Vol 14 No 6, June 2005
1009-1963/2005/14(06)/1174-07
Chinese Physics
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and IOP Publishing Ltd

Theory of passively mode-locked lasers with self-frequency shift and saturable absorber response*

Song Li-Jun(宋丽军), Shi Xiao-Juan(师晓娟), Li Lu(李 录),
Li Zhong-Hao(李仲豪), and Zhou Guo-Sheng(周国生)†

College of Physics and Electronics Engineering, Shanxi University, Taiyuan 030006, China

State Key Subject of Optics, Shanxi University, Taiyuan 030006, China

(Received 11 August 2004; revised manuscript received 5 February 2005)

A general theoretical model for passively mode-locked lasers is presented, in which both the self-frequency shift and either a fast or a slow saturable absorber response are taken into account. An exact soliton-like solution and condition for its existence are obtained under a definite compatible condition. The stability of the solution is analyzed by using a variational method, and a parameter region, in which the solution is linearly stable, is acquired theoretically. To verify the theoretical predictions, a typical example is given for stable pulse propagation over a long distance. The numerical results show that the soliton-like solution is stable under some perturbations within the linearly stable region and an arbitrary Gaussian pulse converges to the exact soliton-like solution after evolution in a distance.

Keywords: passively mode-locked laser, saturable absorber, variational method
PACC: 4255, 4265

1. Introduction

In the past decades, several passively mode-locking techniques have been developed for solid-state lasers whereby fast saturable absorber-like action is achieved in solids. Most of these novel optical modulators rely on the non-resonant Kerr effect. In contrast to the absorption in bleachable dyes, the non-resonant Kerr effect is extremely fast and wavelength-independent, and allows the generation of a continuous train of mode-locked pulses from a cw-pumped laser. But the Kerr nonlinearity is usually not strong enough for the cw mode-locking process to self-start. In order to initiate Kerr-lens mode-locking (KLM), a strong fluctuation is usually introduced by either perturbing the cavity[1−3] or adding another nonlinear device to the system.[4−7] Among these, directly employing a semiconductor saturable absorber (SESAM) in the resonator has been recognized as a practical way to obtain stable femtosecond pulses in cw-pumped self-starting lasers.[6,7] The use of semiconductors as saturable absorbers is attractive because they are inexpensive and compact; the important operation parameters can be designed generally.[8] Besides, the semiconductor saturable absorber can produce a strong nonlinearity to start the mode-locking operation.

The promise of compact, reliable laser sources with output pulses in the femtosecond regime has motivated research efforts in the world. As a result, over a hundred articles are published every year about experimental[1−11] studies and theoretical[12−16] analysis of passively mode-locked lasers. Among them, most of the theoretical papers are generally based on two approaches. In the first approach, which is computational, one simulates the entire evolution of light pulse within the laser, starting by noise. In the second one, which is analytical and was first introduced by Haus,[14] one adopts a simplified method in which the pulse change during a signal passing through the laser is assumed to be small in order to obtain a simple equation. On this assumption, Li *et al*[15] and Akhmediev *et al*[16] described the evolution of laser pulses with a higher-order nonlinear Schödinger equation and a modified complex Ginzberg–Landau equation, respectively. Li *et al* took into account the self-frequency shift, which arises from the stimulated Ra-

*Project supported by the National Natural Science Foundation of China (Grant No 60477026).
†E-mail: zhougs@sxu.edu.cn or songlij@sxu.edu.cn

man scattering, but Akhmediev *et al* introduced the saturable absorption effect into the latter model. They also found the exact solutions of the models and analysed their stability numerically. In KLM systems, the nonlinear phase response is used to realize the mode-locking operation, in which an artificial fast absorber action is caused by Kerr-self-focusing. In this system, the total gain (or loss) always depends on the pulse energy, and an SESAM is often introduced to start mode-locking. If the recovery time of the SESAM is long compared with the pulse width, it acts as a slow variation of loss in the mode-locking mechanism.

