Fig.6, one can find for all the phonon frequencies $\omega_q$, the probability of the electron at the 0th lattice site $P_0$ decays with the time and does not return to 1 again. That means the property of the dynamical localization of the electron is destroyed. When $D_q/\omega = 0.7$, though the resonant peaks decay with the dimensionless time $t^* = \omega t$, one can clearly observe them in the first several periods. When $D_q/\omega = 0.5$ and $D_q/\omega = 1.0$, the resonant peaks rapidly decay with the dimensionless time. After one period of time, the resonant peaks cannot be found. That means the decay rates of $P_0$ are not the same. The electron initially localized at the 0th site will be more quickly scattered to the other sites under the interactions of the electron with the phonons with frequencies $\omega_q/\omega = 0.5$ and 1.0 than with 0.7. The reason is that when $\omega_q/\omega = 0.5$, the resonance takes place between the dc field (or the Bloch oscillator) and the phonons. Under this resonant field, the electron initially located at site 0 is rapidly scattered to the other sites. When $\omega_q/\omega = 1.0$, the resonance takes place between the electric ac field (i.e. the photon field) and the phonon. Under this photon–phonon resonant field, the electron initially located at site 0 is also rapidly scattered to the other sites.

Figure 7 shows that the probability of the electron at the 0th site $P_0$ versus the dimensionless time $t^* = \omega t$ under different strengths of the electron–phonon interaction $D_q$. The other parameters are set as: $V/\omega = 1.0$, the Bloch frequency $\Omega_{\rm B}/\omega = 0.5$, the strength of the electric ac field $eE_1/\omega = 3.832$, the phonon wave vector $q = \pi$ and the phonon frequency $\omega_q/\omega = 0.7$. The strength of the electron–phonon interaction $D_q/\omega = 0.0$, 0.02, 0.05, and 0.08 which correspond to the solid, dashed, dotted, dash-dotted line, respectively. When there is no electron–phonon interaction, i.e., $D_q/\omega = 0.0$, one can find that the probability of the electron at site 0 $P_0$ will return to 1 after a period of time (the solid line). That means that the electron initially localized remains localized with time. When there are electron–phonon interactions, i.e. $D_q/\omega \neq 0.0$, $P_0$ does not return to 1 with the time (the dashed, dotted and dash-dotted lines). This shows that the interactions between the electron and the phonons destroy the property of the dynamical localization of the electron. From Fig.7, one can find that, with the strength of the electron–phonon interactions increasing, the dynamical localization of the electrons is more and more easily destroyed.

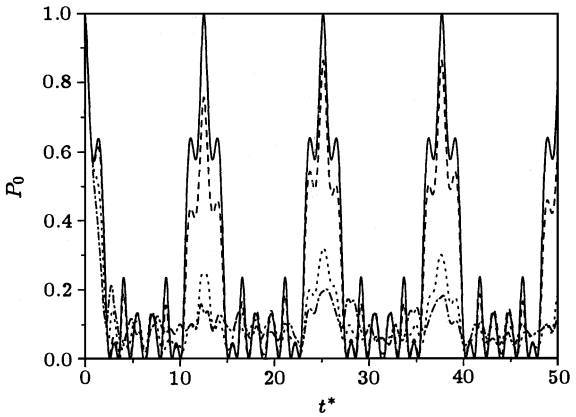



**Fig.7.** Probability of the electron at the 0th lattice $P_0$ versus the dimensionless time $t^* = \omega t$. The superlattice and electric field parameters are chosen to be $V/\omega = 1.0$, $\Omega_B/\omega = 0.5$, $eE_1/\omega = 3.832$, $\omega_q/\omega = 0.7$, and the wave vector $q = \pi$. The strength of the electron–phonon interactions $D_q/\omega = 0.00$, 0.02, 0.05, and 0.08 which correspond to the solid, dashed, dotted, dashed-dotted line, respectively.

# 4. Conclusions

In this paper, we investigate the effect of the electron–phonon interactions on the property of dynamical localization of semiconductor superlattices in a single-band model under the actions of the dc and ac electric field. We find that the electron–phonon interactions can destroy the dynamical localization of the electron. When the electron–phonon interactions become stronger, the electron is more quickly scattered. When the phonon frequency equals the Bloch frequency $\Omega_B$ in dc field or $\omega$ in ac field, the electron initially localized at the 0th site can be rapidly scattered to the other sites. The property of the dynamical localization of the electron is seriously destroyed by the resonant photon–phonon field. Therefore, in order to observe the property of the dynamical localization of the electron, one should have control over the resonance between the phonon field and the external electric field.

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