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Linear entropies in the Jaynes–Cummings model with intensity-dependent coupling in a phase-damped cavity*

Zhou Qing-Chun(周青春)[a)b)†] and Zhu Shi-Ning(祝世宁)[b)]

[a)] *Department of Mathematics and Physics, Jiangsu University of Science and Technology, Zhenjiang 212003, China*

[b)] *Department of Physics, Nanjing University, Nanjing 210093, China*

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We investigate the evolution of a quantum system described by the Jaynes–Cummings model with an arbitrary form of intensity-dependent coupling by displaying the linear entropies of the atom, field and atom–field system in the large detuning approximation. The cavity field is assumed to be coupled to a reservoir with a phase-damping coupling. The effects of cavity phase damping on the entanglement and coherence loss of such a system are studied.

Keywords: phase damping, JCM with intensity-dependent coupling, linear entropy, coherence loss
PACC: 4250

1. Introduction

The usual Jaynes–Cummings model (JCM) with intensity-dependent coupling was proposed by Buck and Sukumar[1] to reflect the dependence of the interaction between an atom and a single-mode optical field. The model was generalized by Buzek[2] to include an arbitrary form of intensity-dependent coupling. Collapses and revivals of the atomic inversion,[3] the light squeezing,[4] phase properties,[5] field entropy[6] and entanglement between the atom and the field[7] in the model have been studied by some authors. These works ignored the damping of the cavity field. The damping is important, however, because realistic cavities have finite Q and measuring processes will cause photons to leak. Recent studies have shown that amplitude damping of the cavity field affects markedly the behaviour of the atom–field system.[8−10] Li and Fang handled the dynamics of the usual JCM with intensity-dependent coupling in the dispersive approximation in the presence of energy dissipation of the cavity by calculating the field linear entropy.[10] But the atomic linear entropy in that situation is difficult to work out. Another model describing a system interacting with an environment is the phase-damping model.[11−13] One advantage of this model is that its mathematical treatment is straightforward while capable of revealing the decoherence process. In this paper, we will investigate the dynamical properties of the JCM with an arbitrary form of intensity-dependent coupling by considering the time evolution of its linear entropy in a phase-damped cavity.

2. Effective Hamiltonian

We take the Hamiltonian for the JCM with an arbitrary form of intensity-dependent coupling in the dipole and rotating-wave approximations to be ($\hbar = 1$)[2]

$$H = \omega a^{+}a + \frac{1}{2}\omega_{eg}\sigma_z + g[\sigma_{+}af(N) + f(N)a^{+}\sigma_{-}], \quad (1)$$

where σ_z, $\sigma_{\pm}$ are the atomic pseudospin operators, $a(a^{+})$ is the creation (annihilation) operator of the single-mode optical field, $N = a^{+}a$ is the photon number operator, $f(N)$ is an arbitrary function of N, and ω and ω_{eg} are the frequencies of the field and the atom, respectively. The constant g is assumed to be real. Eq.(1) describes, respectively, the JCM and the usual JCM with intensity-dependent coupling for $f(N) = 1$ and $f(N) = \sqrt{N}$. In an invariant subspace spanned

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†E-mail: dg0222076@yahoo.com

by $|e\rangle\otimes|n\rangle$ and $|g\rangle\otimes|n+1\rangle$, the eigenvalues of H are

$$E_{\pm}=\left(n+\frac{1}{2}\right)\omega\pm\frac{1}{2}\Delta_n, \tag{2a}$$

where

$$\Delta_n=[\delta^2+4g^2(n+1)f^2(n+1)]^{1/2}, \tag{2b}$$

with $\delta=\omega_{eg}-\omega$ (e and g denote the excited and ground levels, respectively) being the detuning of the field. The corresponding eigenstates are

$$|+,n\rangle=\sin\theta_n\,|e,n\rangle+\cos\theta_n\,|g,n+1\rangle\,, \tag{3a}$$

$$|-,n\rangle=\cos\theta_n\,|e,n\rangle-\sin\theta_n\,|g,n+1\rangle\,, \tag{3b}$$

with

$$\sin\theta_n=2g\left[\frac{(n+1)f^2(n+1)}{(\Delta_n-\delta)^2+4g^2(n+1)f^2(n+1)}\right]^{1/2}, \tag{4a}$$

$$\cos\theta_n=(\Delta_n-\delta)\times\left[\frac{1}{(\Delta_n-\delta)^2+4g^2(n+1)f^2(n+1)}\right]^{1/2}. \tag{4b}$$