In this paper, we present a general model for passively mode-locked lasers, in which both the self-frequency shift and either a fast or a slow saturable absorber response are taken into account. We also obtain an exact soliton-like solution and condition for its existence. By using a variational method and numerical simulation, the stability of the solution is analyzed. The results indicate that the soliton-like pulses can be stable in a definite parameter region, and an arbitrary initial Gaussian pulse can converge to the exact solution after evolution in a distance. The paper is organized as follows. In Section 2 we present the model and the exact soliton-like solution. In Section 3 the variational method is introduced. In Section 4 shown as a typical example are the linear stability analysis of the soliton-like solution and the evolution under the perturbations in amplitude and chirp. We conclude the paper in Section 5.

2. Model and solutions

When the bandwidth-limited gain, self-frequency shift, and saturable absorber response are all taken into account, a so-called distributed system can be used when the field changes only slightly on each round trip. The resulting model, which describes the passively mode-locked lasers, can by written as[15,16]

$$\mathrm{i}U_z + \frac{1}{2}U_{tt} + |U|^2U = \mathrm{i}R(U, U^*), \tag{1}$$

where

$$R(U,U^*) = \delta U + \mathrm{i}\sigma U_t + \varepsilon U_{tt} + \chi|U|^2U + \lambda U_{ttt} + \mu(|U|^2U)_t + \nu U(|U|^2)_t + \alpha U\int_{-\infty}^{t}|U|^2\mathrm{d}t,$$

and $U(z,t)$ is the normalized envelope of the optical field; z is the cavity round-trip number treated as a continuous variable; t is the retarded time. The model parameters δ, σ, ε, χ and α are real constants, λ, μ and ν can be complex. The parameter δ is the total gain (or loss) in the cavity; σ and the imaginary part λ_i of λ result from the difference between the pulse carrier frequency and the gain-centre frequency, which are ignored in Ref.[17]; ε describes spectral filtering; χ denotes the fast saturable absorption; the real part λ_r of λ represents the net third-order dispersion (TOD); the real part μ_r of μ is the nonlinear dispersion; the imaginary part μ_i of μ describes the combined effect of nonlinear gain; the parameter ν is the nonlinear gradient term which results from stimulated Raman scattering. In fact, the imaginary part ν_i of ν is usually responsible for the soliton self-frequency shift and the real part ν_r of ν could be zero.[18] The term with α $(=\delta_0/E_A)$ is responsible for the slow part of loss variation, which is introduced by the saturable absorber. Here δ_0 is the loss introduced by the saturable absorber in equilibrium, E_A is the saturation energy of the absorber.[16]

When the limiting case, in which the pulse amplitude is well below the saturation level, is considered, we find that Eq.(1) has a solution in the following form:

$$U(z,t) = A_0\{\mathrm{sech}[\eta_0(t-\rho_0 z)]\}^{1+\mathrm{i}\beta_0} \times \exp[\mathrm{i}(\Omega_0 t - K_0 z)], \tag{2}$$

where A_0, η_0, β_0, Ω_0, ρ_0 and K_0 are the parameters describing the amplitude, inverse pulse width, chirp, frequency shift, inverse group velocity and phase of the soliton-like solution, respectively. By substituting the solution (2) into Eq.(1), we can write the amplitude A_0, the inverse pulse width η_0 and the frequency shift Ω_0 of the soliton-like solution as

$$A_0 = \eta_0\left(\frac{6\lambda_r b_3 - \beta_0\lambda_i b_7}{b_1}\right)^{1/2}, \tag{3}$$

$$\eta_0 = \left(\frac{b_1 b_4 b_9 - \alpha\beta_0(6\lambda_r b_3 - \beta_0\lambda_i b_7)}{b_1 b_4(\lambda_i b_3 + 2\beta_0\lambda_r)}\right)^{1/2}, \tag{4}$$

