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Time evolution of the intensity correlation function in a single-mode laser driven by both the coloured pump noise with signal modulation and the quantum noise with cross-correlation between the real and imaginary parts*

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Using the linear approximation, we have studied the time evolution of intensity correlation function $C(t)$ in a single-mode laser driven by both the coloured pump noise with signal modulation and the quantum noise with cross-correlation between its real and imaginary parts. In the case of the pump noise with self-correlation time $\tau \ll 1$, we find that the time evolution of $C(t)$ varies with modulation signal frequency Ω, amplitude B, and net gain a_0. (i) As Ω increases, the time evolution of $C(t)$ changes from monotonically descending to flat descending, and finally to damply oscillating; (ii) as B increases, it changes from monotonically descending to maximal; (iii) as net gain $a_0$increases, it changes repeatedly from monotonically descending to monotonically ascending, and to maximal, finally to monotonically descending again. However, in the case of $\tau \gg 1$, the time evolution of $C(t)$ only exhibits a form of damping oscillation.

Keywords: noise, single-mode laser, intensity correlation function
PACC: 4250, 0540, 4260K

1. Introduction

Because of the great influences of noise and input signal on the dynamic behaviour of a non-linear stochastic system, the study of dynamic behaviour for such a system has attracted researchers' extensive attention.[1−8] Since Fulinski and Telejko first studied the influence of cross-correlation noise on the statistical properties of a non-linear system,[9] Cao and Wu have made more detailed investigations of the one-dimensional stochastic system driven by cross-correlation noise,[10] and Zhu introduced a cross-correlation noise into a laser system and studied the property of the laser system in a steady state driven by both pump noise and quantum noise with cross-correlation between its real and imaginary parts.[11] We have studied the dynamic property of a single-mode laser system driven by both pump noise and quantum noise with cross-correlation between its real and imaginary parts.[12] However, in the above-mentioned studies, one emphasized the influence of cross-correlation noise on the dynamic behaviour of a non-linear stochastic system, but given little consideration to the influence of the signal on the dynamic behaviour of the laser system. Dykman has introduced into the laser system the modulation of noise by the signal.[13] The signal modulated noise can better describe the laser system than the ordinary noise, because when a laser system is modulated by signal, the noise of the system will simultaneously be modulated by the signal. As a result, the modulated noise will highly affect the dynamic behaviour

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of the laser system. The study of the function of the modulation signal in the time evolution of the laser intensity correlation function in a single-mode laser system driven by the modulated noise can show, in a certain aspect, how the modulation signal affects the dynamic behaviour of the laser system. Thus it is of great significance theoretically. The intensity correlation function is a basic statistic quantity in describing the dynamic property of a laser system, and it can also reveal statistical properties of the system (e.g. statistical fluctuation, time evolution of the intensity correlation function). From the intensity correlation function can be deduced the power spectrum, power output, correlation time and variance of laser intensity, and also the output single-to-noise ratio (SNR) of a laser system. With the output SNR of a laser system we can further study the phenomenon of stochastic resonance (SR) to provide a theoretic base for optimizing the laser system. Therefore, our study is of certain guiding significance in the field of laser communication.

In this paper, by adopting the linear approximation, we study the time evolution of intensity correlation function for a loss-noise model of a single-model laser system driven by both coloured pump noise with signal modulation and the quantum noise with cross-correlation between its real and imaginary parts. We also provide a detailed analysis of the influences of the net gain a_0, the amplitude B and frequency Ω of modulation signal on the time evolution of the intensity correlation function. Because of the application of the unified coloured noise approximation in the process of solving the Langevin equation, we will discuss the time evolution of the intensity correlation function in the case of the pump noise self-correlation time $\tau \ll 1$ and $\tau \gg 1$ only. Consequently, we find that the signal frequency Ω has a great influence on the process of evolution.

2. The intensity correlation function

The Langevin equation of amplitude for a loss-noise model of a single-mode laser is given by[14]

$$\frac{\mathrm{d}r}{\mathrm{d}t'} = a_0 r - Ar^3 + \frac{Q}{2r}(1-|\lambda_q|) + r p_R(t') + \varepsilon_r(t'). \quad (1)$$

If we consider that the pump noise is modulated by a periodic signal $B\cos\Omega t'$, thus the Langevin equation of intensity for a loss-noise model of a single-mode laser with an input signal is given by

$$\begin{aligned}\frac{\mathrm{d}I}{\mathrm{d}t'} =& 2a_0 I - 2AI^2 + Q(1-|\lambda_q|) \\ &+ 2Ip_R(t')B\cos\Omega t' + 2\sqrt{I}\varepsilon_r(t'), \end{aligned} \quad (2)$$

where noises $p_R(t')$ and $\varepsilon_r(t')$ are correlated in the following forms:

$$\begin{aligned}&\langle p_R(t')\rangle = \langle \varepsilon_r(t')\rangle = 0, \\ &\langle p_R(t')p_R(s)\rangle = \frac{P}{2\tau}e^{-\frac{|s-t'|}{\tau}}, \\ &\langle \varepsilon_r(t')\varepsilon_r(s)\rangle = Q(1+|\lambda_q|)\delta(t'-s), \\ &\langle p_R(t')\varepsilon_r(s)\rangle = 0. \end{aligned} \quad (3)$$

