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Influence of atomic motion on squeezing of double-mode field in interaction with a moving V-type three-level atom

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The influence of atomic motion on squeezing of the double-mode field in the interaction with a moving V-type three level atom is investigated by means of the quantum theory and the method of numerical analysis. The results show, on the one hand, when the initial photon field is in non-correlative coherent state of double-mode, only the quadrature U_1 is squeezed and the squeezed degree of U_1 increases with the increasing of the parameter β which describes the atomic motion and structure of the field mode. On the other hand, when the initial field is in the vacuum squeezed state of double-mode, the quadrature U_2 is squeezed principally and we can obtain a steady squeezed light by choosing an appropriate parameter β.

Keywords: atomic motion, V-type three-level atom, squeezing effect
PACC: 4250

1. Introduction

Since the Jaynes–Cummings model (JCM) was proposed in 1963,[1] people have extensively studied it and revealed many striking nonclassical effects such as squeezing,[2−4] anti-bunching, sub-Poisson statistics[5,6] and the collapse–revivals of atomic population.[7] In recent years, some extended JCMs have been proposed, such as three-level atom interacting with double-mode field,[8] JCM in the presence of a nonlinear medium,[9] and many others.[10−12] But it has been noticed that the atom is always regarded as motionless in all the above researches and the influence of atomic motion on photon field is neglected. Recently, with the development of cavity quantum electrodynamics (QED) the interaction between a moving atom and the field has attracted much attention.[13−18] Researches show that the atomic motion can bring about the nonlinear effect of atomic population,[13−15] change the creating time of Schrödinger cat state[19,20] and enhance the fidelity of quantum state.[21] Furthermore, it has become an active research subject to prepare squeezed light using the interaction between the photon field and atom,[22] which has potential application in detecting weak signals and light communications. It not only can reveal the quantum features of photon field but also has practical significance.

The model of a V-type three level atom interacting with double-mode field is the fundament for preparing macroscopical quantum state, Schrödinger cat state and quantum hidden teleportation. We will start from this model to investigate the influence of atomic motion on squeezing effect in cases of the initial field being in non-correlative coherent state and vacuum squeezed state of double-mode, respectively. At first, the state vector of the system is obtained by solving the Schrödinger equation in interactional picture. Then we investigate the influences of atomic motion and structure of cavity mode on squeezing effect using numerical analysis.

2. The theoretical model and state vector

We consider a system in which a V-type three level atom interacts with double-mode field as shown in Fig.1. The transitions between levels $|a\rangle$ and $|c\rangle$,

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$|b\rangle$ and $|c\rangle$ are connected, respectively, to the photons with frequencies ν_1 and ν_2, while that between $|a\rangle$ and $|b\rangle$ is dipole forbidden. We consider only the double photon resonance condition, that is $\omega_a - \omega_c = \nu_1$, $\omega_b - \omega_c = \nu_2$. The Hamiltonian of the system in rotating wave approximation can be written as

$$H = H_0 + V, \tag{1}$$

$$H_0 = \sum_{i=a,b,c} \omega_i |i\rangle\langle i| + \sum_{j=1}^{2} \nu_j a_j^\dagger a_j, \quad (\hbar = 1) \tag{2}$$

$$V = g_1(a_1|a\rangle\langle c| + a_1^\dagger|c\rangle\langle a|) + g_2(a_2|b\rangle\langle c| + a_2^\dagger|c\rangle\langle b|), \tag{3}$$

where H_0 stands for the Hamiltonian of bare atom and photon field, V the interactional Hamiltonian. ω_i represents the eigenenergy of the ith level of the atom, $a_j(a_j^\dagger)(j = 1, 2)$ the annihilation (creation) operators and g_j the coupling constant between the atom and the ith mode field.

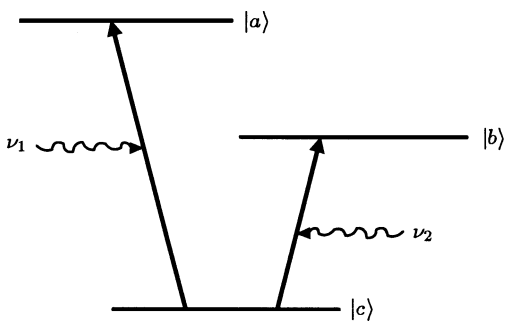


Fig.1. The model of V-type three level atom interacting with double-mode photon field (in case of resonance).