Following Peixoto *et al*,[8] in the large-detuning approximation

$$\frac{\delta}{g}\gg[(n+1)f^2(n+1)]^{1/2}, \tag{5}$$

we arrive at the effective Hamiltonian

$$\begin{aligned}H_{\rm eff}=&\omega a^+a+\frac{1}{2}(\omega+\delta)\sigma_z\\&+\Omega[(a^+a+1)f^2(a^+a+1)\,|e\rangle\langle e|\\&-a^+af^2(a^+a)\,|g\rangle\langle g|],\end{aligned} \tag{6}$$

with $\Omega\equiv g^2/\delta$.

3. Density operator and linear entropy

In the interaction picture, the master equation for the density operator of the atom–field system can be written as

$$\begin{aligned}\frac{{\rm d}\rho(t)}{{\rm d}t}=&-{\rm i}\Omega[(a^+a+1)f^2(a^+a+1)\,|e\rangle\langle e|\\&-(a^+a)f^2(a^+a)\,|g\rangle\langle g|\,,\rho(t)]\\&+\lambda[2a^+a\rho(t)a^+a-\rho(t)a^+aa^+a\\&-a^+aa^+a\rho(t)].\end{aligned} \tag{7}$$

The last term in Eq.(7) describes the phase damping of the cavity field, and λ is related to temperature by $\lambda=\lambda' k_{\rm B}T$($\lambda'$ is the constant of decay, and $k_{\rm B}$ is the Boltzmann constant).[13]

If we introduce superoperators $\mathcal{M}$ and $\mathcal{P}$ such that

$$\mathcal{M}O=a^+aO,\ \mathcal{P}O=Oa^+a,$$

where O is an arbitrary operator, then the Liouvillians corresponding to the matrix elements ρ_{gg}, ρ_{ee}, ρ_{eg} and ρ_{ge} take the following forms:

$$\begin{aligned}\mathcal{L}_{ee}=&-{\rm i}\Omega[(\mathcal{M}+1)f^2(\mathcal{M}+1)-(\mathcal{P}+1)f^2(\mathcal{P}+1)]\\&-\lambda(\mathcal{M}^2+\mathcal{P}^2-2\mathcal{M}\mathcal{P}),\end{aligned} \tag{8a}$$

$$\begin{aligned}\mathcal{L}_{gg}=&{\rm i}\Omega[\mathcal{M}f^2(\mathcal{M})-\mathcal{P}f^2(\mathcal{P})]\\&-\lambda(\mathcal{M}^2+\mathcal{P}^2-2\mathcal{M}\mathcal{P}),\end{aligned} \tag{8b}$$

$$\begin{aligned}\mathcal{L}_{eg}=&-{\rm i}\Omega[(\mathcal{M}+1)f^2(\mathcal{M}+1)+\mathcal{P}f^2(\mathcal{P})]\\&-\lambda(\mathcal{M}^2+\mathcal{P}^2-2\mathcal{M}\mathcal{P}),\end{aligned} \tag{8c}$$

$$\begin{aligned}\mathcal{L}_{ge}=&{\rm i}\Omega[\mathcal{M}f^2(\mathcal{M})+(\mathcal{P}+1)f^2(\mathcal{P}+1)]\\&-\lambda(\mathcal{M}^2+\mathcal{P}^2-2\mathcal{M}\mathcal{P}).\end{aligned} \tag{8d}$$

The master equation (7) can be solved to obtain

$$\rho_{ij}(t)={\rm e}^{\mathcal{L}_{ij}t}\rho_{ij}(0)\ \ (i,j=e,g). \tag{9}$$

We suppose that the atom is initially in a superposition of the excited and the ground state

$$|\Psi_A(0)\rangle=C_e\,|e\rangle+C_g\,|g\rangle\,, \tag{10}$$

and the field in a coherent state

$$|\Psi_F(0)\rangle=\sum_{n=0}^{\infty}F_n\,|n\rangle, \tag{11}$$

where $F_n=\exp(-|\alpha|^2/2)\dfrac{\alpha^n}{\sqrt{n!}}$, and α is a complex number (we will take it to be real for simplicity). Suppose that at the moment the interaction between the atom and the field starts, the state of the atom–field system is the direct product of Eqs.(10) and (11), then we have