$$\Omega_0 = \frac{b_2(b_5 + 6\varepsilon\beta_0) - 12\lambda_i b_3 - 2\beta_0\lambda_r b_7}{6(b_2 - \beta_0\mu_r)(3\beta_0\lambda_i - \lambda_r b_5) + 2\mu_r(3\lambda_i b_6 + \beta_0\lambda_r b_8)}, \tag{5}$$

and the chirp parameter β_0 together with model parameters δ, σ, ε, χ, α, λ, μ and ν satisfy the compatibility conditions as follows:

$$\lambda_i = \frac{\lambda_r(6b_2b_3 - \beta_0 b_1 b_7)}{6b_1b_3 + \beta_0 b_2 b_7}, \tag{6}$$

$$\delta = -\frac{A_0^2\alpha}{\eta_0} + \Omega_0 b_{10} + \eta_0^2[\beta_0(1 + 6\lambda_r\Omega_0) - b_3(\varepsilon - 3\lambda_i\Omega_0)], \tag{7}$$

$$\chi = \mu_i\Omega_0 + \frac{\eta_0^2}{2A_0^2}[2b_5(\varepsilon - 3\lambda_i\Omega_0) - 3\beta_0(1 + 6\lambda_r\Omega_0)], \tag{8}$$

whereas the inverse group velocity ρ_0 and phase parameter K_0 can be determined by the following equations directly:

$$\rho_0 = \frac{A_0^2\alpha}{\eta_0^2 b_4} + \Omega_0(1 + 3\lambda_r\Omega_0) + \eta_0^2(2\beta_0\lambda_i - \lambda_r b_3), \tag{9}$$

$$K_0 = \frac{1}{2}\Omega_0^2 + \lambda_r\Omega_0^3 - \frac{1}{2}\eta_0^2[b_3(1 + 6\lambda_r\Omega_0) + 4\beta_0(\varepsilon - 3\lambda_i\Omega_0)], \tag{10}$$

with the notation

$$\begin{aligned} b_1 &= 3\mu_r - \beta_0\mu_i + 2\nu_r, \\ b_2 &= 3\mu_i + \beta_0\mu_r + 2\nu_i, \\ b_3 &= 1 - \beta_0^2, \\ b_4 &= 1 + \beta_0^2, \\ b_5 &= 2 - \beta_0^2, \\ b_6 &= 2 + \beta_0^2, \\ b_7 &= 11 - \beta_0^2, \\ b_8 &= 5 + 2\beta_0^2, \\ b_9 &= -\sigma - 2\varepsilon\Omega_0 + 3\lambda_i\Omega_0^2, \\ b_{10} &= \sigma + \varepsilon\Omega_0 - \lambda_i\Omega_0^2. \end{aligned} \tag{11}$$

For different values of α, the dependences of A_0^2 and η_0^2 on the chirp parameter β_0 are both shown in Fig.1, in which other parameter values are σ=0.0592, ε=0.4749, λ_r=–0.03147, μ_r=–0.04173, ν_i=–0.02609, and μ_i=ν_r=0. Curves 1, 3, 4 and 2, 5, 6 denote A_0^2 and η_0^2 for α=0.1, 0.01, 0.001 respectively. They indicate the regions where the solution of Eq.(1) in the form (2) exists. We can see that, the regions for solution existence are $\beta_0 \in$ [–0.907, –0.278] ∪ [0.384, 0.44] for α=0.001 and the regions will be extended with the increasing of parameter α.

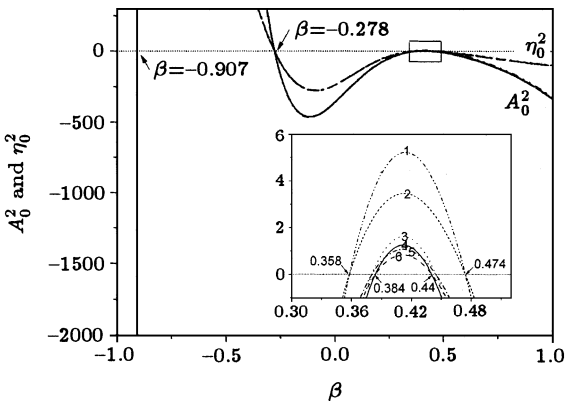


Fig.1. Some regions of the chirp parameter β_0 where the soliton-like solution (2) exists.

