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Zeeman effects on the coherent tunnelling conductance in normal-metal/superconductor/normal-metal double tunnel junctions*

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The coherent quantum transport is investigated in normal-metal/superconductor/normal-metal (N/S/N) double tunnel junctions under a Zeeman magnetic field on the S. Taking simultaneously into account the electron-injected current from one N electrode and the hole-injected current from the other N electrode, we derive a general formula for the differential conductance in the N/S/N system. It is shown that the conductance spectrum exhibits oscillatory behaviour with the bias voltage, and the oscillation amplitude is reduced with increasing temperature and Zeeman magnetic field, the Zeeman energy can lead to the Zeeman splitting of conductance peaks. In the tunnel limit, a series of bound states of quasiparticles will form in the S.

Keywords: Zeeman effect, coherent effect, normal-metal/superconductor/normal-metal tunnel junctions
PACC: 7210, 7430

1. Introduction

Coherent effects of the quasiparticle transport in double tunnel junctions consisting of superconductor (S) and normal metal (N) have attracted much attention since early experiments by Tomasch.[1] The coherent tunnelling has been studied in S/N/S double tunnel junctions by considering current-carrying Andreev bound states,[2,3] and multiple Andreev reflection (AR),[4−7] The geometric resonance nature of differential conductance oscillations in the S/N/S[8−10] and ferromagnet (F)/F/S[11,12] double tunnel junctions have been ascribed to the quasiparticle interference in the central film. Recently,the study of coherent quantum transport has been extended to F/S/F[13−15] double tunnel junctions. It was pointed out[14] that for an F/S/F double tunnel junction, if only the injection of electrons from the left F to S is taken into account, the current continuous condition cannot be satisfied, which arises from the creation and annihilation of Cooper pairs in superconductor. To solve this difficulty, in the presence of a voltage drop between two F electrodes, it is necessary to take into account not only the electron injection from one F electrode to S, but also the hole injection from the other F electrode to S.[14] Several important features have been revealed. The quantum interference effects of quasiparticle in the S interlayer give rise to oscillations of reflection and transmission probabilities as well as conductances with energy above the superconducting gap, and the AR and corresponding transmission coefficients show periodic vanishing phenomenon. In the tunnel limit, all the reflection and transmission coefficients exhibit some sharp peaks, corresponding to a series of bound states of quasiparticles in S. A similar but somewhat different approach[15] was applied

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to the same F/S/F structure, in which both electron and hole injections from the left F to S were taken into account. If the exchange splitting of F is taken to be zero, both the approaches[14,15] are equivalent to each other, reducing to the Lambert approach to the N/S/N structures.[16]

Very recently, the coexistence of superconductivity and ferromagnetism has been a most interesting subject in condensed matter physics.[17−27] Following the original work of Clogston,[28] and Chandrasekhar,[29] the modification of superconductivity by the Zeeman coupling between the spins of the electrons and an applied magnetic field has attracted intermittent attention.[30] It is predicted in early 1960s by Fulde and Ferrell[31] and Larkin and Ovchinnikov[32] (FFLO) that a finite momentum pairing state can occur under magnetic fields when electron momenta at the Fermi energy are different for the two spin directions. Experimentally, the classic predictions on the nature of the s-wave phase boundary have been confirmed by the work on thin Al films.[33] Unlike the conventional Cooper pairs in which two electrons have opposite spins and momenta $(K\uparrow, -K\downarrow)$, the Cooper pair in the FFLO state has a finite centre-of-mass momentum Q of the order of $2h_0/\hbar v_{\rm F}$, and consequently leads to a spatially modulated superconducting order parameter, where h_0 is the magnetic exchange energy corresponding to the half of the difference in the energy between the spin-up and spin-down bands, and $v_{\rm F}$ is the Fermi velocity. It was shown that the coexistence of superconductivity and ferromagnetism is given by $h_0 < 0.707\Delta_0$ for zero centre-of-momentum Cooper pairs and $0.707\Delta_0 < h_0 < 0.754\Delta_0$ for the FFLO states for the s-wave S,[31,32] or $h_0 < 0.56\Delta_0$ for zero centre-of-momentum Cooper pairs and $0.56\Delta_0 < h_0 < 1.06\Delta_0$ for the FFLO states for the d-wave S,[34] where Δ_0 is the energy gap in a nonmagnetic S.