Supposing that the atom moves along the axis z of the cavity with velocity v and interacts with double-mode field. Then the velocity and the structure of the cavity can influence the interaction. The Hamiltonian of the system in the interactional picture can be expressed as[14,15]

$$V^I = g_1 Q_1(z)(a_1|a\rangle\langle c| + a_1^\dagger|c\rangle\langle a|) + g_2 Q_2(z)(a_2|b\rangle\langle c| + a_2^\dagger|c\rangle\langle b|), \tag{4}$$

where $Q_j(z)$ is the form function of cavity mode, which we have defined as TEM_{mnp} mode and can be expressed as[14,15]

$$Q_j(z) \to Q_j(vt) = \sin\left(\frac{p_j \pi v t}{L}\right), \tag{5}$$

where p_j stands for the half-wave number of the jth mode field in the cavity with length L.

We suppose that the atom is initially in the upper state $|a\rangle$ and the field is initially in a certain superposition state

$$|\Psi_F(0)\rangle = \sum_{n_1,n_2} F_{n_1,n_2}|n_1, n_2\rangle. \tag{6}$$

Then the initial state of the system can be written as

$$|\Psi(0)\rangle = \sum_{n_1,n_2} F_{n_1,n_2}|a, n_1, n_2\rangle. \tag{7}$$

In the interactional picture, the state of the system at an arbitrary moment t evolves as

$$|\Psi^I(t)\rangle = \sum_{n_1,n_2} F_{n_1,n_2}[C_{a,n_1,n_2}(t)|a, n_1, n_2\rangle + C_{b,n_1,n_2}(t)|b, n_1+1, n_2-1\rangle + C_{c,n_1,n_2}(t)|c, n_1+1, n_2\rangle]. \tag{8}$$

Substituting Eqs.(4), (5) and (8) into the following Schrödinger equation of interactional picture:

$$\mathrm{i}\frac{\partial}{\partial t}|\Psi^I(t)\rangle = V^I(t)|\Psi^I(t)\rangle, \tag{9}$$

we get the coefficients as follows:

$$C_{a,n_1,n_2} = \frac{1}{g_1^2(n_1+1) + g_2^2 n_2} \times \left\{ g_2^2 n_2 + g_1^2(n_1+1)\cos\left[\frac{\sqrt{g_1^2(n_1+1) + g_2^2 n_2}}{p\pi v/L}\left(1 - \cos\frac{p\pi v t}{L}\right)\right]\right\}, \tag{10}$$

$$C_{b,n_1,n_2} = \frac{g_1 g_2\sqrt{(n_1+1)n_2}}{g_1^2(n_1+1) + g_2^2 n_2}\left[-1 + \cos\frac{\sqrt{g_1^2(n_1+1) + g_2^2 n_2}}{p\pi v/L}\left(1 - \cos\frac{p\pi v t}{L}\right)\right], \tag{11}$$

$$C_{c,n_1,n_2} = -\,\mathrm{i}\frac{g_1\sqrt{n_1+1}}{\sqrt{g_1^2(n_1+1) + g_2^2 n_2}}\sin\left[\frac{\sqrt{g_1^2(n_1+1) + g_2^2 n_2}}{p\pi v/L}\left(1 - \cos\frac{p\pi v t}{L}\right)\right]. \tag{12}$$

In the above equations we have supposed $p_1 = p_2 = p$ in order to simplify the problem.

From Eq.(8) and Eqs. (10)–(12), the state vector of the system at an arbitrary moment t can be determined. It can be seen that the state of the system is obviously influenced by the atomic motion and the structure of field mode.

In order to investigate the squeezing effect of the photon field in the case of atomic motion, we define the following two dimensionless quadrature amplitudes U_1, U_2 for double-mode field:

$$U_1 = \frac{1}{2^{3/2}}\left(a_1^\dagger + a_1 + a_2^\dagger + a_2\right), \tag{13}$$

$$U_2 = \frac{\mathrm{i}}{2^{3/2}}\left(a_1^\dagger - a_1 + a_2^\dagger - a_2\right). \tag{14}$$

They satisfy the commutation relation

$$[U_1, U_2] = \frac{\mathrm{i}}{2}, \tag{15}$$

and the uncertainty relation

$$\langle(\Delta U_1)^2\rangle\langle(\Delta U_2)^2\rangle \geq \frac{1}{16}. \tag{16}$$

For a given state of the system, if the component U_i satisfies the following expression:

$$P_i(t) = \langle(\Delta U_i)^2\rangle - \frac{1}{4} < 0, \tag{17}$$

we say that the ith component of the photon field is squeezed.

