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Thermodynamics of squeezed states in mesoscopic circuits*

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Based on the information theory, we present a density matrix to discuss coherent and squeezed states in a mesoscopic LC circuit with time-dependent frequency. With the relevant operators included in the density matrix, a connection between the appearance of coherent and squeezed states is established, i.e., the quantum state evolution of the system is closely related to the initial state. Generally speaking, due to the effect of environment temperature, the mesoscopic LC circuit will evolve to a squeezed state when it initially lies in an excited state. In particular, at a low temperature, step changes of circuit parameters will result in a squeezed minimum uncertainty state if the resonance frequency remains the same after the change.

Keywords: mesoscopic LC circuit, thermodynamics, squeezed state, coherent state
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1. Introduction

Technological miniaturization of circuits is of increasing concern and their mesoscopic aspects become more prominent. The in-depth study interests people in three aspects: nanometer electronic elements acting as quantum dot or quantum wire, non-classical quantum effect in mesoscopic quantum circuits, and electronic approaches to quantum state.[1−5] Some nonclassical states, such as coherent states, squeezed states, etc., have been studied extensively.[6,7] The squeezed states are important because they can achieve lower quantum noise than the zero point fluctuations associated with vacuum or coherent states. Thus they provide a way of manipulating quantum fluctuations and a promising future in different applications ranging from optical communication to gravitational wave detection. For example, the idea of squeezed states has also been used in the field of condensed matter physics; squeezed phonon states have been observed;[8] and it is also shown that squeezing effects can exist in mesoscopic circuits.[9−12]

Practically, since a circuit system always works at certain temperatures, quantum statistic mechanics, which is based on quantum mechanics, should be used to study the qualities of mesoscopic circuits. Then, how to construct a density matrix becomes a key point for a system to be studied. The information theory approach based on the maximum entropy theorem provides us with a new perspective to study quantum properties of a system.[13] In the information theory, it can be found that the appearance of squeezing is a direct property of dynamical Lie-group structure (i.e. the Hamiltonian and the relevant operators are taken into account); and the thermodynamical approach to quantal systems that can be achieved within the maximum-entropy principle (MEP) formalism allows for a straightforward analysis of squeezing in quantal systems with nonzero temperature. It can be said that the MEP approach makes evident the intrinsic connection among the appearance of coherent or squeezed states, the different "knowledge" introduced in a general density matrix and the Hamiltonian dynamics. In this paper, the information theory is applied to study the quantum state evolution and the squeezing effect in a mesoscopic LC circuit with variable frequency.

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2. Basic concepts of information theory

The information theory approach provides us with a definite prescription to construct the density matrix $\hat{\rho}$, starting from the knowledge of the expectation values of, say, M operators $\hat{O}_j(\hat{O}_0 = \hat{I})$ defined as usual,

$$\left\langle \hat{O}_j \right\rangle = \mathrm{Tr}(\hat{\rho}\hat{O}_j), \quad j = 0, 1, \ldots, M. \tag{1}$$

The identity operator is included in the original set in order to fulfill the normalization condition $\mathrm{Tr}\hat{\rho} = 1$. The MEP version of $\hat{\rho}$ is

$$\hat{\rho} = \exp\left[-\lambda_0 \hat{I} - \sum_{j=1}^{M} \lambda_j \hat{O}_j\right], \tag{2}$$

where λ_j are $M+1$ Lagrange multipliers determined so as to satisfy Eq.(1) and the maximization of the entropy $S[\hat{\rho}]$ (taking the Boltzmann constant equal to unity),

$$S[\hat{\rho}] = -\mathrm{Tr}(\hat{\rho}\ln\hat{\rho}) = \lambda_0 + \sum_{j=1}^{M} \lambda_j \left\langle \hat{O}_j / \hat{\rho} \right\rangle. \tag{3}$$