$$\rho_{ee}(0)=|C_e|^2\sum_{m,n=0}^{\infty}F_mF_n^*\,|m\rangle\langle n|, \tag{12a}$$

$$\rho_{gg}(0)=|C_g|^2\sum_{m,n=0}^{\infty}F_mF_n^*\,|m\rangle\langle n|, \tag{12b}$$

$$\rho_{eg}(0)=C_eC_g^*\sum_{m,n=0}^{\infty}F_mF_n^*\,|m\rangle\langle n|, \tag{12c}$$

$$\rho_{ge}(0)=C_gC_{\rm e}^*\sum_{m,n=0}^{\infty}F_mF_n^*\,|m\rangle\langle n|. \tag{12d}$$

Substituting Eqs.(8) and (12) into Eq.(9), we obtain

$$\rho_{ee}(t) = |C_e|^2 \sum_{m,n=0}^{\infty} F_m F_n^* \exp\left\{-\mathrm{i}\Omega t\left[(m+1)f^2(m+1) - (n+1)f^2(n+1)\right] - \lambda t(m-n)^2\right\} |m\rangle\langle n|, \quad (13a)$$

$$\rho_{gg}(t) = |C_g|^2 \sum_{m,n=0}^{\infty} F_m F_n^* \exp\left\{\mathrm{i}\Omega t\left[mf^2(m) - nf^2(n)\right] - \lambda t(m-n)^2\right\} |m\rangle\langle n|, \quad (13b)$$

$$\rho_{eg}(t) = C_e C_g^* \sum_{m,n=0}^{\infty} F_m F_n^* \exp\left\{-\mathrm{i}\Omega t\left[(m+1)f^2(m+1) + nf^2(n)\right] - \lambda t(m-n)^2\right\} |m\rangle\langle n|, \quad (13c)$$

$$\rho_{ge}(t) = C_g C_{\mathrm{e}}^* \sum_{m,n=0}^{\infty} F_m F_n^* \exp\left\{\mathrm{i}\Omega t\left[(n+1)f^2(n+1) + mf^2(m)\right] - \lambda t(m-n)^2\right\} |m\rangle\langle n|. \quad (13d)$$

The reduced atomic density operator can be obtained from Eqs.(13) by

$$\begin{aligned}\rho_A(t) =& \mathrm{Tr}_F\left[\rho(t)\right] \\ =& |C_e|^2 |e\rangle\langle e| + |C_g|^2 |g\rangle\langle g| + \left\{C_e C_g^* \sum_{n=0}^{\infty} |F_n|^2 \mathrm{e}^{-\mathrm{i}\Omega t\left[(n+1)f^2(n+1) + nf^2(n)\right]} |e\rangle\langle g| + \mathrm{h.c.}\right\}. \end{aligned} \quad (14)$$

The reduced field density operator is

$$\rho_F(t) = \rho_{ee}(t) + \rho_{gg}(t). \quad (15)$$

The linear entropies of the system, of the atom and of the field are, respectively,

$$S(t) = 1 - \mathrm{Tr}[\rho^2] = 1 - \sum_{m,n} |F_n F_m|^2 \mathrm{e}^{-2\lambda t(m-n)^2}, \quad (16)$$

$$S_A(t) = 1 - \mathrm{Tr}[\rho_A^2] = 2|C_e C_g|^2 \left\{1 - \sum_{m,n} |F_m F_n|^2 \mathrm{e}^{\mathrm{i}\Omega t[(m+1)f^2(m+1) + mf^2(m) - (n+1)f^2(n+1) - nf^2(n)]}\right\}, \quad (17)$$

$$\begin{aligned} S_F(t) =& 1 - \mathrm{Tr}[\rho_F^2] = 1 - \sum_{m,n} |F_m F_n|^2 \mathrm{e}^{-2\lambda t(m-n)^2} \\ & \times \left\{1 - 4|C_e C_g|^2 \sin^2\left[\frac{(m+1)f^2(m+1) + mf^2(m) - (n+1)f^2(n+1) - nf^2(n)}{2}\right] \Omega t\right\}. \end{aligned} \quad (18)$$