Moreover, to simplify the discussion, we introduce the parameter β to describe the effect of atomic motion and structure of field mode hereinafter, namely

$$\beta = \frac{p\pi v}{L}, \tag{18}$$

and let $g_1 = g_2 = 1$.

3. The initial photon field in non-correlative coherent state of double-mode

We firstly discuss the situation that the photon field is initially in the double-mode coherent state, then

$$F_{n_1,n_2} = \frac{\alpha_1^{n_1}\alpha_2^{n_2}}{\sqrt{n_1!n_2!}}\mathrm{e}^{-(|\alpha_1|^2+|\alpha_2|^2)/2}. \tag{19}$$

From Eqs.(8), (13), and (14), we can obtain the expression of $P_i(t)$. Figure 2 shows the evolution of $P_i(t)$ when the atomic motion is neglected. We can see that both quadratures cannot be squeezed and the amplitudes of P_1 and P_2 decrease gradually. To compare with this, we discuss the squeezing effect with atomic motion considered. The quadrature squeezing in case of atomic motion is shown in Fig.3. It can be seen that, as a result of the atomic motion, the quantum fluctuations of both quadratures exhibit the periodical evolution with time and have the same peak values in every period. Furthermore, because of the influence of atomic motion, the quadrature U_1 is squeezed (as shown in Fig.3(a) (c) (d)), and the squeezed degree become deeper as well as the squeezing time longer with the increasing of β. However, the quadrature U_2 cannot be squeezed (as shown in Fig.3(b)). Increasing β can only make the frequency of P_2 increase and the intensity decrease (this is not shown). So we confirm that when the field is initially in non-correlative coherent state the atomic motion makes the quadrature U_1 squeezed. We give the curves of P_1, P_2 versus β at $t = 10$ as shown in Fig.4. We can see with the increasing of β, the squeezed degree of U_1 tends to a steady value, but the non-squeezed state always appears periodically. Meanwhile the amplitude of P_2 is always greater than 0 and decreases gradually.

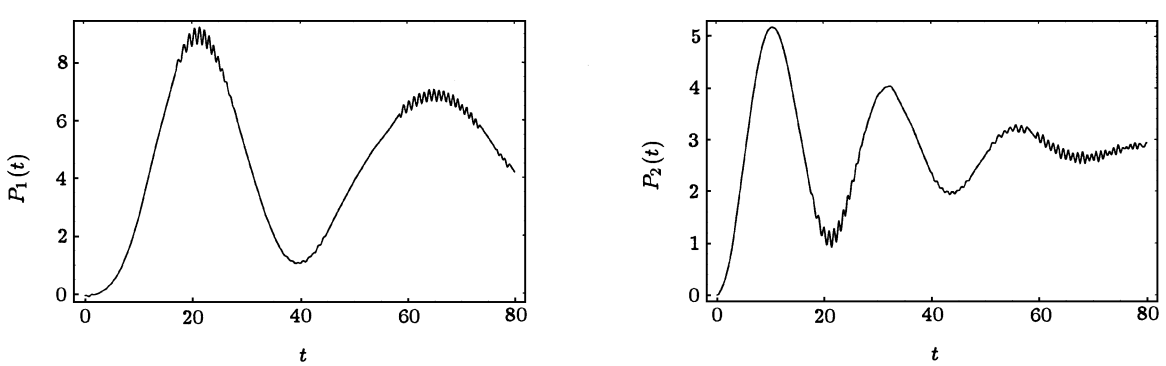


Fig.2. There are no squeezing for both quadratures of the field when the atomic motion is neglected and $\bar{n}_1 = \bar{n}_2 = 5$.