In Eqs.(1), (2) and (3), a_0 and A represent the net gain coefficient and self-saturation coefficient, respectively; I is the laser intensity; B is the amplitude of a modulation signal, Ω is the signal frequency; $p_R(t')$ is the real part of the pump noise, and $\varepsilon_r(t')$ is the quantum noise of phase lock; Q and P are the intensities of the pump noise and the quantum noise, respectively; τ is the self-correlation time of pump noise; λ_q is the coefficient of cross-correlation between the real part and the imaginary part of the quantum noise, and $-1 \le \lambda_q \le 1$. Let

$$I = I_0 + \delta(t'),$$

where $I_0 = \dfrac{a_0}{A}$ is the deterministic steady-state intensity, and $\delta(t')$ is the perturbation term. We linearize Eq.(2) around the deterministic steady-state intensity I_0, thus the linear equation of the laser intensity is found to be

$$\begin{aligned}\frac{\mathrm{d}\delta(t')}{\mathrm{d}t'} =& -\gamma\delta(t') + 2I_0 p_R(t')B\cos\Omega t' \\ &+ 2\sqrt{I_0}\varepsilon_r(t') + Q(1-|\lambda_q|), \end{aligned} \quad (4)$$

where $\gamma = 2a_0$.

The solution of Eq.(4) has the following form:

$$\begin{aligned}\delta(t') =& \mathrm{e}^{\gamma t'}\left[\frac{P(1-|\lambda_q|)}{\gamma}(\mathrm{e}^{\gamma t'}-1)\right. \\ &+ 2I_0 B\int_0^{t'} p_R(s)\cos\Omega s\mathrm{e}^{\gamma s}\mathrm{d}s \\ &\left.+ 2\sqrt{I_0}\int_0^{t'}\varepsilon_r(s)\mathrm{e}^{\gamma s}\mathrm{d}s\right]. \end{aligned} \quad (5)$$

According to the steady-state mean intensity correlation function (SSMICF) defined by

$$C(t)=\lim_{t'\to\infty}\overline{\langle I(t')I(t'+t)\rangle}=\lim_{t'\to\infty}\left(\frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}}\langle I(t')I(t'+t)\rangle \mathrm{d}t'\right),$$

the SSMICF is obtained by solving Eq.(5)

$$\begin{aligned}C(t)=&\left(I_0+\frac{Q(1-|\lambda_q|)}{\gamma}\right)^2\\&+\left(\frac{2I_0Q(1+|\lambda_q|)}{\gamma}+\frac{I_0^2B^2P(2\pi\gamma^2+\Omega^3)(\Omega^2-\gamma^2+\tau^{-2})}{\tau^2\Omega\gamma(k_1^2+\Omega^2)(k_2^2+\Omega^2)(\gamma^2+\Omega^2)}\right)\mathrm{e}^{-\gamma|t|}\\&+\frac{I_0^2B^2P(\Omega^2+\gamma^2-\tau^{-2})}{\tau(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-\frac{|t|}{\tau}}\cos\Omega t+\frac{2\Omega I_0^2B^2P}{\tau^2(k_1^2+\Omega^2)(k_2^2+\Omega^2)}\mathrm{e}^{-\frac{|t|}{\tau}}\sin\Omega|t|,\end{aligned}\tag{6}$$

where $k_1=\gamma-\tau^{-1}$, $k_2=\gamma+\tau^{-1}$, and $\tau\neq\dfrac{1}{2a_0}$.

We can see from Eq.(6) that there appear two features of the intensity correlation function $C(t)$: one is the existence of two timescales of the $C(t)$ because of pump noise being "coloured"; the other is the $C(t)$ being a periodic function of the signal frequency Ω due to the pump noise being modulated by signal.

In the following, we will discuss the influences of the net gain a_0, the amplitude B and the frequency Ω of modulation signal on the time evolution of the $C(t)$ based on Eq.(6). Furthermore, due to the use of unified coloured noise approximation in solving Eq.(1), we will only discuss the time evolution of $C(t)$ in the cases of $\tau\ll 1$ and $\tau\gg 1$.

3. The influence of frequency Ω of modulation signal on the time evolution of the intensity correlation function $C(t)$

3.1. In the case of $\tau\ll 1$

When $\tau=0.04$, the three-dimensional plots in Figs.1(a)–(c) exhibit how the frequency Ω of modulation signal influences the time evolution of $C(t)$. Figures 1(d)–(g) are its sectional drawings. As shown in Figs.1(a) and (d), within the range $2\leq\Omega\leq 6$, $C(t)$ vs t curves each descend monotonically, which is a common descending form. From Figs.1(b) and (e), we can see that when the frequency Ω increases to the range $6\leq\Omega\leq 30$, the $C(t)$ vs t curves each have a flat appearing at $t=0$ and then descend monotonically. Figures 1(c), (f) and (g) show that when $\Omega\geq 30$, the initial flat disappears and the $C(t)$ vs t curves each change in a form of damping oscillation. At the same time, their amplitudes and frequencies increase with Ω. When the evolution time $t>0.3$, all values of $C(t)$ in different evolution processes tend to be identical, as shown in Fig.1. In summary, as Ω increases, $C(t)$ vs t curve changes from monotonically descending to flat descending, and finally to damply oscillating.