It can be proven that, due to the fact that $\hat{\rho}(t)$ obeys the Liouville equation and Eq.(3), the set of operators $\hat{O}_j$ originally considered must be eventually extended to a set of q relevant operators selected in order to close a partial Lie algebra under commutation with the Hamiltonian $\hat{H}$

$$[\hat{H}, \hat{O}_k] = \mathrm{i}\hbar \sum_{j=0}^{q} \hat{O}_j g_{jk}, \tag{4}$$

where g_{jk} are the elements of a $q \times q$ matrix G (which may depend upon the time if $\hat{H}$ is time-dependent). The closure condition given by Eq.(4) allows us to solve the dynamical problem using a set of coupled equations for the $\lambda_j' s$

$$\frac{\mathrm{d}\lambda_j}{\mathrm{d}t} = \sum_{k=0}^{q} g_{jk}\lambda_k, \tag{5}$$

which is formally equivalent to the time-dependent Schrödinger equation. It is possible to obtain the temporal evolution of the expectation values of the relevant operators, resulting in

$$\frac{\mathrm{d}\left\langle \hat{O}_j \right\rangle}{\mathrm{d}t} = -\sum_{k=0}^{q} \left\langle \hat{O}_k \right\rangle g_{kj}, \quad i = 1, 2, \ldots, q. \tag{6}$$

3. Density matrix for a mesoscopic LC circuit

It is Louisell who first quantized a single LC (L stands for the inductance, and C, the capacitance) circuit

$$H = \frac{1}{2L}p^2 + \frac{1}{2C}q^2,$$

$p = L\dot{q}$. According to the canonical quantization principle, we can quantize the system by operators $\hat{q}$ and $\hat{p}$, which satisfy the following commutation relation: $[\hat{q}, \hat{p}] = \mathrm{i}\hbar$. We define the annihilation and creation operators $\hat{a}$ and $\hat{a}^+$ by the relations

$$\hat{q} = \sqrt{\frac{\hbar}{2L\omega(t)}}\left(\hat{a}^+ + \hat{a}\right),$$

$$\hat{p} = \mathrm{i}\sqrt{\frac{\hbar L\omega(t)}{2}}\left(\hat{a}^+ - \hat{a}\right),$$

$[\hat{a}, \hat{a}^+] = 1$, where $\omega(t) = 1/\sqrt{LC}$ is the frequency of the oscillator. It is well known that the operators of the mesoscopic LC circuit closes a partial Lie algebra either with the Heisenberg algebra H_4, i.e., $\{\hat{I}, \hat{q}, \hat{p}\}$, or with $SU(1,1)$, i.e., $\{\hat{q}^2, \hat{p}^2, \hat{l}\}$, where $2\hat{l} = \hat{q}\hat{p} + \hat{p}\hat{q}$. Thus the operators belonging to these algebras can be naturally selected as relevant operators. As is required, to obtain states with a very special fluctuation relation between $\hat{q}$ and $\hat{p}$, we need to include the information regarding not only these operators, but also their squares. Thus, we will consider the relevant operators as those belonging to H_4 and $SU(1,1)$. The information theory prescribes that, in this case, the density matrix is

$$\hat{\rho} = \exp(-\lambda_0\hat{I} - \lambda_1\hat{q} - \lambda_2\hat{p} - \lambda_3\hat{q}^2 - \lambda_4\hat{p}^2 - \lambda_5\hat{l}). \tag{7}$$

We also define a new creation operator

$$\hat{b}^+ = |\cosh r|\,\mathrm{e}^{\mathrm{i}\varphi}\hat{a}^+ + |\sinh r|\,\mathrm{e}^{-\mathrm{i}\theta}\hat{a} + |\gamma|, \tag{8}$$

with $[b, \hat{b}^+] = 1$. In Eq.(8), r represents the squeezed parameter. In order to gain the diagonalization, one should include the Hamiltonian as a relevant operator. Therefore, the density matrix reads

$$\begin{aligned}\hat{\rho} =& \exp(-\lambda_0\hat{I} - \lambda_1\hat{q} - \lambda_2\hat{p} - \lambda_3\hat{q}^2 \\ & - \lambda_4\hat{p}^2 - \lambda_5\hat{l} - \beta\hat{H}).\end{aligned} \tag{9}$$