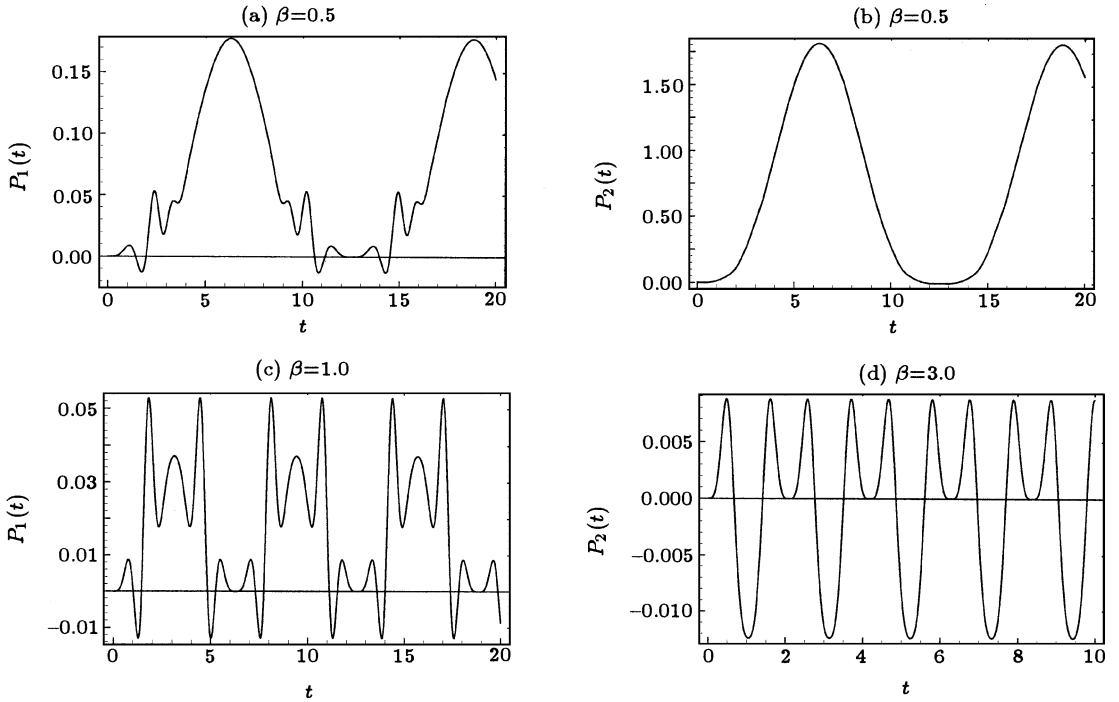


Fig.3. Quadrature squeezing effect for $\bar{n}_1 = \bar{n}_2 = 5$ when the initial field is in non-correlative coherent state of double-mode.

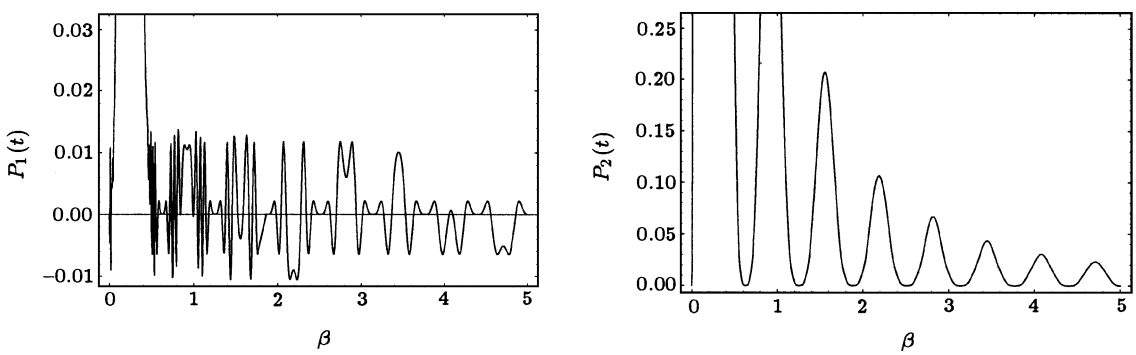


Fig.4. Evolution of quadrature squeezing with β when $\bar{n}_1 = \bar{n}_2 = 20$ and $t = 10$.

4. The initial photon field is vacuum squeezed state of double-mode

Secondly, we investigate the situation that the initial field is in the vacuum squeezed state of double-mode. In this case we have $n_1 = n_2 = n$ and the coefficient F_{n_1,n_2} in Eq.(6) is

$$F_{n,n} = \frac{(\tanh r)^n}{\cosh r} \mathrm{e}^{-\mathrm{i}n\theta}, \tag{20}$$

where r is the squeezing factor and θ is the angle of squeezing direction. To simplify the discussion, we let $\theta = 0$.