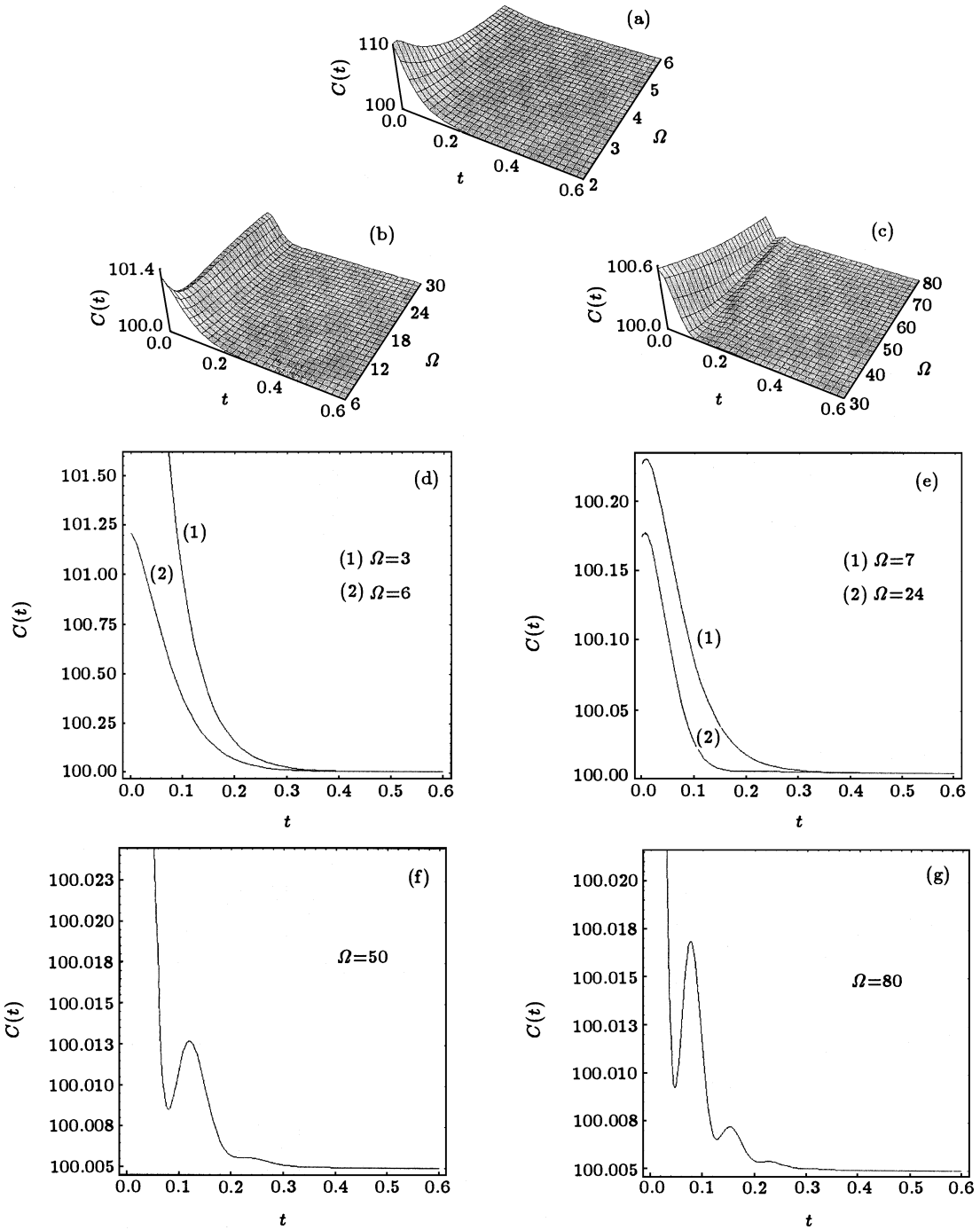


Fig.1. When $\tau \ll 1$, the influence of Ω on time evolution of $C(t)$ with the parameters being $A = 1$, $B = 20$, $P = 0.001$, $Q = 0.001$, $a_0 = 10$, $\lambda_q = 0.5$, $\tau = 0.04$.