In this paper, we present a theory of the coherent quantum transport in N/S/N double tunnel junctions under a Zeeman magnetic field on the S, study the response of S to a magnetic field, and derive a general formula for the differential conductance in terms of reflection and transmission coefficients by taking simultaneously into account the electron-injected current from one N electrode and the hole-injected current from the other N electrode, as shown in Figs.1(a) and (b) (or (d) and (c)). In this case, the chemical potential in S is determined by the current continuity condition, i.e. the current from the left N to S via the left N/S interface must be equal to that from the S to the right N via the right interface. In the present coherent transport, the coexistence state between superconductivity and ferromagnetism means that there is an exchange splitting in the S caused by the Zeeman magnetic field, the quasiparticle interference in the S and the resonant tunnelling play an important role, exhibiting new quantum effects on the tunnelling conductance in the N/S/N structures.

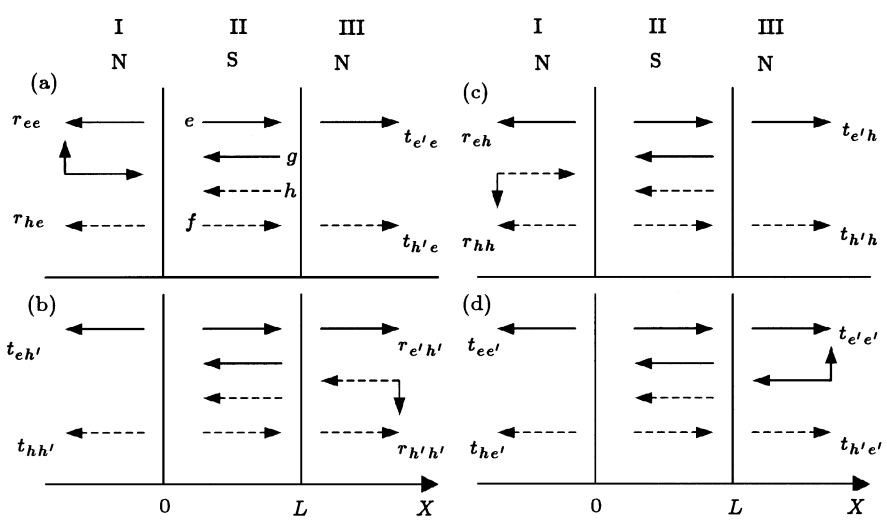


Fig.1. Schematic illustration of reflections and transmissions of quasiparticles in an N/S/N structures.

2. Ferromagnetic superconductor and quasiparticle transport coefficients

Consider an N/S/N double tunnel junctions, in which the left and right electrodes are made of the same N, and they are separated from the central S by two thin insulating interfaces, separately. There is a uniform magnetic field $\boldsymbol{B}$ applied to the S, the Zeeman energy $h_0 = \mu B$, where $\mu = g\mu_{\rm B}/2$ is the magnetic moment of an electron, $\mu_{\rm B}$ is the Bohr magneton, g is the g factor of the paramagnetic centre. We adopt the Bogoliubov–de Gennes (BdG) approach[35] to study the superconducting order parameter Δ and the transport of quasiparticles in the N/S/N structure in a magnetic field. In the absence of spin-flip scattering, the four-component BdG equations may be decoupled into two sets of two-component equations: one for spin-up electron-like and spin-down hole-like quasiparticle wavefunctions $(u_\uparrow, v_\downarrow)$, the other for $(u_\downarrow, v_\uparrow)$. The decoupled BdG equation has the form[11,35]

$$\begin{pmatrix} H_0 - \eta_\sigma h_0 & \Delta(T,h_0) \\ \Delta^*(T,h_0) & -H_0 + \eta_{\bar\sigma} h_0 \end{pmatrix} \begin{pmatrix} u_\sigma \\ v_{\bar\sigma} \end{pmatrix} = E \begin{pmatrix} u_\sigma \\ v_{\bar\sigma} \end{pmatrix}. \tag{1}$$

Here the excitation energy E is measured relative to Fermi energy $E_{\rm F}$, $\Delta(T,h_0)$ is the effective superconducting order parameter, and depends on the temperature T and Zeeman energy h_0, $\eta_\sigma = 1$ for $\sigma = \uparrow$, $\eta_{\bar\sigma} = -1$ for $\sigma = \downarrow$, $\bar\sigma$ standing for the spin opposite to σ. $H_0(\boldsymbol{r}) = -\hbar^2\nabla_r^2/2m + V(\boldsymbol{r}) - E_{\rm F}$ with $V(\boldsymbol{r})$ the usual static potential. In our calculations, the two very thin insulating layers at $x = 0$ and $x = L$ can be modelled to be two δ-type barrier potentials: $U(x) = U_0[\delta(x) + \delta(x - L)]$, where L is the thickness of the interlayer S, and U_0 depends on the product of barrier height and width. From the BdG equation, we obtain

$$u_\sigma^2 = \frac{1}{2}\left[1 + \sqrt{1 - \Delta^2(T,h_0)/(E + \eta_\sigma h_0)^2}\right], \tag{2}$$

$$v_{\bar\sigma}^2 = \frac{1}{2}\left[1 - \sqrt{1 - \Delta^2(T,h_0)/(E - \eta_{\bar\sigma} h_0)^2}\right]. \tag{3}$$