3. Variational method

To study the stability of the soliton-like solution, we analyse it by evaluating the evolution of the solution parameters. Or, rather, based on the solution (2), we assume that the solution parameters are slowly varying functions of a propagating distance z. In addition, considering the possible shift of the centre frequency due to stimulated Raman scattering, we take the generalized trying function as

$$U(z,t) = A(z)[\mathrm{sech}(\tau)]^{1+\mathrm{i}\beta(z)}\exp(\mathrm{i}\phi), \tag{12}$$

where $\tau = \eta(z)[t-\rho(z)]$, and $\phi = \Omega(z)[t-\rho(z)]-K(z)$; A, η, β, Ω, ρ and K are the amplitude, the inverse pulse width, chirp, frequency shift, inverse group velocity and phase of the soliton-like solution, respectively, which are functions of the propagating distance z. We use a variational method for solving Eq.(12) to derive dynamic equations for the parameters $A(z)$, $\eta(z)$, $\beta(z)$, $\Omega(z)$, $\rho(z)$ and $K(z)$. Firstly, we introduce the Lagrangian[18]

$$L = \int_{-\infty}^{+\infty} L_0(U, U^*)\mathrm{d}t, \tag{13}$$

where the Lagrangian density is given by

$$L_0(U, U^*) = \frac{1}{2}[\mathrm{i}(U^*U_z - UU_z^*) - |U_t|^2 + |U|^4]. \tag{14}$$

It is easy to verify that Eq.(1) can be recovered by

$$\frac{\partial L_0}{\partial U^*} - \frac{\partial}{\partial t}\left(\frac{\partial L_0}{\partial U_t^*}\right) - \frac{\partial}{\partial z}\left(\frac{\partial L_0}{\partial U_z^*}\right) = \mathrm{i}R(U, U^*). \tag{15}$$

In the presence of the perturbation terms $R(U, U^*)$, the Euler–Lagrange equation can be extended to the following form:[19]

$$\frac{\partial L}{\partial P} - \frac{\partial}{\partial z}\left(\frac{\partial L}{\partial P_z}\right) = \mathrm{i}\int_{-\infty}^{+\infty}\left(R\frac{\partial U^*}{\partial P} - R^*\frac{\partial U}{\partial P}\right)\mathrm{d}t, \tag{16}$$

where P represents one of the six pulse parameters. Substituting $A(z)$, $\eta(z)$, $\beta(z)$, $\Omega(z)$, $\rho(z)$ and $K(z)$ for P in (16), we can obtain the following set of evolution equations:

$$\frac{\mathrm{d}A}{\mathrm{d}z} = \frac{A^3\alpha}{\eta} + \frac{A\eta^2}{9}[(3\beta - 7\varepsilon - \varepsilon\beta^2) + 3\Omega(7\lambda_i + \beta^2\lambda_i + 6\beta\lambda_r)] + A(\delta - \sigma\Omega - \varepsilon\Omega^2 + \lambda_i\Omega^3) + \frac{8}{9}A^3(\chi - \mu_i\Omega), \tag{17}$$

$$\frac{\mathrm{d}\eta}{\mathrm{d}z} = \frac{4}{9}A^2\eta(\chi - \mu_i\Omega) + \frac{2}{9}\eta^3(3\beta - 4\varepsilon + 12\lambda_i\Omega + 18\beta\lambda_r\Omega + 2\beta^2\varepsilon - 6\beta^2\lambda_i\Omega), \tag{18}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}z} = -\frac{2A^2\alpha\beta}{3} + \frac{2}{15}(1+\beta^2)(7+3\beta^2)\eta^4\lambda_i + \frac{2}{3}\eta^2(1+\beta^2)(3\lambda_i\Omega^2 - 2\varepsilon\Omega - \sigma) + \frac{4}{15}A^2\eta^2(2\beta\mu_r + 2\beta\nu_r - \beta^2\mu_i - 3\mu_i - 2\nu_i), \tag{19}$$