3.2. In the case of $\tau \gg 1$

When $\tau = 50$, the influence of frequency Ω of modulation signal on time evolution of curve $C(t)$ can be seen from the three-dimensional plot (Fig.2(a)), the $C(t)$ vs t curves of the sectional drawing (Figs.2(b) and (c)), and the $C(t)$ vs Ω curve of the sectional drawing (Fig.2(d)). Figures 2(b) and (c) (The sectional drawing plot) show that $C(t)$ vs t curves each exhibit a form of damping oscillation. At a given time t, $C(t)$ is also displays a periodic change with Ω, as shown in Fig.2(d). It can be also found in Figs.2(b)–(d) that as Ω increases, the periodic oscillation frequency of $C(t)$ becomes higher while the amplitude becomes smaller, that is, the curve descends rapidly. When the evolution time is long enough, the values of $C(t)$ tend to be identical, as shown in Fig.2.

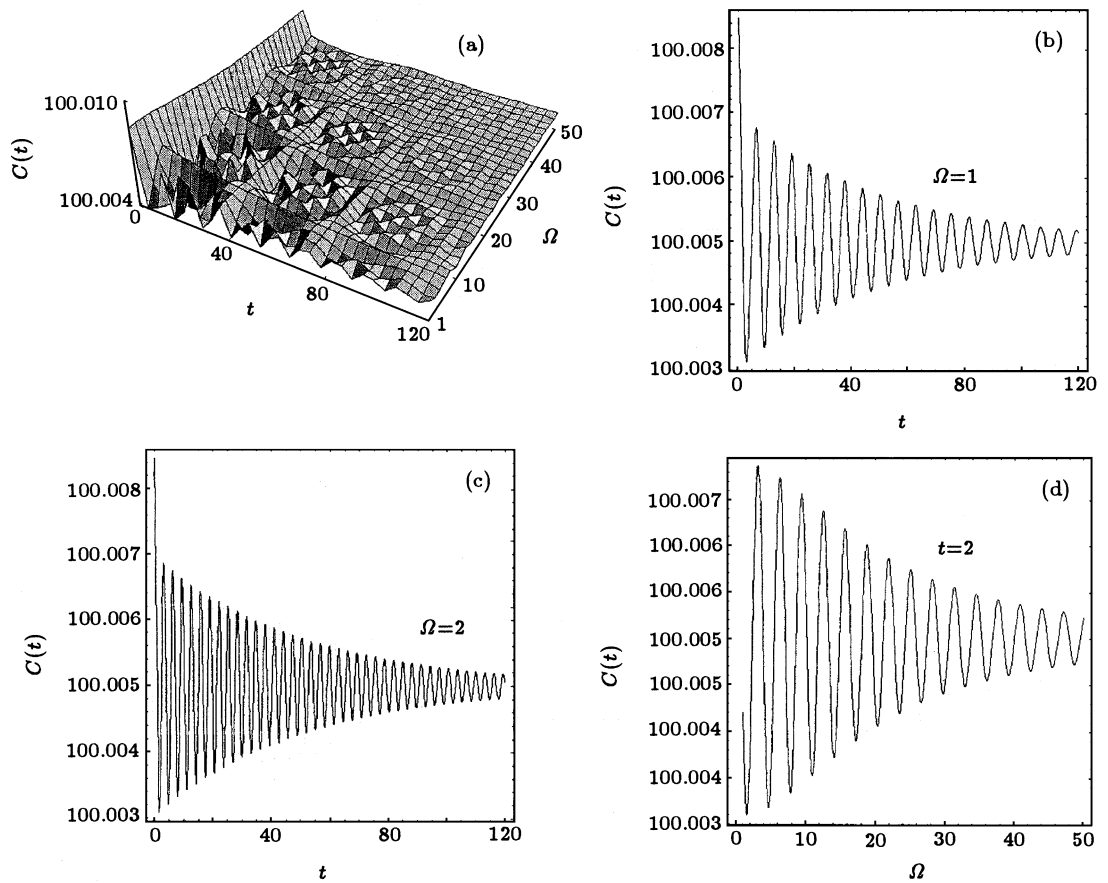


Fig.2. When $\tau \gg 1$, the influence of Ω on time evolution of $C(t)$ with the parameters being $A = 1$, $B = 20$, $P = 0.001$, $Q = 0.001$, $a_0 = 10$, $\lambda_q = 0.5$, $\tau = 50$.

In the cases of $\tau \ll 1$ and $\tau \gg 1$, the time evolution processes of $C(t)$ differ greatly from one another. Since $C(t)$ has two timescales and is a periodic function of Ω, both the modulation signal frequency Ω and the pump noise self-correlation time τ greatly affect the time evolution of $C(t)$. A detailed analysis is as follows.