Then, it is easy to prove that the density matrix can be written in terms of the new operators $\hat{b}^+$ and $\hat{b}$ (using the normalization condition) as

$$\begin{aligned}\hat{\rho} =& 2\sinh[\beta\hbar\omega(t)/2]\cdot\exp[-\beta\hbar\omega(t)/2] \\ & \cdot\exp[-\beta\hbar\omega(t)\hat{b}^+\hat{b}],\end{aligned} \tag{10}$$

if the Lagrange multipliers verify

$$\lambda_1\sqrt{\frac{\hbar}{2L\omega(t)}} =\beta\hbar\omega(t)\,|\gamma|\,(|\sinh r|\cos\theta + |\cosh r|\cos\varphi), \tag{11}$$

$$\lambda_2\sqrt{\frac{\hbar L\omega(t)}{2}} =\beta\hbar\omega(t)\,|\gamma|\,(|\sinh r|\sin\theta - |\cosh r|\sin\varphi), \tag{12}$$

$$\lambda_3\frac{1}{L\omega(t)} + \lambda_4 L\omega(t) = 2\beta\omega(t)(\sinh r)^2, \tag{13}$$

$$\lambda_3\frac{1}{L\omega(t)} - \lambda_4 L\omega(t) =2\beta\omega(t)\,|\sinh r|\,|\cosh r|\cos(\theta+\varphi), \tag{14}$$

$$\lambda_5 = 2\beta\omega(t)\,|\sinh r|\,|\cosh r|\sin(\theta+\varphi). \tag{15}$$

From Eqs.(1) and (10), we obtain their expectation values

$$\langle\hat{q}\rangle =2\,|\gamma|\sqrt{\frac{\hbar}{2L\omega(t)}}\,(|\sinh r|\cos\theta - |\cosh r|\cos\varphi), \tag{16}$$

$$\langle\hat{p}\rangle =2\,|\gamma|\sqrt{\frac{\hbar L\omega(t)}{2}}\,(|\sinh r|\sin\theta - |\cosh r|\sin\varphi), \tag{17}$$

$$\langle\Delta\hat{q}\rangle^2 =\frac{\hbar}{2L\omega(t)}\,[|\cosh 2r| - |\sinh 2r|\cos(\theta+\varphi)] \times\coth\left(\frac{\beta\hbar\omega(t)}{2}\right), \tag{18}$$

$$\langle\Delta\hat{p}\rangle^2 =\frac{\hbar L\omega(t)}{2}\,[|\cosh 2r| + |\sinh 2r|\cos(\theta+\varphi)] \times\coth\left(\frac{\beta\hbar\omega(t)}{2}\right), \tag{19}$$

$$\langle\Delta\hat{q}\rangle\langle\Delta\hat{p}\rangle =\frac{\hbar}{2}\left\{1+[|\sinh 2r|\sin(\theta+\varphi)]^2\right\} \times\coth\left(\frac{\beta\hbar\omega(t)}{2}\right). \tag{20}$$

It can be proved that $\theta+\varphi = \theta_0+\varphi_0-2\omega t$ and r is a time-dependent invariant of motion.[14] From Eqs.(18)–(20), at T=0K, if $(\sinh r)^2 \le |\sinh r|\,|\cosh r|$, we can always find $(2\omega_0 t-\theta_0-\varphi_0)=0$ or π, which means that charge or current is squeezed. It is shown that during the process that a mesoscopic LC circuit evolves to a squeezed state if it initially lies in an excited state ($\lambda_i', s\neq 0$) with charge and current being squeezed periodically.

Notice that, when the system initially lies in the ground state, the knowledge about the system at $t=0$ is (1) $\omega(t)=\omega_0$, i.e. the Hamiltonian of the mesoscopic circuit with a constant frequency, and (2) restraint to $\langle\hat{q}(0)\rangle$ and $\langle\hat{p}(0)\rangle$ and β. Then, the expectation values $\langle\hat{q}^2(0)\rangle$ and $\langle\hat{p}^2(0)\rangle$ and $\left\langle\hat{l}(0)\right\rangle$ should be obtained using Eq.(4), which means that we can obtain the same values of $\langle\hat{q}^2(0)\rangle$ and $\langle\hat{p}^2(0)\rangle$ and $\left\langle\hat{l}(0)\right\rangle$ through the method of quantum mechanics or that of quantum statistic mechanics. So, the information theory formalism determines that the density matrix is, from $\lambda_3(0)=0$ and $\lambda_4(0)=0$ and $\lambda_5(0)=0$,

$$\hat{\rho} = \exp(-\lambda_0\hat{I} - \lambda_1\hat{q} - \lambda_2\hat{p} - \beta\hat{H}).$$