$$\frac{\mathrm{d}\beta}{\mathrm{d}z} = \frac{2A^2}{3}(1 - \beta\chi + \beta\mu_i\Omega + \mu_r\Omega) + \frac{2}{3}\eta^2(1+\beta^2)(3\beta\lambda_i\Omega - 6\lambda_r\Omega - \beta\varepsilon - 1), \tag{20}$$

for the amplitude $A(z)$, the inverse pulse width $\eta(z)$, the frequency shift $\Omega(z)$ and the chirp $\beta(z)$, respectively, and the equations

$$\frac{\mathrm{d}\rho}{\mathrm{d}z} = \frac{\alpha A^2}{\eta^2} - \frac{\eta^2}{3}(1+\beta^2)(2\beta\lambda_i - 3\lambda_r) + \frac{A^2}{3}(\beta\mu_i - 3\mu_r - 2\nu_r) + \beta(\sigma + 2\varepsilon\Omega - 3\lambda_i\Omega^2) + \Omega(1 + 3\lambda_r\Omega), \tag{21}$$

$$\frac{\mathrm{d}K}{\mathrm{d}z} = -\frac{\alpha A^2\Omega}{\eta^2} + \frac{2}{9}\eta^2(5\beta\varepsilon + 2\beta^3\varepsilon - 12\beta\lambda_i\Omega - 3\beta^3\lambda_i\Omega + 18\lambda_r\Omega + 9\beta^2\lambda_r\Omega) + \frac{\eta^2}{6}(5 + 3\beta^2) + \frac{1}{9}A^2(-12 + 4\beta\chi - 7\beta\mu_i\Omega - 3\mu_r\Omega + 6\nu_r\Omega) - \frac{2\ln 2}{3}[\eta^2(1+\beta^2)(1 + \beta\varepsilon - 3\beta\lambda_i\Omega + 6\lambda_r\Omega) + A^2(\beta\chi - \beta\mu_i\Omega - \mu_r\Omega - 1)] - \frac{1}{2}\Omega(2\beta\sigma + \Omega + 4\beta\varepsilon\Omega - 6\beta\lambda_i\Omega^2 + 4\lambda_r\Omega^2), \tag{22}$$

for the inverse group velocity $\rho(z)$ and the phase shift $K(z)$, respectively, which are dependent on the parameters A, η, Ω and β. It is easy to verify that the parameters A_0, η_0, Ω_0 and β_0 satisfy the conditions (3)–(8) presenting the steady-state solution of Eqs.(17)–(20). When only the effects of group velocity dispersion and Kerr nonlinearity are taken into account, namely $\delta = \sigma = \varepsilon = \chi = \alpha = \lambda = \mu = \nu = 0$, and in the absence of chirp, Eqs.(17)–(22) can be reduced to

$$\frac{\mathrm{d}A}{\mathrm{d}z} = 0, \quad \frac{\mathrm{d}\eta}{\mathrm{d}z} = 0, \quad \frac{\mathrm{d}\Omega}{\mathrm{d}z} = 0, \quad |A| = \eta, \quad \frac{\mathrm{d}\rho}{\mathrm{d}z} = \Omega, \quad \frac{\mathrm{d}K}{\mathrm{d}z} = -\frac{1}{2}(\eta^2 + \Omega^2).$$

Then Eq.(2) presents the exact soliton solution of the standard nonlinear Schrödinger equation.