In the $\tau \ll 1$ case, for example $\tau = 0.04$, and by using $A = 1$, $B = 20$, $P = 0.001$, $Q = 0.001$, $a_0 = 10$, and $\lambda_q = 0.5$, Eq.(6) can be expressed in the following form:

$$C(t) = D_1 + D_2 \mathrm{e}^{-20|t|} + (D_3 \cos \Omega t + D_4 \sin \Omega |t|)\mathrm{e}^{-25|t|} . \quad (7)$$

In Eq.(7), coefficients D_i (i=1,2,3,4) each are a function of Ω. The exponent factor $\mathrm{e}^{-25|t|}$ comes from the case of $\tau = 0.04$, and the factor $(D_3 \cos \Omega t + D_4 \sin \Omega |t|)\mathrm{e}^{-25|t|}$ plays an important role in the time evolution process of $C(t)$. When Ω is very small (e.g, Ω= 1), the exponent factor $\mathrm{e}^{-25|t|}$ descends very fast with t. On the other hand, because the frequency Ω in the factor $(D_3 \cos \Omega t + D_4 \sin \Omega |t|)$ is low, the exponent factor $\mathrm{e}^{-25|t|}$ can not be modulated by the factor within its descending time. As a result, $C(t)$ vs t curve behaves in a form of monotonic descent. When Ω increases enough (e.g, Ω=50), although the exponent factor $\mathrm{e}^{-25|t|}$ descends fast with t, the rapid change of the factor $(D_3 \cos \Omega t + D_4 \sin \Omega |t|)$ arising from larger Ω makes it possible to modulate the exponent factor $\mathrm{e}^{-25|t|}$ within its descending time. Therefore, the $C(t)$ vs t curve changes from monotonically descending to damply oscillating.

In the $\tau \gg 1$ case, for example $\tau = 50$, and also by using $A = 1$, $B = 20$, $P = 0.001$, $Q = 0.001$, $a_0 = 10$, and $\lambda_q = 0.5$, we obtain from Eq.(6)

$$C(t) = D_5 + D_6 \mathrm{e}^{-20|t|} + (D_7 \cos \Omega t + D_8 \sin \Omega |t|)\mathrm{e}^{-0.02|t|} . \quad (8)$$

In Eq.(8), the coefficients $D_i(5, 6, 7, 8)$ each are a function of Ω, and the exponent factor $\mathrm{e}^{-0.02|t|}$ comes from the case of $\tau = 50$. In Eq.(8), because of the slow descent of the exponent factor $\mathrm{e}^{-0.02|t|}$, if $\Omega \geq 1$, $\mathrm{e}^{-0.02|t|}$ can be modulated by the factor $(D_7 \cos \Omega t + D_8 \sin \Omega |t|)$ within its descending time, and so the $C(t)$ vs t curve exhibits a form of damping oscillation.

From the analysis above, we can see that when $\tau \ll 1$ and the value of Ω is small ($\Omega \leq 30$), the time evolution of $C(t)$ is in a form of monotonic descent; when $\tau \ll 1$ and the value of Ω is large enough, the time evolution of $C(t)$ exhibits a form of damping oscillation; when $\tau \gg 1$, whatever the value of Ω is, the time evolution of $C(t)$ is always in a form of damping oscillation. Obviously, the coloured pump noise modulated by periodic signal is a major reason why the time evolution of intensity correlation function has the above-mentioned characteristics. This shows that the interaction between modulation signal and the "colour" pump noise has a great influence on the dynamic behaviour of a laser system.

4. The influence of amplitude B of a modulation signal on time evolution of $C(t)$

4.1. In the case of $\tau \ll 1$ and $\Omega = 1$

When $\tau = 0.04$, the three-dimensional plot in Fig.3(a) exhibits how the amplitude B of modulation signal influences the time evolution of $C(t)$, whose sectional figures are shown in Figs.3(b) and (c). As shown in Fig.3, because of the low values of τ and Ω, as the amplitude B increases, the time evolution of $C(t)$ changes from monotonically descending to maximal, and $C(t)$ vs t curves shift towards the direction of t increasing. When the evolution time $t > 0.7$, the values of $C(t)$ in different evolution processes tend to be identical.