From Eqs.(8), (13) and (14), we have

$$\begin{aligned} P_1 =& \langle U_1^2 \rangle - \langle U_1 \rangle^2 - \frac{1}{4} \\ =& \frac{1}{4}\left(\langle a_1^\dagger a_1 \rangle + \langle a_2^\dagger a_2 \rangle + \langle a_1^\dagger a_2^\dagger \rangle + \langle a_1 a_2 \rangle \right), \end{aligned} \tag{21}$$

$$\begin{aligned} P_2 =& \langle U_2^2 \rangle - \langle U_2 \rangle^2 - \frac{1}{4} \\ =& \frac{1}{4}\left(\langle a_1^\dagger a_1 \rangle + \langle a_2^\dagger a_2 \rangle - \langle a_1^\dagger a_2^\dagger \rangle - \langle a_1 a_2 \rangle \right). \end{aligned} \tag{22}$$

According to the above equations, we can investigate the squeezing effect.

Figures 5 and 6 show the evolutions of squeezing with t for different squeezing factor r when the atomic motion is neglected. It can be seen that the squeezing of quadratures U_1 and U_2 appears only when the squeezing factor r takes a small value and the durative

time of squeezing is very short (as shown in Fig.5(a) and Fig.6(a)). When r increases from 0.1 to 0.5, the quadrature U_1 is not squeezed (as shown in Fig.5(b)), while the opportunity of squeezing for U_2 decreases but its squeezing degree deepens a little (as shown in Fig.6(b)). With the value of r increased to 1, the fluctuation of U_1 continuously increases (as shown in Fig.5(c)), while U_2 can only be squeezed in a very short time at the beginning (as shown in Fig.6(c)).

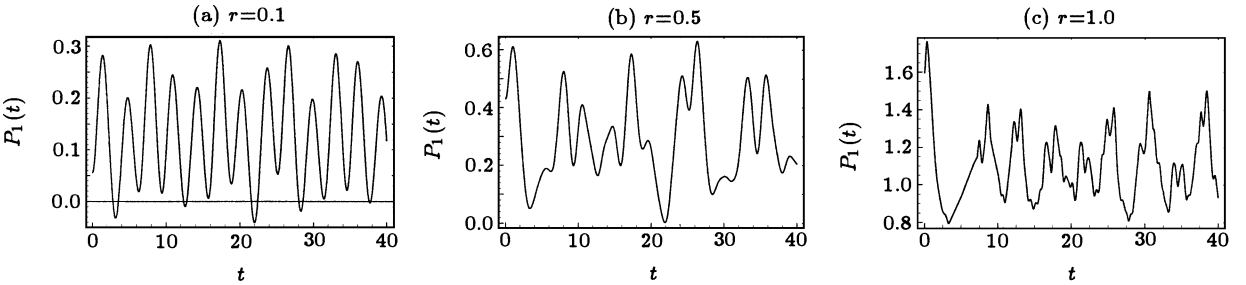


Fig.5. Evolution of squeezing for quadrature U_1 when the initial field is in vacuum squeezed state of double-mode and the atomic motion is neglected.

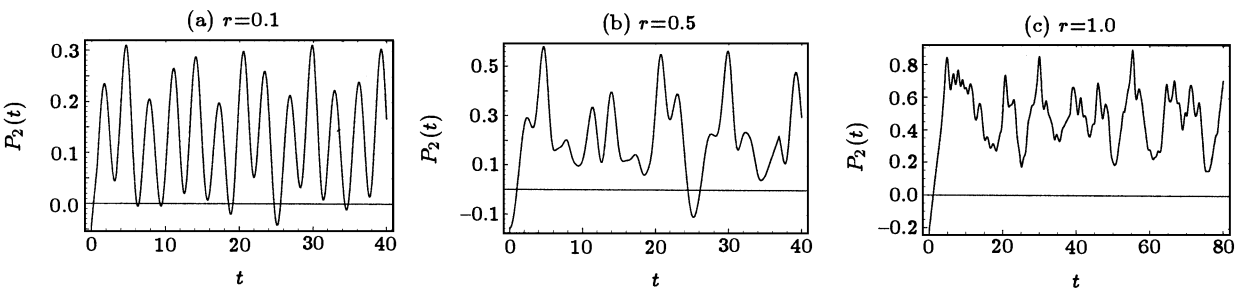


Fig.6. Evolution of squeezing for quadrature U_2 when the initial field is in vacuum squeezed state of double-mode and the atomic motion is neglected.