The wave vectors of the electron-like and hole-like quasiparticles are given by

$$k_\sigma^e = \frac{\sqrt{2m}}{\hbar}\left[E_{\rm F} + \sqrt{(E + \eta_\sigma h_0)^2 - \Delta^2(T,h_0)}\right]^{1/2}, \tag{4}$$

$$k_{\bar\sigma}^h = \frac{\sqrt{2m}}{\hbar}\left[E_{\rm F} - \sqrt{(E - \eta_{\bar\sigma} h_0)^2 - \Delta^2(T,h_0)}\right]^{1/2}. \tag{5}$$

In the S the effective order parameter $\Delta(T,h_0)$ can be determined by the self-consistent equation[35]

$$\Delta = V_0(\psi_\uparrow\psi_\downarrow), \tag{6}$$

where V_0 is the strength of the effective attractive potential between the conducting electrons, and

$$\psi_\sigma = \sum_k \left(\gamma_{k\sigma} u_{k\sigma} - \gamma_{k\bar\sigma}^+ v_{k\sigma}^*\right), \tag{7}$$

with $\gamma_{k\sigma}$ being the Bogoliubov transformation operators. With the help of Eqs.(2), (3), (6) and (7) along with the equation $\gamma_{k\sigma}$ obeys, we obtain

$$1 = \frac{V_0}{2}\sum_k \left(\frac{1 - f_{k_\uparrow}}{\sqrt{\varepsilon_{k_\uparrow}^2 + \Delta^2(T,h_0)}} - \frac{f_{k_\downarrow}}{\sqrt{\varepsilon_{k_\downarrow}^2 + \Delta^2(T,h_0)}}\right), \tag{8}$$

where

$$f_{k\sigma} = \frac{1}{\exp\left[\beta\left(\sqrt{\varepsilon_{k\sigma}^2 + \Delta^2(T,h_0)} - \eta_\sigma h_0\right)\right] + 1} \tag{9}$$

with $\epsilon_{k\sigma} = \hbar^2(k_\sigma^e)^2/(2m) - E_{\rm F}$ and $\beta = 1/k_{\rm B}T$ being the inverse temperature. From Eqs.(8) and (9), we obtain

$$\ln\frac{\Delta_0}{\Delta(T,h_0)} = \int_0^{\hbar\omega_{\rm D}} \frac{{\rm d}\varepsilon}{\sqrt{\varepsilon^2 + \Delta^2(T,h_0)}}\left(\frac{1}{\exp[\beta(\sqrt{\varepsilon^2 + \Delta^2(T,h_0)} - h_0)] + 1} + \frac{1}{\exp[\beta(\sqrt{\varepsilon^2 + \Delta^2(T,h_0)} + h_0)] + 1}\right). \tag{10}$$

Here Δ_0 is the superconducting gap at zero temperature and in the absence of magnetic field, and $\omega_{\rm D}$ is the Debye frequency.

By solving Eq.(10) self-consistently, the dependence of the effective order parameter $\Delta(T, h_0)$ on the temperature and Zeeman energy is obtained. At zero temperature, $\Delta(0, h_0) = \Delta_0$ remains unchanged for $h_0 < \Delta_0$, independent of increasing h_0; as h_0 is increased to Δ_0, Δ suddenly drops to zero, exhibiting a first-order phase transition from the superconducting state to normal state. This zero-temperature solution that $\Delta(0, h_0) = \Delta_0$ for $h_0 < \Delta_0$ and $\Delta(0, h_0) = 0$ for $h_0 > \Delta_0$ has been obtained previously.[31,32] At finite temperatures, it is found that for a Zeeman energy, the order parameter decreases with increasing temperature and has a sudden drop from a finite value to zero at the critical temperature $T_{\rm c}(h_0)$, as shown in Fig.2. Such a first-order phase transition arises from the presence of Zeeman energy. While the Zeeman energy $h_0 = 0$, the superconducting transition remains second-order as shown by the solid line in Fig.1. With increasing Zeeman energy, the critical temperature $T_{\rm c}(h_0)$ also decreases and is always lower than $T_{\rm c}(h_0 = 0)$.