4. Results and discussions

Numerical calculations can be performed based on Eqs.(16)–(18), but let us first have a look at some general results. We conclude from Eq.(16) that the time evolution of the entropy of the total system does not depend on the form of the function $f(N)$, and it is independent of the initial atomic state also. The linear entropy of the atom and that of the field, however, are determined by both how the atom–field coupling relates to the field intensity and what the initial atomic state is. If the atom is initially in the excited or ground state, the atomic linear entropy is always zero, and the field linear entropy and the linear entropy of the atom–field system are equal. This is because, in the large detuning approximation, the atom dose not transit between the two atomic levels, and remain on the excited level or on the ground level, which is a pure atomic state. In this case, the atom and the field are not entangled, and the field loses coherence monotonically. For the initial atomic state $(|e\rangle + |g\rangle)/\sqrt{2}$, we display in Fig.1 the evolutions of $S(t)$, $S_A(t)$, $S_F(t)$ with the scaled time Ωt in the special cases $f(N) = 1$, $f(N) = N^{1/2}$, $f(N) = N$, and $f(N) = N^{3/2}$, respectively, for the same parameters $\alpha = 1$, $\lambda/\Omega = 0.1$. We see that the atom–field system has the same time-evolution curve of $S(t)$ for all the cases and the field linear entropy has damped oscillations. The atomic linear entropy turns back to zero periodically with a period π/Ω, $\pi/2\Omega$, π/Ω, $\pi/8\Omega$ in Figs.1(a)–(d), respectively. The atom is disentangled from the field at times when $S_A(t) = 0$. In the case of amplitude damping of the field, the atomic coherence is affected by the reservoir through the factor $\Gamma(t)$, the real part of the argument of the exponential in ρ_{eg}, and lost completely as time increases. The atomic coherence, however, is independent of the cavity decay parameter λ in Eq.(17) in the presence of phase damping of

the field, and lost and regained periodically even as $t \to \infty$. According to Eq.(14), only diagonal matrix elements of ρ_{ee}, ρ_{gg}, ρ_{eg} and ρ_{ge} in the photon number representation contribute to the reduced atomic density operator, while phase damping has no effect on these diagonal matrix elements from Eqs.(13). This explains why the atom is entangled with and disentangled from the field as if phase dissipation of the cavity did not exist. When the atom arrives at the zero value of its linear entropy (disentanglement occurs), the field linear entropy takes a local minimum, which is just the value of the linear entropy of the atom–field system at the same moment. The oscillations of $S_F(t)$ between local maxima and local minima are due to its interaction with the atom, for $S_F(t)$ would evolve as $S(t)$ without such interaction and oscillate as $S_A(t)$ without damping. The phase damping causes the amplitude suppression of the oscillations of $S_F(t)$. At the maxima, the field loses coherence, and partially gain coherence at the minima. But the field loses coherence more than it gains due to phase damping, and finally completely loses coherence and falls into a statistically mixed state. Both $S(t)$ and $S_F(t)$ approach the same steady value $S^{\max}$ as $t \to \infty$ for a given amplitude of the coherent field α. Apart from the initial moment, neither the total system nor the field subsystem can be in a pure state. The atom, however, returns to a pure state regularly. These results are quite different from those in the JCM with amplitude damping of the cavity field.[8] There, the field linear entropy returns to zero periodically while the atom will never be in a pure state as long as the interaction between the field and the atom begins.