The squeezing effects with atomic motion considered are shown in Fig.7 and Fig.8. From them we can see the squeezing of U_1 only appears when both β and r have small values. When β increases to 0.8, there is no squeezing for U_1, however, the quadrature U_2 steadily exhibits periodical squeezing, and the duration is longer than that without atomic motion (as shown in Fig.8(b)). When the parameter β is equal to 4.5, we find the quadrature U_2 is completely in squeezed state (as shown in Fig.8(c)). After this the increasing of β can hardly deepen the squeezing degree of U_2.

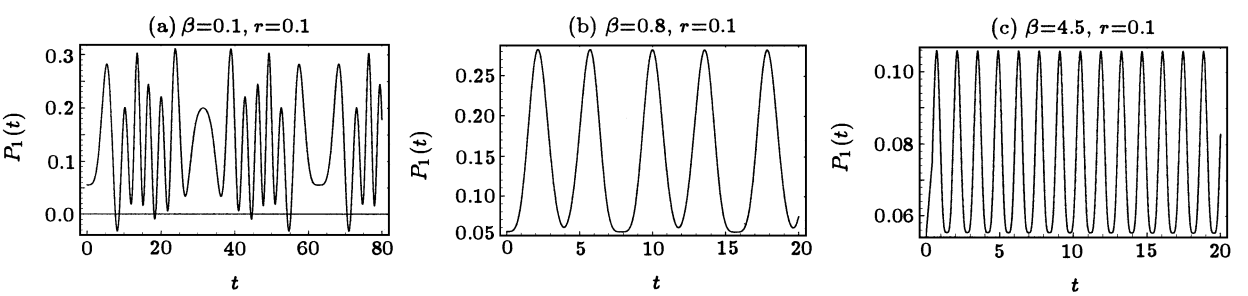


Fig.7. Evolution of squeezing for quadrature U_1 when the initial field is in vacuum squeezed state of double-mode and the atomic motion is considered.

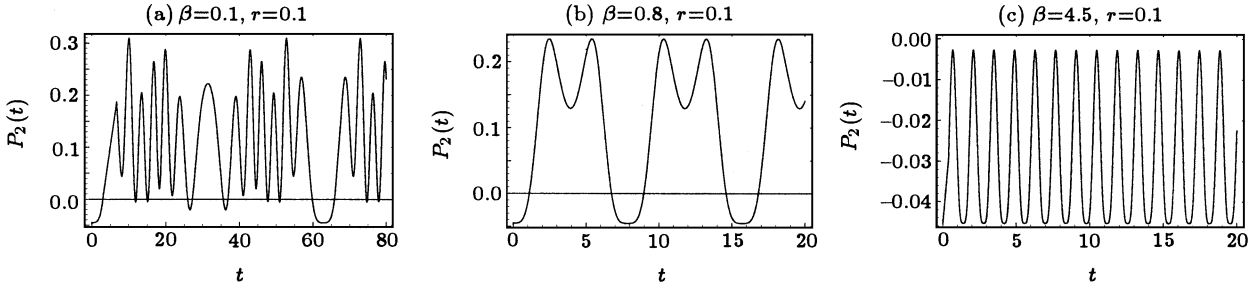


Fig.8. Squeezing for quadrature U_2 when the initial field is in vacuum squeezed state of double-mode and the atomic motion is considered.

The influences of atomic motion and structure of field mode on the quadrature squeezing are shown vividly in the Fig.9 and Fig.10. From the Fig.9, we see that the squeezing effects only appear when squeezing factor is in the range of 0–1 and the squeezing degree is very small. The influence of atomic motion on quadrature U_2 is stronger than on U_1. This can be seen from the Fig.10, in which the maximum squeezing degree of U_2 reaches 0.25 or so at the point $r \approx 1.8$. After this the squeezing degree of U_2 decreases with the increasing of r. We have noticed a flat area in the Fig.10(b) which is always below $P_2 = 0$, and the larger the squeezing factor, the more closer to $P_2 = 0$ the flat area is. Combining with the Fig.8, we can confirm that a steady squeezed light with the optimal squeezed degree can be obtained by choosing appropriate parameters β and r. But the squeezing of quadrature U_1 cannot be obviously seen (as shown in Fig.10(a)).