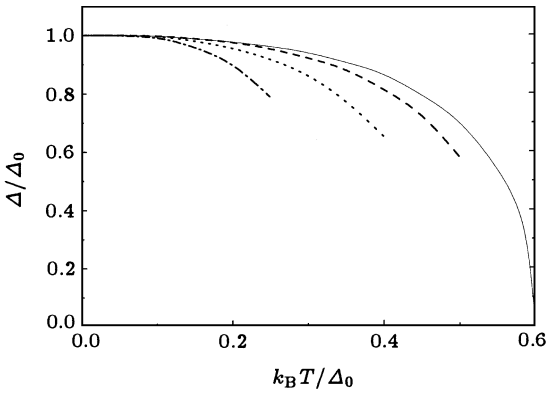


Fig.2. Dependence of the effective order parameter in S on the temperature for different Zeeman energies: $h_0/\Delta_0 = 0$ (solid line), $h_0/\Delta_0 = 0.2$ (dashed line), $h_0/\Delta_0 = 0.4$ (dotted line) and $h_0/\Delta_0 = 0.6$ (dot-dashed line).

Consider an electron of spin σ incident on the interface at $x = 0$ from the left N. As shown in Fig.1(a),there are four possible trajectories: normal reflection (r_{ee}), AR (r_{he}), transmission to the right electrode as an electron-like quasiparticle ($t_{e'e}$) and as a hole-like quasiparticle ($t_{h'e}$), where subscripts e (h) and e' (h') indicate the electron (hole) in the left and right N electrodes, respectively. With general solutions of the BdG equation (1), the wavefunctions in the three regions have the following form:

$$\Psi_{\rm I} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} {\rm e}^{{\rm i}q_+ x} + r_{he} \begin{pmatrix} 0 \\ 1 \end{pmatrix} {\rm e}^{{\rm i}q_- x} + r_{ee} \begin{pmatrix} 1 \\ 0 \end{pmatrix} {\rm e}^{-{\rm i}q_+ x}, \tag{11}$$

for $x < 0$;

$$\Psi_{\rm II} = e \begin{pmatrix} u_\sigma \\ v_{\bar\sigma} \end{pmatrix} {\rm e}^{{\rm i}k^e_\sigma x} + f \begin{pmatrix} v_{\bar\sigma} \\ u_\sigma \end{pmatrix} {\rm e}^{-{\rm i}k^h_{\bar\sigma} x} + g \begin{pmatrix} u_\sigma \\ v_{\bar\sigma} \end{pmatrix} {\rm e}^{-{\rm i}k^e_\sigma x} + h \begin{pmatrix} v_{\bar\sigma} \\ u_\sigma \end{pmatrix} {\rm e}^{{\rm i}k^h_{\bar\sigma} x}, \tag{12}$$

for $0 < x < L$; and

$$\Psi_{\rm III} = t_{e'e} \begin{pmatrix} 1 \\ 0 \end{pmatrix} {\rm e}^{{\rm i}q_+ x} + t_{h'e} \begin{pmatrix} 0 \\ 1 \end{pmatrix} {\rm e}^{-{\rm i}q_- x}, \tag{13}$$

for $x > L$. Here $q_\pm = \sqrt{2m(E_{\rm F} \pm E)}/\hbar$ are the different wave vectors for the electron and hole in N. All the coefficients in Eqs.(11)–(13) can be determined by the boundary conditions at $x = 0$ and $x = L$. The matching conditions for the wavefunction are given by

$$\Psi_{\rm II}(0) = \Psi_{\rm I}(0),$$

$$({\rm d}\Psi_{\rm II}/{\rm d}x)_{x=0} - ({\rm d}\Psi_{\rm I}/{\rm d}x)_{x=0} = 2mU_0\,\Psi_{\rm I}(0)/\hbar^2,$$

$$Ps_{\rm III}(L) = \Psi_{\rm II}(L),$$

and

$$({\rm d}\Psi_{\rm III}/{\rm d}x)_{x=L} - ({\rm d}\Psi_{\rm II}/{\rm d}x)_{x=L} = 2mU_0\,\Psi_{\rm III}(L)/\hbar^2.$$

Since analytical results for these coefficients are tedious, we only give expressions for r_{he}, r_{ee}, $t_{e'e}$ and $t_{h'e}$ in the Appendix. From these expressions we obtain $R_{he} = q_-|r_{he}|^2/q_+$, $R_{ee} = |r_{ee}|^2$, $T_{e'e} = |t_{e'e}|^2$ and $T_{h'e} = q_-|h_{h'e}|^2/q_+$, respectively, for the AR, the normal reflection coefficients, the transmission coefficients of electron-like and hole-like quasiparticle. For a spin $\bar\sigma$ hole incident on the interface at $x = L$