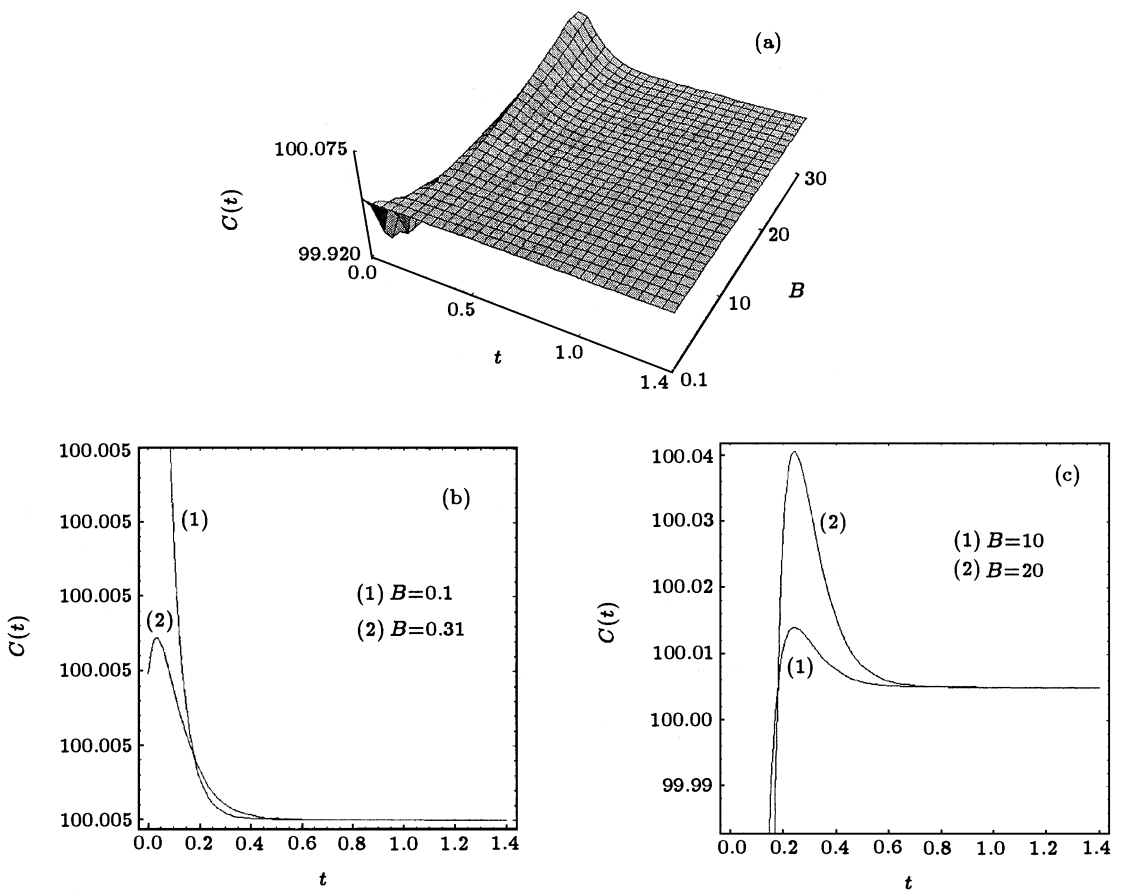


Fig.3. When $\tau \ll 1$, the influence of B on time evolution of $C(t)$ with the parameters being $A = 1$, $P = 0.001$, $Q = 0.001$, $a_0 = 10$, $\lambda_q = 0.5$, $\tau = 0.08$, $\Omega = 1$.

4.2. In the case of $\tau \gg 1$

When $\tau = 50$, the three-dimensional plot in Fig.4(a) exhibits how the amplitude B of modulation signal influences the time evolution of the $C(t)$, whose sectional figure is shown in Fig.4(b). Figure 4 shows that, like the situation discussed in Section 3.2, the time evolution of $C(t)$ behaves in a form of damping oscillation. As t increases, the periodic oscillation frequency of $C(t)$ does not change while the amplitude

of the curve increases, that is, the attenuation process becomes slow. When the evolution time is long enough, the values of $C(t)$ in different evolution processes finally tend to be identical.

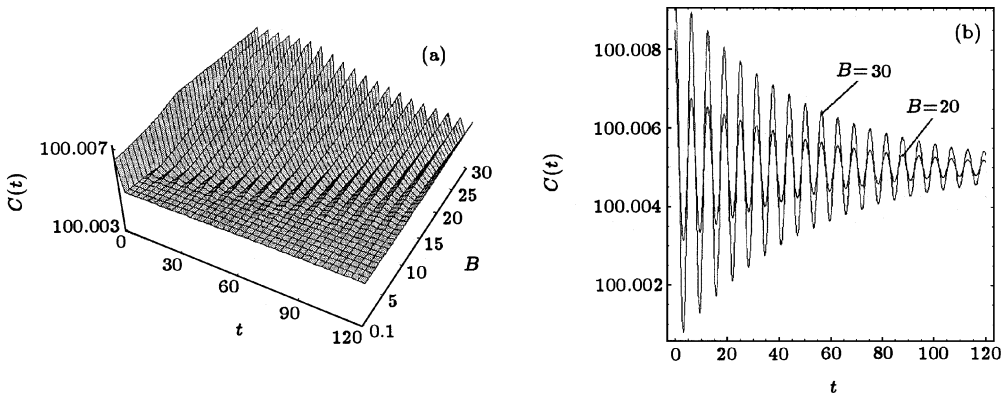


Fig.4. When $\tau \gg 1$, the influence of B on time evolution of $C(t)$ with the parameters being $A = 1$, $P = 0.001$, $Q = 0.001$, $a_0 = 10$, $\lambda_q = 0.5$, $\tau = 50$, $\Omega = 1$.