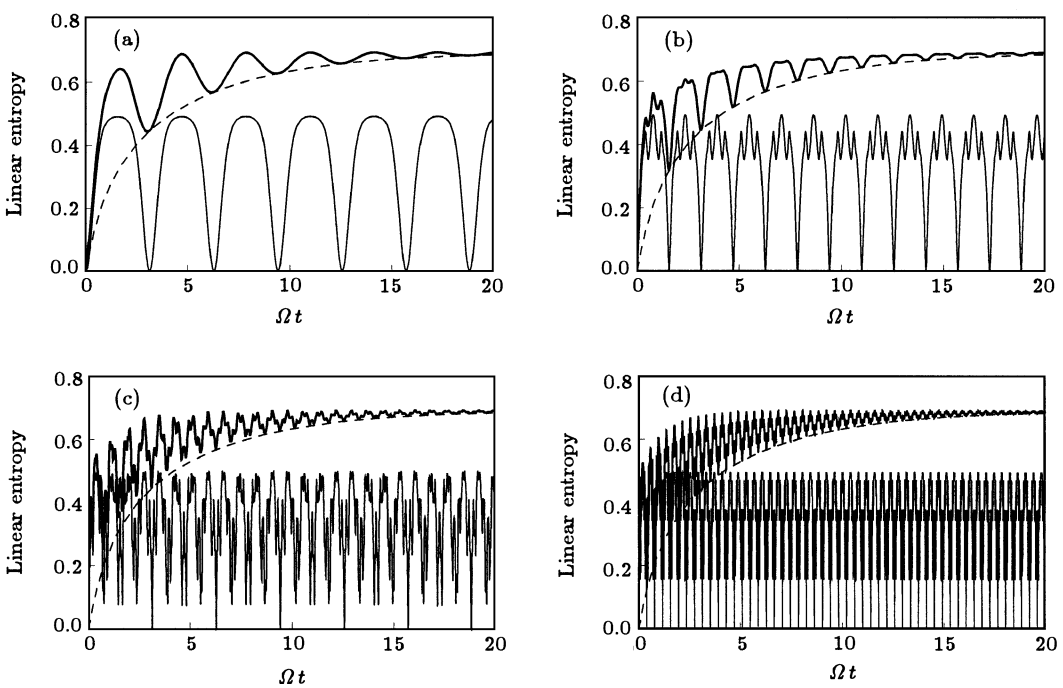


Fig.1. Time evolutions of $S(t)$ (dashed line), $S_A(t)$ (light solid line), and $S_F(t)$ (heavy solid line) for (a) $f(N)$=1, (b) $f(N) = N^{1/2}$, (c)$f(N) = N$, and (d) $f(N) = N^{3/2}$. Parameters: α=1, λ/Ω=0.1, $C_e = C_g$=$1/\sqrt{2}$.

Let us concentrate on the case of $f(N) = N^{1/2}$, for it is the most frequently used intensity-dependent coupling. If the initial field is very weak ($\alpha \ll 1$), we obtain the approximate expressions for $S(t)$, $S_A(t)$ and $S_F(t)$ as follows:

$$\begin{aligned} S(t) =& 2\alpha^2(1 - \mathrm{e}^{-2\lambda t}), \\ S_A(t) =& 8\alpha^2 \left|C_e C_g\right|^2 \sin^2(2\Omega t), \\ S_F(t) =& 2\alpha^2(1 - \mathrm{e}^{-2\lambda t}) \\ &+ 8\alpha^2 \left|C_e C_g\right|^2 \mathrm{e}^{-2\lambda t} \sin^2(2\Omega t). \end{aligned} \quad (19)$$

Both the field linear entropy and the atomic linear entropy reach their local maxima at times $T_0 = (n + 1/2)(\pi/2\Omega)$ $(n = 0, 1, 2, ...)$. For the case $C_e = C_g = 1/\sqrt{2}$, these maxima are of the same value $2\alpha^2$. Fig.2(a) shows the evolutions of the linear entropies based on Eqs.(16)–(18) for $\alpha = 0.1$, $C_e = C_g = 1/\sqrt{2}$. It is clear that $S_F(t)$ oscillates harmonically and the evolution of $S_F(t)$ is the evolution of S plus damped harmonic oscillations, consistent with Eqs.(19). As α

increases, the maxima of $S_F(t)$ increase more rapidly than those of $S_A(t)$, and the maxima of $S_F(t)$ become larger then those of $S_A(t)$ for α large enough (Fig.2(b)).

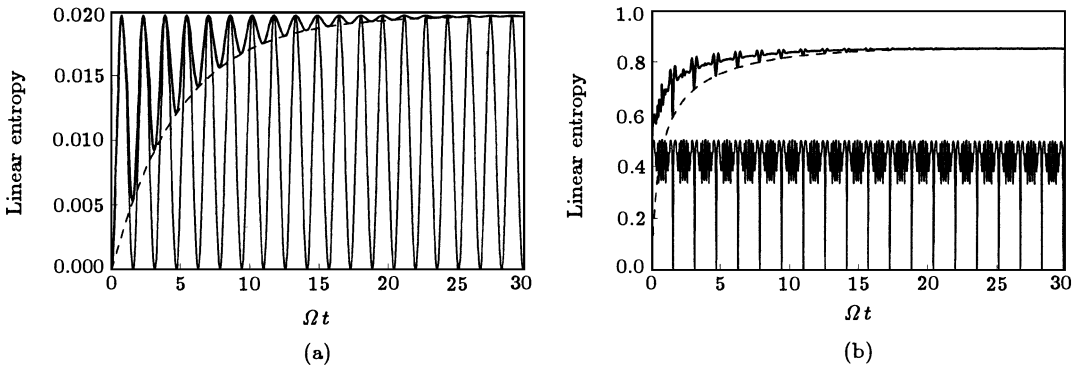


Fig.2. Time evolutions of $S(t)$ (dashed line), $S_A(t)$ (light solid line), and $S_F(t)$ (heavy solid line) in the usual JCM with intensity-dependent coupling for λ/Ω=0.1 and different α ($C_e = C_g = 1/\sqrt{2}$). (a) $\alpha = 0.1$, (b) α=2.