from the right N, as shown in Fig.1(b), $r_{e'h'}$, $r_{h'h'}$, $t_{eh'}$ and $t_{hh'}$ can be obtained by a similar calculation and their expressions are also given in the Appendix. As a result, $R_{e'h'} = q_+|r_{e'h'}|^2/q_-$, $R_{h'h'} = |r_{h'h'}|^2$, $T_{eh'} = q_+|t_{eh'}|^2/q_-$ and $T_{h'h'} = |h_{h'h'}|^2$. From electron-hole scattering symmetries as well as our calculation results, we have $R_{ee} = R_{e'e'}$, $R_{hh} = R_{h'h'}$, $R_{eh} = R_{he} = R_{e'h'} = R_{h'e'}$, and $T_{e'e} = T_{ee'}$, $T_{h'h} = T_{hh'}$, $T_{h'e} = T_{eh'} = T_{e'h} = T_{he'}$. It is easily shown analytically that all the coefficients of electron–hole transformation such as R_{he} and $T_{h'e}$ are proportional to $\sin^2[(k^e_\sigma - k^h_{\bar\sigma})L/2]$, which vanish if $(k^e_\sigma - k^h_{\bar\sigma})L = 2n\pi$ with n being arbitrary positive integer. From the expressions for k^e_σ and $k^h_{\bar\sigma}$ given above, this condition is equivalent to

$$\left[\frac{E+\eta_\sigma h_0}{\Delta(T,h_0)}\right]^2 = \left[\frac{2\pi n E_{\rm F}/\Delta(T,h_0)}{k_{\rm F}L}\right]^2 + 1, \qquad (14)$$

under which there is neither AR nor hole (electron) transmission so that the quasiparticles pass directly from one N electrode to the other, not converting to the Cooper pair in S.

3. Tunnelling conductance

Once all the transmission and reflection probabilities are obtained, we can calculate currents in response to a difference in chemical potentials between the two Ns. Assume $\mu_{\rm L}$ and $\mu_{\rm R}$ to be the chemical potentials of the left- and right- N electrodes, respectively, and $\mu_{\rm S}$ the chemical potential of S. Under the bias voltage $V(eV = \mu_{\rm L} - \mu_{\rm R})$ applied to the N/S/N structure, and taking into account the four processes shown in Fig.1, we obtain the current from the left N into S

$$\begin{aligned} I_{\rm L} =& \frac{e}{h}\sum_\sigma \int_0^\infty {\rm d}E[f_0(E-e\phi_1)(1-R_{ee}+R_{he}) \\ &+ f_0(E-e\phi_2)(T_{hh'}-T_{eh'}) \\ &+ f_0(E+e\phi_1)(-1-R_{eh}+R_{hh}) \\ &+ f_0(E+e\phi_2)(T_{he'}-T_{ee'})], \end{aligned} \qquad (15)$$

where h is the Planck constant, $f_0(E)$ is the Fermi distribution function, $e\phi_1 = \mu_{\rm L} - \mu_{\rm S}$, and $e\phi_2 = \mu_{\rm S} - \mu_{\rm R}$. Similarly, the current from S to the right N is given by

$$\begin{aligned} I_{\rm R} =& \frac{e}{h}\sum_\sigma \int_0^\infty {\rm d}E[f_0(E-e\phi_1)(T_{e'e}-T_{h'e}) \\ &+ f_0(E-e\phi_2)(1-R_{h'h'}+R_{e'h'}) \\ &+ f_0(E+e\phi_1)(T_{e'h}-T_{h'h}) \\ &+ f_0(E+e\phi_2)(-1-R_{h'e'}+R_{e'e'})]. \end{aligned} \qquad (16)$$

The current continuity condition requires $I_{\rm L} = I_{\rm R}$, from which $\mu_{\rm S}$ is determined to be $\mu_{\rm S} = (\mu_{\rm L} + \mu_{\rm R})/2$. Using the property of probability $R_{ee} + R_{he} + T_{e'e} + T_{h'e} = 1$, $R_{eh} + R_{hh} + T_{e'h} + T_{h'h} = 1$, $R_{e'e'} + R_{h'e'} + T_{ee'} + T_{he'} = 1$, and $R_{e'h'} + R_{h'h'} + T_{eh'} + T_{hh'} = 1$, we obtain

$$\begin{aligned} I =& \frac{e}{h}\sum_\sigma \int_0^\infty {\rm d}E(R_{he}+R_{eh}+T_{e'e}+T_{h'h}) \\ &\times \left[f_0\left(E-\frac{eV}{2}\right) - f_0\left(E+\frac{eV}{2}\right)\right]. \end{aligned} \qquad (17)$$

The differential conductance is given by

$$G = \frac{G_0}{16k_{\rm B}T}\sum_\sigma \int_0^\infty {\rm d}E(R_{he}+R_{eh}+T_{e'e}+T_{h'h})W, \qquad (18)$$

with $G_0 = 2e^2/h$ and

$$W = \cosh^{-2}\left(\frac{2E-eV}{4k_{\rm B}T}\right) + \cosh^{-2}\left(\frac{2E+eV}{4k_{\rm B}T}\right). \qquad (19)$$