5. The influence of net gain a_0 on time evolution of $C(t)$

5.1. In the case of $\tau \ll 1$ and $\Omega = 1$

When $\tau = 0.06$, the influence of net gain a_0 on the time evolution of $C(t)$ is shown in Fig.5. The figure shows that because of the low values of τ and Ω, as the net gain a_0 increases, the shape of curve cyclically

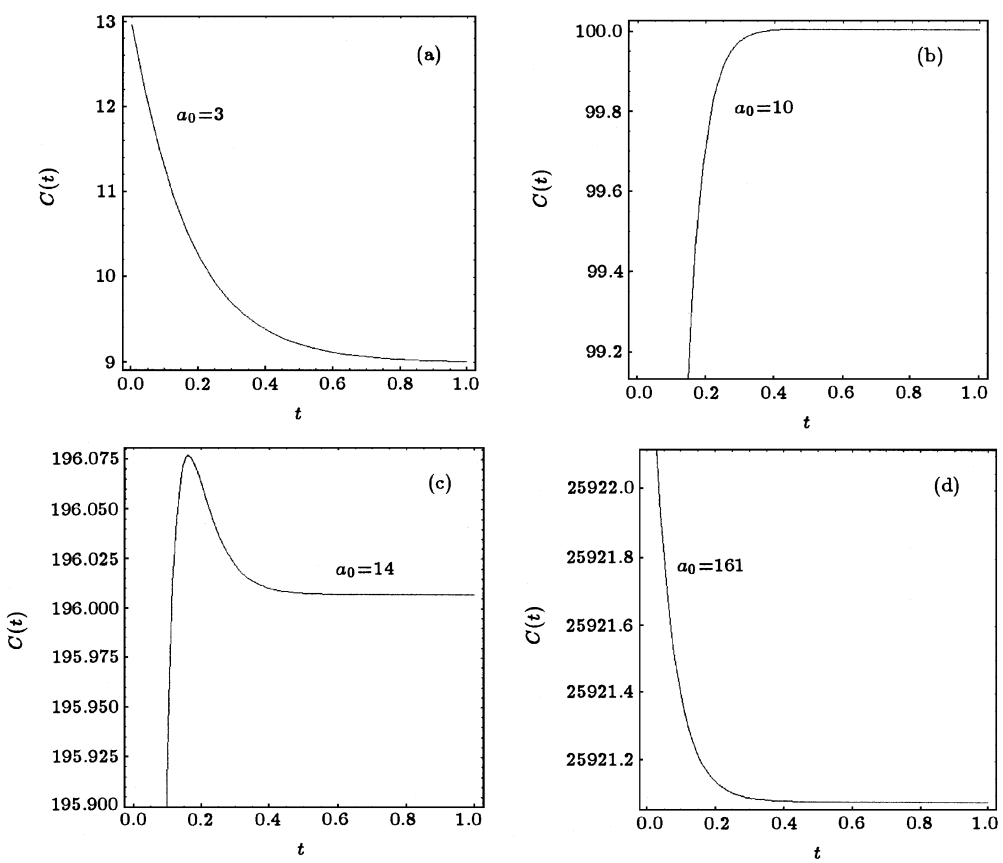


Fig.5. When $\tau \ll 1$, the influence of a_0 on time evolution of $C(t)$ with the parameters being $A = 1$, $B = 20$, $P = 0.001$, $Q = 0.001$, $\lambda_q = 0.5$, $\tau = 0.06$, $\Omega = 1$.

changes from monotonically descending to monotonically ascending, then to maximal, and finally to monotonically descending again, and the $C(t)$ vs t curve shifts upwards. When the evolution time $t >$ 0.5, the final values of $C(t)$ in different evolution processes are different as shown in Fig.5.

5.2. In the case of $\tau \gg 1$

Figure 6 shows how the net gain a_0 influences the time evolution of $C(t)$ when $\tau = 50$. In Fig.6, the time evolution of $C(t)$ exhibits a form of damping oscillation. As a_0 increases, both the amplitude and the frequency of periodic oscillation of $C(t)$ do not change. However, when the evolution time is long enough, the values of $C(t)$ in different evolution processes are different as shown in Fig.6.

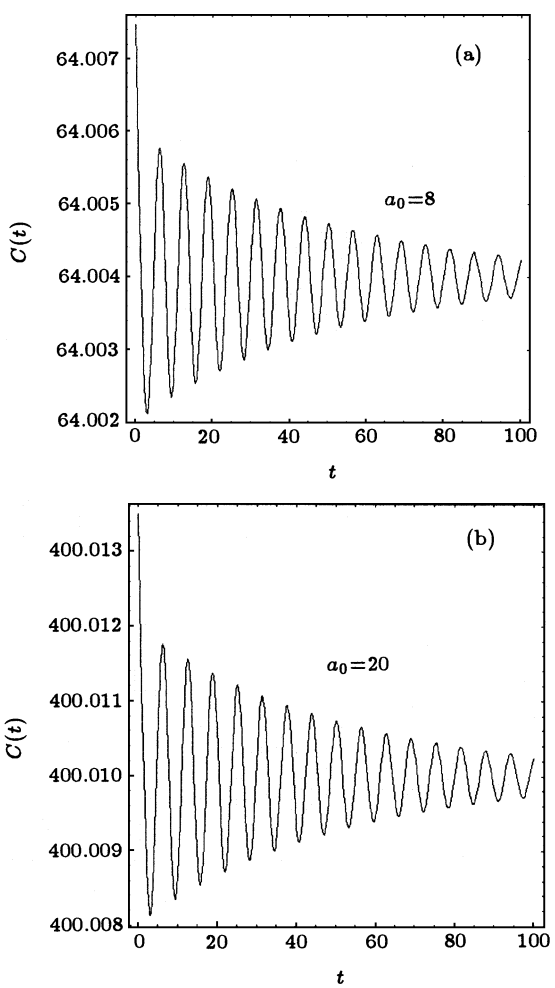


Fig.6. When $\tau \gg 1$, the influence of a_0 on time evolution of $C(t)$ with the parameters being $A = 1$, $B = 20$, $P = 0.001$, $Q = 0.001$, $\lambda_q = 0.5$, $\tau = 50$, $\Omega = 1$.