4. Linear stability analysis and results

In order to understand the stability of soliton-like solution (2), setting $A = A_0 + \Delta A$, $\eta = \eta_0 + \Delta\eta$, $\Omega = \Omega_0 + \Delta\Omega$ and $\beta = \beta_0 + \Delta\beta$ and linearizing Eqs.(17)–(20) around the steady-state solution (A_0, η_0, Ω_0, β_0), we derive the evolution equations for the small deviations ΔA, $\Delta\eta$, $\Delta\Omega$, and $\Delta\beta$ in the forms

$$\begin{aligned}
\frac{\mathrm{d}\Delta A}{\mathrm{d}z} &= C_{11}\Delta A + C_{12}\Delta\eta + C_{13}\Delta\Omega + C_{14}\Delta\beta,\\
\frac{\mathrm{d}\Delta\eta}{\mathrm{d}z} &= C_{21}\Delta A + C_{22}\Delta\eta + C_{23}\Delta\Omega + C_{24}\Delta\beta,\\
\frac{\mathrm{d}\Delta\Omega}{\mathrm{d}z} &= C_{31}\Delta A + C_{32}\Delta\eta + C_{33}\Delta\Omega + C_{34}\Delta\beta,\\
\frac{\mathrm{d}\Delta\beta}{\mathrm{d}z} &= C_{41}\Delta A + C_{42}\Delta\eta + C_{43}\Delta\Omega + C_{44}\Delta\beta,
\end{aligned} \tag{23}$$

where the coefficients are given by

$$
\begin{aligned}
C_{11} =& \frac{2\alpha A_0^2}{\eta_0} + \frac{16}{9}A_0^2(\chi - \mu_i\Omega),\\
C_{12} =& -\frac{3\alpha A_0^3}{\eta_0^2} - \frac{2A_0}{9\eta_0}[9(\delta - \Omega b_{10})\\
&+ 8A_0^2(\chi - \mu_i\Omega)],\\
C_{13} =& \frac{\alpha\beta_0 A_0^3}{\eta_0^2 b_4} + \frac{1}{9}A_0[6\eta_0^2(5\lambda_i - \beta_0^2\lambda_i\\
&+ 6\beta_0\lambda_r) - 8A_0^2\mu_i],\\
C_{14} =& \frac{A_0\eta_0^2}{9}(3 - 2\beta_0\varepsilon + 6\beta_0\lambda_i\Omega_0 + 18\mu_r\Omega_0),\\
C_{21} =& \frac{8}{9}A_0\eta_0(\chi - \mu_i\Omega_0),\\
C_{22} =& -\frac{8}{9}A_0^2(\chi - \mu_i\Omega_0),\\
C_{23} =& -\frac{4}{9}A_0^2\eta_0\mu_i + \frac{4}{3}\eta_0^3(\lambda_i b_5 + 3\lambda_r\beta_0),\\
C_{24} =& -\frac{2}{9}\eta_0^3(3 + 4\beta_0\varepsilon - 12\lambda_i\beta_0\Omega_0 + 18\lambda_r\Omega_0),\\
C_{31} =& -\frac{4}{3}\alpha A_0\beta_0 - \frac{8}{15}A_0\eta_0^2(b_2 - \beta_0 b_1),\\
C_{32} =& \frac{4}{3}\eta_0 b_4 b_9 - \frac{8}{15}a_0^2\eta_0(b_2 - \beta_0 b_1)\\
&+ \frac{8}{15}\lambda_i\eta_0^3 b_4(7 + 3\beta_0^2),\\
C_{33} =& -\frac{4}{3}\eta_0^2 b_4(\varepsilon_3\lambda_i\Omega_0),\\
C_{34} =& -\frac{2\alpha b_3 A_0^2}{3b_4} + \frac{4}{15}\eta_0^2[\beta_0\eta_0^2(15\lambda_i + \beta_0^2\lambda_i + 10\beta_0\lambda_r)\\
&- 2A_0^2(\beta_0\mu_i - \mu_r - \nu_r)],\\
C_{41} =& \frac{4}{3}A_0(1 - \beta_0\chi + \beta_0\Omega_0\mu_i + \Omega_0\mu_r),\\
C_{42} =& -\frac{4A_0^2}{3\eta_0}(1 - \beta_0\chi + \beta_0\Omega_0\mu_i + \Omega_0\mu_r),\\
C_{43} =& \frac{2}{3}A_0^2(\beta_0\mu_i + \mu_r) + 2\eta_0^2 b_4(\lambda_i\beta_0 - 2\lambda_r),\\
C_{44} =& -\frac{1}{3}\eta_0^2[\beta_0(1 + 6\lambda_r\Omega_0)\\
&+ (6 + 4\beta_0^2)(\varepsilon - 3\lambda_i\Omega_0)].
\end{aligned}
$$