At zero temperature, the differential conductance is reduced to

$$G = \frac{G_0}{2}\sum_\sigma (R_{he}+R_{eh}+T_{e'e}+T_{h'h})_{E=eV}. \qquad (20)$$

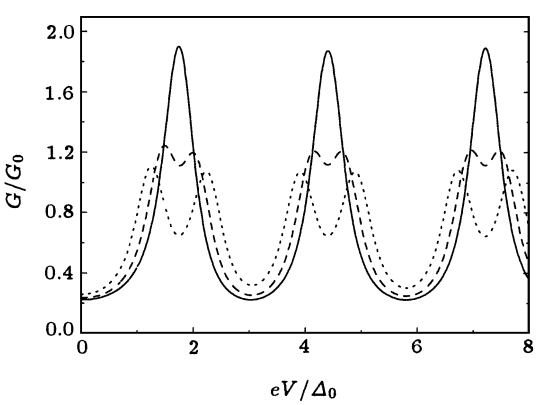


Fig.3. Differential conductance as a function of the bias voltage at zero temperature for different Zeeman energies: $h_0/\Delta_0 = 0$ (solid line), $h_0/\Delta_0 = 0.3$ (dashed line) and $h_0/\Delta_0 = 0.5$ (dotted line). Here $E_{\rm F}/\Delta_0 = 2000$, $k_{\rm F}L = 1000$ and $z = 1.0$.

Figures 3 and 4 show the differential conductance G as a function of bias voltage eV/Δ_0 for different Zeeman energy h_0/Δ_0 and different barrier strength z at zero temperature, respectively. Several interesting features can be found. First, the conductance exhibits oscillatory behaviour, and the oscillation period is given by Eq.(14). These oscillation phenomena arise from the quantum interference effects of quasiparticles

in S at the Fermi level. Second, with increasing Zeeman energy, the conductance peak is gradually split into two peaks, as shown in Fig.3, and the energy difference between the two split peaks is equal to $2h_0$. For the case of a finite Zeeman energy, there is exchange splitting in the energy spectrum between the spin-up and spin-down energy subbands, and consequently it leads to the Zeeman splitting of conductance peaks. Third, with increasing barrier strength, each split peak becomes two sharp peaks, corresponding to a series of bound states of quasiparticles in S. In the tunnel limit for large z, the positions of these peaks are determined by $k_\sigma^e L = n\pi$ and $k_{\bar{\sigma}}^h = n\pi$, as has been discussed in Ref.[14]. These bound states are the results of quantum interference between electron-like quasiparticles in the S well and those between hole-like ones.

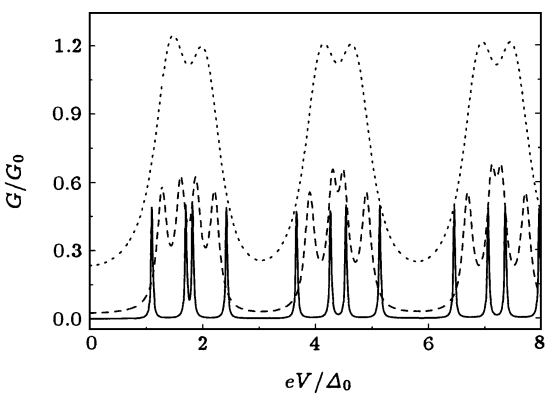


Fig.4. Differential conductance as a function of the bias voltage at zero temperature for different barrier strength: $z = 1.0$ (dotted line), $z = 2.0$ (dashed line) and $z = 3.0$ (solid line). Here $E_F/\Delta_0 = 2000$, $k_F L = 1000$ and $h_0/\Delta_0 = 0.3$.

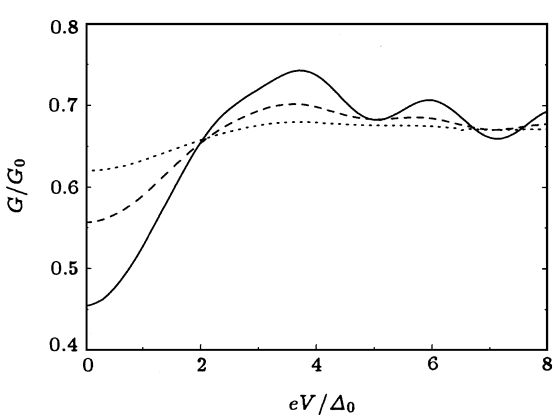


Fig.5. Differential conductance as a function of the bias voltage for different temperatures: $k_B T/\Delta_0 = 0.3$ (solid line), $k_B T/\Delta_0 = 0.4$ (dashed line) and $k_B T/\Delta_0 = 0.5$ (dotted line). Here $E_F/\Delta_0 = 2000$, $k_F L = 1000$, $z = 1.0$ and $h_0/\Delta_0 = 0.4$.