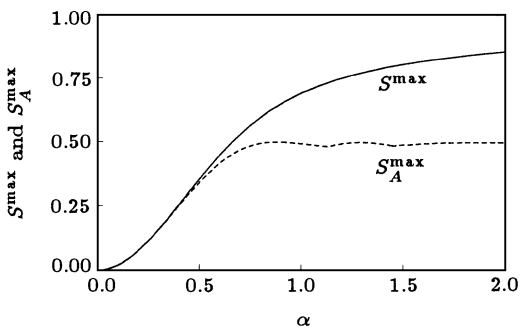


Fig.3. Asymptotic linear entropy of the atom–field system and that of the atom as a function of α (λ/Ω=0.1, $C_e = C_g = 1/\sqrt{2}$, $f(N) = N^{1/2}$).

From Fig.2, we also see other effects of the intensity of the initial coherent field. The time for the $S(t)$ to get to its steady value shortens for large α, which is a reflection of the argument that the intensity of the coherent field is a measure of its "classicality". Eqs.(19) tell us explicitly that the increasing rate of the linear entropy of the atom–field system is proportional to α^2 at the beginning of the interaction. Small oscillations take place during each period of the abovementioned entanglement and disentanglement in both $S(t)$ and $S_F(t)$. This phenomenon is the result of the nonlinearity of the interaction between the atom and the field. Figure 3 shows that the steady value of $S(t)$ and $S_F(t)$ (both equal to $S^{\max}$) rises with increasing α. For large $\alpha(\alpha > 4)$, the atom, except at time $t_0 = n\pi/2\Omega$ $(n = 0, 1, 2, ...)$, is almost always in mixed states with $S_A(t \neq t_0) \approx 0.5$, characteristic of a statistical mixture. It is obvious from Fig.2 that the entanglement of the atom with the field goes from harmonic to inharmonic as the intensity of the field grows.

5. Conclusion

We have studied the linear entropy properties of the field, the atom and the combined system in a JCM with intensity-dependent coupling in the presence of phase damping of the cavity field. The following conclusions can be drawn from the above discussions. (1) The time evolution of the linear entropy of the combined system depends on the phase damping parameter λ, and it is not affected by the initial atomic state and the form of intensity-dependent coupling. (2) The atomic linear entropy evolves periodically with time, and it is not influenced by phase damping of the cavity field. (3) During its time evolution, the cavity field will not turn back to a pure state; the coherence of the field is lost and regained through its interaction with the atom, and completely lost finally due to cavity damping.

References

[1] Buck B and Sukumar C V 1981 *Phys. Lett.* A **81** 132
[2] Buzek V 1990 *J. Mod. Opt.* **37** 1033
[3] Zhou P, Hu Z L and Peng J S 1992 *J. Mod. Opt.* **39** 49
[4] Buzek V 1989 *J. Mod. Opt.* **36** 1151
[5] Zhou P, Peng J S and Li G X 1993 *Acta Opt. Sin.* **13** 444
[6] Fang M F 1995 *Acta Opt. Sin.* **15** 296
[7] Huang Y X and Guo G C 1999 *Acta Phys. Sin.* **48** 49 (in Chinese)
[8] Peixoto J G and Names M C 1999 *Phys. Rev.* A **59** 3918
[9] Zhou L *et al* 2001 *Phys. Lett.* A **284** 156
[10] Li C X and Fang M F 2003 *Chin. Phys.* **12** 866
[11] Zhang L H *et al* 2002 *Acta Phys. Sin.* **51** 541 (in Chinese)
[12] Zhang L H *et al* 2003 *Acta Phys. Sin.* **52** 1168 (in Chinese)
[13] Walls D F, Milburn G J 1985 *Phys. Rev.* A **31** 2403