In Eq.(23), there exist four unknown quantities that determine the small deviations from the steady-state solution. The complete stability analysis of the four quantities is complicated both analytically and numerically. Here we investigate an example, in which β_0=0.398, and other parameters of the equation are the same as those used previously. The soliton-like pulse is determined by Eqs.(3)–(11) depending on the chirp parameter β_0. We know that if the steady-state solution (A_0, η_0, Ω_0, β_0) is linearly stable, the real parts of all eigenvalues for Eq.(23) are negative. The maxima of the real parts of the four eigenvalues for different chirps β_0 are shown in Fig.2. We can see that only when $\beta_0 \in [0.398, 0.44]$ for α=0.001, they are all negative and the steady-state solution (A_0, η_0, Ω_0, β_0) may be linearly stable.

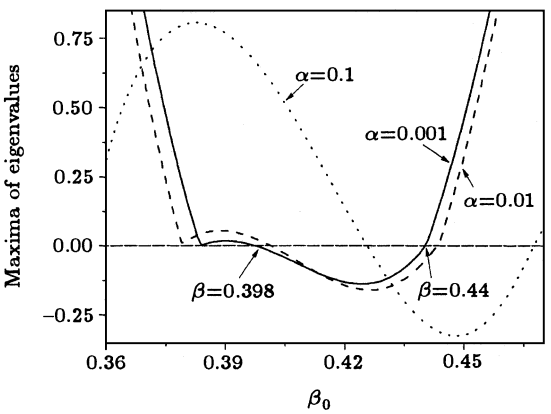


Fig.2. Maxima of the four eigenvalues of Eq.(23) and linearly stable regions of chirp parameter β_0 for Eq.(3).

Moreover, if the parameters in Eq.(1), including δ, are fixed, the pulse is unstable. Any increase in amplitude of the pulse relative to the exact solution can augment the total gain across the pulse, and then the amplitude increases exponentially. Any decrease in the pulse amplitude appearing in the opposite direction can result in the pulse decay. For a proper choice of parameters, the pulse can become stable if the gain (or loss) δ depends on the total energy of the pulse Q, as $\delta = g(Q) - \delta_0$, the gain term $g(Q)$ describes a gain medium with a recovery time T_L, which is much longer than the round-trip time of the cavity and therefore the gain term does not depend explicitly on t. It can be described by the following rate equation[16]

$$\frac{\partial g(Q)}{\partial z} = -\frac{g(Q) - g_0}{T_L} - \frac{Q}{T_R E_L}g(Q), \qquad (24)$$

where g_0 is the small signal gain, E_L is the gain saturation energy, and $T_R = 1$ is the round-trip time. The dependence of δ on the pulse energy Q serves as a feedback mechanism that stabilizes the pulse for some values of g_0, E_L, T_L, and δ_0. When β_0=0.398, the maxima of real parts of the four eigenvalues take the value –0.00224. Then we have δ=–0.00217, λ_i=0.00653, χ=0.18319, and the other corresponding pulse parameters A_0 =0.98226, η_0 =0.80348, Ω_0 = –0.04953, ρ_0 =–0.02802, and K_0 =–0.51751.