For a finite temperature, the differential conductance is calculated in terms of Eqs.(18) and (10). The dependence of the differential conductance on bias voltage at different temperatures and Zeeman energies are plotted in Figs.5 and 6. It is found that the G exhibits a damped oscillatory behaviour with bias voltage, and the oscillation amplitude is reduced with increasing temperature and Zeeman energy.

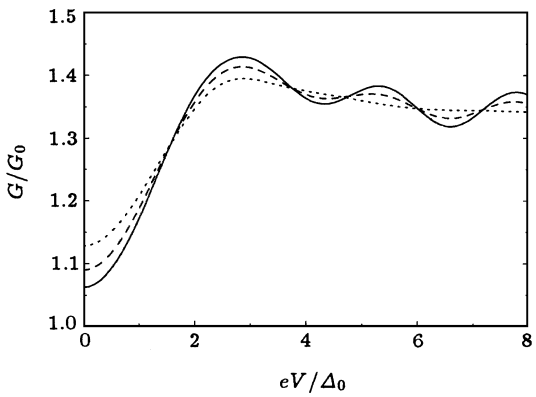


Fig.6. Differential conductance as a function of the bias voltage at finite temperature for different Zeeman energies: $h_0/\Delta_0 = 0$ (solid line), $h_0/\Delta_0 = 0.2$ (dashed line) and $h_0/\Delta_0 = 0.3$ (dotted line). Here $E_F/\Delta_0 = 2000$, $k_F L = 1000$, $z = 0.5$ and $k_B T/\Delta_0 = 0.3$.

4. Conclusion

In summary, we have studied the coherent tunnelling conductance in the N/S/N double tunnel junctions under a Zeeman magnetic field on the S. The expression for the tunnelling current through the junction is derived by simultaneously taking into account the electron-injected current from one N electrode and the hole-injected current from the other. The quantum interference effects of quasiparticle in the S interlayer give rise to oscillations of the tunnelling conductance with bias voltage, and the oscillation amplitude is reduced with increasing temperature and Zeeman energy, the Zeeman energy can lead to the Zeeman splitting of conductance peaks. In the tunnel limits of strong barrier strength, a series of bound states of quasiparticles will form in the S. The N/S/N structures can be made with the development of nanofabrication technique and the improvement of experimental methods. It is expected that the theoretical results obtained will be confirmed in future experiments.

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Appendix Expressions for reflection and transmission coefficients

Using the boundary conditions to the wavefunctions given by Eqs.(11)–(13) and carrying out a little of algebra, we find

$$\begin{aligned} r_{he} =& -2u_\sigma v_{\overline{\sigma}} \sin\left[\left(k_\sigma^e - k_{\overline{\sigma}}^h\right) L/2\right] \left\{\left(u_\sigma^2 + v_{\overline{\sigma}}^2\right) \sin\left[\left(k_\sigma^e - k_{\overline{\sigma}}^h\right) L/2\right]\right. \\ & \left. + \mathrm{i}\left(u_\sigma^2 - v_{\overline{\sigma}}^2\right)\left(1+2z^2\right) \cos\left[\left(k_\sigma^e - k_{\overline{\sigma}}^h\right) L/2\right]\right\}/M, \end{aligned} \tag{A1}$$

$$\begin{aligned} r_{ee} =& z\{\mathrm{i}\left(u_\sigma^2 - v_{\overline{\sigma}}^2\right)^2 \left[(\mathrm{i}z-1)^3 \, \mathrm{e}^{\mathrm{i}\left(k_\sigma^e + k_{\overline{\sigma}}^h\right)L} - z^2\left(1+\mathrm{i}z\right)\mathrm{e}^{-\mathrm{i}\left(k_\sigma^e + k_{\overline{\sigma}}^h\right)L}\right] \\ & + 2u_\sigma^2 v_{\overline{\sigma}}^2 \left[\cos\left(k_\sigma^e - k_{\overline{\sigma}}^h\right) L - 1\right](\mathrm{i}+z)\left(1+2z^2\right) \\ & + z^2(\mathrm{i}+z)Q + (\mathrm{i}-z)(1-\mathrm{i}z)^2 W\}/M, \end{aligned} \tag{A2}$$