We conclude that this is equivalent to taking $r=0$ (and this condition will remain for all t as r is an invariant).

$$S = -\ln\left[2\sinh\left(\frac{\beta\hbar\omega_0}{2}\right)\right] + \frac{\beta\hbar\omega_0}{2}\coth\left(\frac{\beta\hbar\omega_0}{2}\right).$$

In the limit of T=0K, one can easily obtain that the entropy $S=0$. So, in this case we have a pure state with minimum uncertainty. This pure state thus corresponds to the well-known coherence state. This can be easily proven using Eq.(8) as follows. Because $\lambda_3(0)=0$, $\lambda_4(0)=0$, $\lambda_5(0)=0$, we conclude that this is equivalent to taking $r=0$, which is a time-dependent invariant of motion (and this condition will remain for all t). The transformation given in Eq.(8) becomes $\hat{b}^+ = \mathrm{e}^{\mathrm{i}\varphi}\hat{a}^+ + |\gamma|$. We consider a mixed set of initial conditions. As the density matrix, in terms of the new operators $\hat{b}^+$and $\hat{b}$, corresponds to the canonical one, at T=0K the new Hamiltonian should condense to its ground state $|0\rangle_b$ for which $\hat{b}\,|0\rangle_b = (\mathrm{e}^{\mathrm{i}\varphi}\hat{a} + |\gamma|\,\mathrm{e}^{-\mathrm{i}\psi})\,|0\rangle_b = 0$. So this state is an eigenstate of $\hat{a}$, and this is one of the definitions of the coherence state.

We now consider a special case in which the temporal behaviour, a sudden jump of the frequency at a particular time $t=0$, is given by

$$\omega(t) = \begin{cases} \omega_0 & t\le 0, \\ \omega_1 & t>0. \end{cases}$$

Supposing that the system is in a coherence state before the frequency jump occurs, we obtain

$$\langle\Delta\hat{q}\rangle_0^2 = \frac{\hbar}{2L\omega_0}\coth\left(\frac{\beta_0\hbar\omega_0}{2}\right), \tag{21}$$

$$\langle\Delta\hat{p}\rangle_0^2 = \frac{\hbar L\omega_0}{2}\coth\left(\frac{\beta_0\hbar\omega_0}{2}\right), \tag{22}$$

$$S_0 = -\ln\left[2\sinh\left(\frac{\beta_0\hbar\omega_0}{2}\right)\right] + \frac{\beta_0\hbar\omega_0}{2}\coth\left(\frac{\beta_0\hbar\omega_0}{2}\right). \tag{23}$$

After the jump, from Eqs.(18)–(20), we have

$$\langle\Delta\hat{q}\rangle_1^2 = \frac{\hbar}{2L\omega_1}\{|\cosh 2r_1| - |\sinh 2r_1|\cos[2\omega_1 t - (\theta_0+\varphi_0)]\}\coth\left(\frac{\beta_1\hbar\omega_1}{2}\right), \tag{24}$$

$$\langle\Delta\hat{p}\rangle_1^2 = \frac{\hbar L\omega_1}{2}\{|\cosh 2r_1| + |\sinh 2r_1|\cos[2\omega_1 t - (\theta_0+\varphi_0)]\}\coth\left(\frac{\beta_1\hbar\omega_1}{2}\right), \tag{25}$$

$$\langle\Delta\hat{q}\rangle_1\langle\Delta\hat{p}\rangle_1 = \frac{\hbar}{2}\{1+(\sinh 2r)^2\sin^2[2\omega_1 t - (\theta_0+\varphi_0)]\}^{1/2}\coth\left(\frac{\beta_1\hbar\omega_1}{2}\right). \tag{26}$$

The entropy of the system is written as

$$S_1 = -\ln\left[2\sinh\left(\frac{\beta_1\hbar\omega_1}{2}\right)\right] + \frac{\beta_1\hbar\omega_1}{2}\coth\left(\frac{\beta_1\hbar\omega_1}{2}\right). \tag{27}$$