To verify the predictions, we simulate Eq.(2) numerically by using the split-step Fourier method. The initial conditions are the same as those for the soliton-like solution under disturbances in amplitude and chirp with g_0=1, T_L=100, and E_L=100, which ensure

that the pulse amplitude is well below the saturation level. Firstly, the evolution of the initial pulse with an amplitude A_0 being 10% larger than the theoretical prediction, i.e. 1.08049, is shown in Fig.3. We can see clearly that, after adjustment in a short distance, the theoretically predicted pulse is formed. Secondly, we seek for the evolution of an initial pulse with different initial chirps, which is shown in Fig.4. Finally, the evolution under the perturbation of white noise, whose maximal value is 0.26, is shown in Fig. 5. From the evolution behaviours of the pulse, we can conclude that the soliton-like solution is stable under finite initial perturbations when β_0=0.398. In addition, we investigate the evolution of an arbitrary Gaussian pulse for β_0=0.398, which is shown in Fig.6. We can see that the Gaussian pulse converges to the soliton-like solution after 1000 round trips.

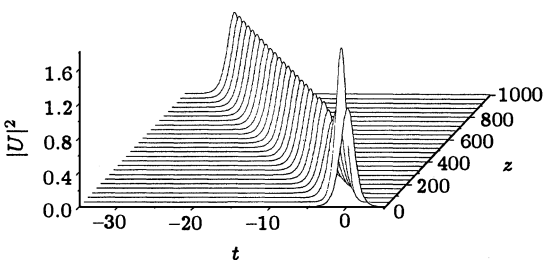


Fig.3. Evolution of an initial pulse whose amplitude is 10% larger than that of the exact solution, i.e. A_0=1.08049.

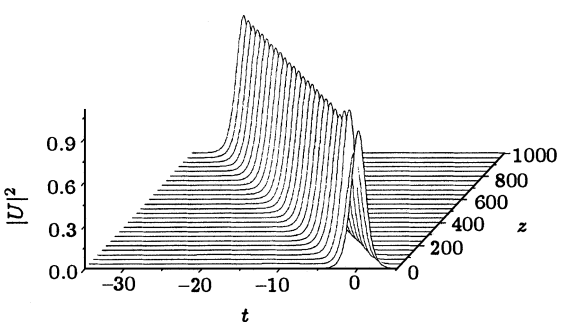


Fig.4. Evolution of an initial pulse with the perturbation of chirp, the initial pulse is $E(0,t) = A_0\mathrm{sech}(\eta_0 t)\exp[\mathrm{i}(\Omega_0 t + 0.5t^2)]$.

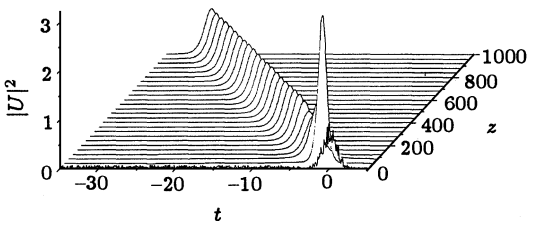


Fig.5. Evolution of an initial pulse with the perturbation of white noise, whose maximal value is 0.26.

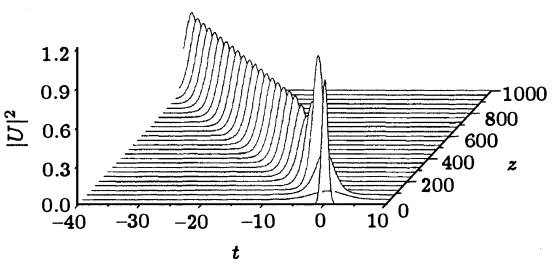


Fig.6. Evolution of an arbitrary Gaussian pulse $E(0,t) = A_0\exp(-t^2)$.

5. Conclusion

We have obtained analytically the soliton-like solution for a general passively mode-locked laser model, in which both the self-frequency shift and either a fast or a slow saturable absorber response are taken into account. The stability of the solution has also been investigated by using both a variational method and numerical simulation. The numerical simulation results match well with the theoretical predictions obtained by use of the variational method. From the numerical results, we can conclude that the soliton-like solution is stable under a definite parameter condition. In addition, we also find that when started with an initial arbitrary Gaussian pulse, the exact soliton-like solution can be converged under the feedback mechanism introduced by the saturable absorber.

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