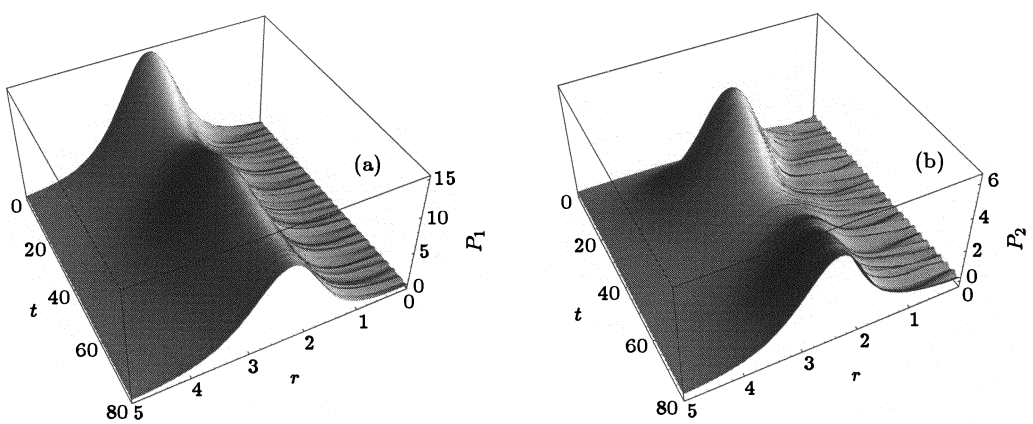


Fig.9. Evolutions of quadrature squeezing with r and t when the initial field is in vacuum squeezed state of double-mode and the atomic motion is neglected.

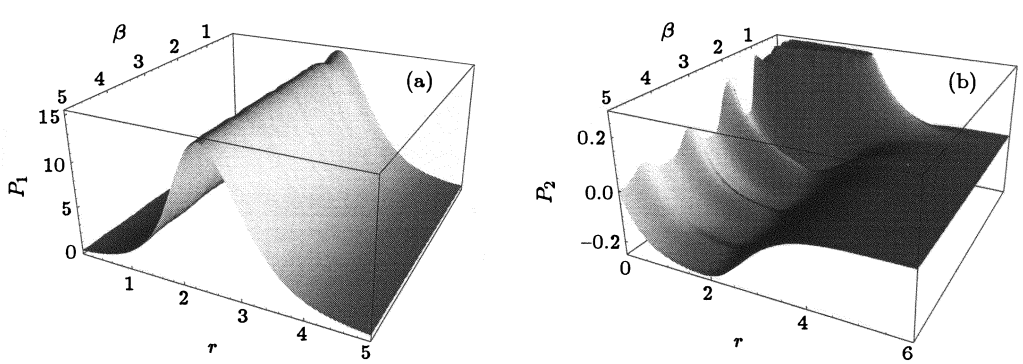


Fig.10. Evolutions of quadrature squeezing with r and β for $t = 5$ when the initial field is in vacuum squeezed state of double-mode.

5. Conclusion

From the above investigation, we reach the conclusion that the atomic motion has prominent impact upon the squeezing property of double-mode photon field. Firstly, when the initial photon field is in non-correlative coherent state of double-mode, the atomic motion results in the squeezing of quadrature U_1 and its squeezing degree increases with the increasing of β (we have shown that the squeezing effect exhibits for neither U_1 nor U_2 when the atomic motion is absent in this initial state). Secondly, when the initial photon field is in the vacuum squeezed state of double-mode, the atomic motion primarily results in the squeezing of quadrature U_2, and the squeezing degree changes with both β and squeezing factor r. What is most important is that we can obtain a steady squeezed light by choosing an appropriate parameter β for a given squeezing factor r. Lastly, by researching on the squeezing effect of double-mode field in the above two situations, we confirm it is a better way to obtain the squeezed light using a moving atom interacting with a photon field of vacuum squeezed state than with one of double-mode coherent state.

In short, we have investigated the influence of atomic motion and structure mode on squeezing effect of the double-mode field on conditions of initial non-correlative coherent state and vacuum squeezed state of double-mode, respectively, and reached some significant results as above-mentioned. We believe these results will be useful to the preparation of squeezed light and to the research of the properties of interaction between the atom and field.

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