$$\begin{aligned} t_{e'e} =& -\left(u_\sigma^2 - v_{\overline{\sigma}}^2\right)\mathrm{e}^{-\mathrm{i}q_+ L}\{[(\mathrm{i}z-1)^2 \, \mathrm{e}^{\mathrm{i}k_{\overline{\sigma}}^h L} + z^2 \mathrm{e}^{-\mathrm{i}k_{\overline{\sigma}}^h L}]u_\sigma^2 \\ & - [(\mathrm{i}z-1)^2 \, \mathrm{e}^{\mathrm{i}k_\sigma^e L} + z^2 \mathrm{e}^{-\mathrm{i}k_\sigma^e L}]v_{\overline{\sigma}}^2\}/M, \end{aligned} \tag{A3}$$

$$\begin{aligned} t_{h'e} =& 4\mathrm{i}z u_\sigma v_{\overline{\sigma}} \left(u_\sigma^2 - v_{\overline{\sigma}}^2\right)\mathrm{e}^{\mathrm{i}q_- L}\{\sin\left[\left(k_\sigma^e + k_{\overline{\sigma}}^h\right) L/2\right] \\ & - z\cos\left[\left(k_\sigma^e + k_{\overline{\sigma}}^h\right) L/2\right]\} \sin\left[\left(k_\sigma^e - k_{\overline{\sigma}}^h\right) L/2\right]/M, \end{aligned} \tag{A4}$$

with

$$Q = 2u_\sigma^2 v_{\overline{\sigma}}^2 - u_\sigma^4 \mathrm{e}^{\mathrm{i}\left(k_\sigma^e - k_{\overline{\sigma}}^h\right)L} - v_{\overline{\sigma}}^4 \mathrm{e}^{-\mathrm{i}\left(k_\sigma^e - k_{\overline{\sigma}}^h\right)L}, \tag{A5}$$

$$W = 2u_\sigma^2 v_{\overline{\sigma}}^2 - u_\sigma^4 \mathrm{e}^{-\mathrm{i}\left(k_\sigma^e - k_{\overline{\sigma}}^h\right)L} - v_{\overline{\sigma}}^4 \mathrm{e}^{\mathrm{i}\left(k_\sigma^e - k_{\overline{\sigma}}^h\right)L}, \tag{A6}$$

$$\begin{aligned} M =& z^2\left(u_\sigma^2 - v_{\overline{\sigma}}^2\right)^2 [(\mathrm{i}+z)^2 \mathrm{e}^{\mathrm{i}\left(k_\sigma^e + k_{\overline{\sigma}}^h\right)L} + (\mathrm{i}-z)^2 \mathrm{e}^{-\mathrm{i}\left(k_\sigma^e + k_{\overline{\sigma}}^h\right)L}] \\ & + 4u_\sigma^2 v_{\overline{\sigma}}^2 z^2 (1+z^2)[\cos\left(k_\sigma^e - k_{\overline{\sigma}}^h\right) L - 1] + Qz^4 + (1+z^2)^2 W. \end{aligned} \tag{A7}$$

The coefficients in Fig.1(b) can be similarly obtained as

$$r_{e'h'} = -r_{he}, \tag{A8}$$

$$\begin{aligned} r_{h'h'} =& z\mathrm{e}^{2\mathrm{i}q_- L}\{\left(u_\sigma^2 - v_{\overline{\sigma}}^2\right)^2 \mathrm{i}[(1-\mathrm{i}z)z^2 \mathrm{e}^{\mathrm{i}\left(k_\sigma^e + k_{\overline{\sigma}}^h\right)L} + (1+\mathrm{i}z)^3 \mathrm{e}^{-\mathrm{i}\left(k_\sigma^e + k_{\overline{\sigma}}^h\right)L}] \\ & + 2u_\sigma^2 v_{\overline{\sigma}}^2 [\cos\left(k_\sigma^e - k_{\overline{\sigma}}^h\right) L - 1](\mathrm{i}-z)(1+2z^2) \\ & + z^2(z-\mathrm{i})Q + (1+\mathrm{i}z)^2(\mathrm{i}+z)W\}/M, \end{aligned} \tag{A9}$$

$$\begin{aligned} t_{hh'} =& -\left(u_\sigma^2 - v_{\overline{\sigma}}^2\right)\mathrm{e}^{\mathrm{i}q_- L}\{[(1+\mathrm{i}z)^2 \mathrm{e}^{-\mathrm{i}k_\sigma^e L} + z^2 \mathrm{e}^{\mathrm{i}k_\sigma^e L}]u_\sigma^2 \\ & - [(1+\mathrm{i}z)^2 \mathrm{e}^{-\mathrm{i}k_{\overline{\sigma}}^h L} + z^2 \mathrm{e}^{\mathrm{i}k_{\overline{\sigma}}^h L}]v_{\overline{\sigma}}^2\}/M, \end{aligned} \tag{A10}$$

and

$$t_{eh'} = t_{h'e}, \tag{A11}$$

where $z = mU_0/(\hbar^2 k_\mathrm{F})$.

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