6. Conclusion and discussion

The analyses and discussion above may be summarized as follows. (1) In the case of $\tau \ll 1$, as modulation signal frequency Ω increases, the time evolution of $C(t)$ changes from monotonically descending to flat descending, and finally to damply oscillating; in the case of $\tau \gg 1$, as the modulation signal frequency Ω increases, the time evolution of $C(t)$ exhibits a form of damping oscillation. What is noteworthy is that only in the case of the $\tau \ll 1$ and $\Omega \leq 30$, the time evolution of $C(t)$ behaves in a form of monotonic descent; but when $\tau \ll 1$ and the value of Ω is high enough or when $\tau >> 1$ (the value of Ω is unrestrained), the time evolution of $C(t)$ exhibits a form of damping oscillation. The physical mechanism of the phenomenon mentioned above is the interaction between the modulation signal and coloured pump noise. This phenomenon also suggests that the modulation of the coloured pump noise by a periodic signal highly affects the dynamic behaviour of the laser system. (2) In the case of $\tau \ll 1$, if the value of Ω is very low (e.g, $\Omega = 1$), the time evolution of $C(t)$ changes from monotonically descending to maximal as the amplitude B increases, but it cannot behave in a form of damping oscillation. In the case of $\tau \gg 1$, the time evolution of $C(t)$ exhibits a form of damping oscillation with the increase of B. In the two cases of $\tau \ll 1$ and $\tau \gg 1$, when the evolution time is long enough, the values of the $C(t)$ corresponding to different amplitudes B tend to be in accord with each other. So we see that the amplitude B and the frequency Ω of the modulation signal have different influences on the time evolution of $C(t)$. (3) The net gain a_0 is an important parameter in describing the statistic property of a laser system. In the case of $\tau \ll 1$, if Ω is taken to be a low value (e.g., $\Omega = 1$), as a_0 increases, the time evolution of $C(t)$ cyclically changes from monotonically descending to monotonically ascending, then to maximal, and finally to monotonically descending again. In the case of $\tau \gg 1$, as a_0 increases, the time evolution of $C(t)$ only exhibits a form of damping oscillation. When the evolution time is long enough, the values of $C(t)$ corresponding to different net gains a_0 are different. This shows a great influence of the a_0 on the dynamic behaviour of the laser system. (4) It is emphasized that the conclusions drawn by the linear approximation method are correct and reliable when the values of the parameters are within the valid range for the approximation. The valid range is determined by the condition of the normalized steady-state mean intensity fluctuation $C(0) \ll 1$. Hence, we calculate the normalized steady-state mean intensity fluctua-

tion under the condition of $C(0) \ll 1$.

In conclusion, the intensity correlation function is a theoretic basis of the study on characteristics of SR in a laser system. Because of the great influences of modulation signal, net gain, the interaction between the modulation signal and the "coloured" of pump noise on the dynamic behaviour of a laser system, the intensity correlation function can also greatly affect the output SNR, power output and variance of laser intensity, which we will further study.

References

[1] Hernandez-Gorcia E, Toral R and San Minguel M 1990 *Phys. Rev.* A **42** 6823

[2] Luo X Q and Zhu S Q 2003 *Phys. Rev.* E **67** 021104-1

[3] Wang J, Cao L and Wu D J 2003 *Phys. Lett.* A **308** 23

[4] Wu D J, Cao L and Ke S Z 1994 *Phys. Rev.* E **50** 2496

[5] Ke S Z, Cao L, Wu D J and Yao K L 2001 *Phys. Lett.* A **281** 113

[6] Cheng Q H, Cao L and Wu D J 2004 *Acta Phys. Sin.* **53** 2556 (in Chinese)

[7] Zhang L, Cao L and Wu D J 2003 *Chin. Phys.* **12** 33

[8] Liang G Y, Cao L and Wu D J 2003 *Chin. Phys.* **12** 1109

[9] Fulinski A and Telejko T 1991 *Phys. Lett.* A **152** 11

[10] Cao L and Wu D J 1994 *Phys. Lett.* A **185** 59

[11] Zhu S Q 1993 *Phys. Rev.* A **47** 2405

[12] Cheng Q H *et al* 2004 *Acta Phys. Sin.* **53** 1675 (in Chinese)

[13] Dykmen M I *et al* 1992 *Phys. Rev.* A **46** 1713

[14] Zhang L, Cao L and Wu D J 2003 *Phys. Lett.* A **30** 29