We may avoid solving the evolution equations, obtaining the constants r_1, $(\theta_0+\varphi_0)$ and β_1 from continuity conditions in $(\Delta\hat{q})^2$, $(\Delta\hat{p})^2$, $(\Delta\hat{q})(\Delta\hat{p})$, and S. Firstly, from $S_1 = S_0$ at $t=0$, we obtain

$$\beta_1\omega_1 = \beta_0\omega_0.$$

Secondly, from the continuity conditions of relation uncertainty of the charge and current at $t=0$, we have

$$\{1+(\sinh 2r_1)^2\sin^2(\theta_0+\varphi_0)\} = 1,$$

which implies

$$(\theta_0+\varphi_0) = 0, \pi.$$

Finally, at $t=0$, the quantum fluctuations of charge and current will be

$$\langle\Delta\hat{q}\rangle_1^2 = \frac{\hbar}{2L\omega_1}[\{|\cosh 2r_1| - |\sinh 2r_1|\cos(\theta_0+\varphi_0)] \times \coth\left(\frac{\beta_1\hbar\omega_1}{2}\right), \tag{28}$$

$$\langle\Delta\hat{p}\rangle_1^2 = \frac{\hbar L\omega_1}{2}[|\cosh 2r_1| + |\sinh 2r_1|\cos(\theta_0+\varphi_0)]\} \times \coth\left(\frac{\beta_1\hbar\omega_1}{2}\right). \tag{29}$$

Equating Eq.(21) to Eq.(28), we obtain

$$2r_1 = \ln\omega_1 - \ln\omega_0, \tag{30}$$

where the minus and plus signs correspond to $(\theta_0+\varphi_0)=0$ and $(\theta_0+\varphi_0)=\pi$, respectively. The minus and plus signs will be determined by the conditions $\omega_0 > \omega_1$ and $\omega_0 < \omega_1$, respectively.

If $\omega_0 > \omega_1$ $(\theta_0+\varphi_0=0)$, at $t=0$, the system will be led to squeezing in charge when

$$\frac{\omega_1}{\omega_0} < \tanh\left(\frac{\beta_0\hbar\omega_0}{2}\right). \tag{31}$$

So, when Eq.(31) is reduced to the case T=0K, it seems that any jump of the frequency will lead to squeezing in charge.

In contrast, if $\omega_0 < \omega_1$ $(\theta_0+\varphi_0=\pi)$, at $t=0$, the system will be led to squeezing in current when

$$\frac{\omega_0}{\omega_1} < \tanh^2\left(\frac{\beta_0\hbar\omega_0}{2}\right). \tag{32}$$

So, when Eq.(32) is reduced to the case T=0K, it seems that any jump of the frequency will lead to squeezing in current.

At T=0K, Ref.[15] discussed the case that the mesoscopic LC circuit can evolve to the squeezed minimum uncertainty state due to a step change in circuit parameters L or C when the resonance frequency is time-independent. From Eqs.(26) and (28)–(30), we can obtain the same results. When $C_1 > C_0$, the quantum fluctuation of charge decreases (lower than that of the limit value of coherent state), and the quantum fluctuation of current increases (higher than that of the limit value of coherent state), which means that the squeezing effect appears only in charge. However, in contrast, when $C_1 < C_0$, the squeezing effect appears only in current. When $\omega_0 = \omega_1 = \cdots = \omega_j$, mesoscopic LC circuits show squeezing and antisqueezing effects if the circuit parameters jump suddenly due to the action of switch. But if $\omega_0 \neq \omega_1$, the mesoscopic LC circuits will evolve to a squeezed state and $\langle\Delta\hat{q}\rangle\langle\Delta\hat{p}\rangle > \hbar/2$.

4. Summary and conclusions

In the present paper, we have analysed the dynamic behaviour of a mesoscopic LC circuit with constant or time-dependent frequency, using the information theory formalism. In both cases we evaluate the density matrix associated with the dynamics of the mesoscopic circuit. For a mesoscopic circuit with a constant frequency we can conclude that: (a) if the only information available, at $t=0$, is related to $\langle\hat{q}\rangle$, $\langle\hat{p}\rangle$, and T=0K, the information theory determines that the density matrix of the system will be the one

corresponding to coherent states; (b) if, on the other hand, we can have any knowledge, at $t = 0$, of $\langle \hat{q}^2 \rangle$, $\langle \hat{p}^2 \rangle$, and $\langle \hat{l} \rangle$, being their mean values different from those emerging as a result of evaluating them using the density matrix under conditions (a), then we always obtain squeezed states.

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