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A complementary least-mean-square algorithm of adaptive filtering for SQUID based magnetocardiography*

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We present a complementary least-mean-square algorithm of adaptive filtering for SQUID-based magnetocardiography, in which both rapid convergence and fine tracking are realized by switching the weight parameters back and forth between two filters according to the least mean square principle.

Keywords: magnetocardiography, convergence, tracking
PACC: 0650D, 0755

1. Introduction

In adaptive filtering, the least-mean-square (LMS) algorithm is very popular for its simplicity and efficiency to eliminate environment noise. The convergence and the tracking behaviours of an algorithm are two fundamental factors of merit in evaluating its performance for signal recovery.[1] In the case of stationary environment, a good tracking that provides a small steady-state error is important, while in a non-stationary situation a fast convergence is necessary to have the tracking back in a short time. Usually, normalized LMS algorithm with a proper parameter setting provides both a fairly good tracking and a rather fast convergence.[2] Generally speaking, the optimum parameters in an algorithm vary from place to place and time to time, according to the situations of input signal and background noise properties.

In SQUID-based magnetocardiography (MCG) as well as other biomagnetic measurements, the magnetic field signals are extremely weak in comparison with ambient field variations. In this situation, the major issue is to obtain a good tracking to recover the signal sequence with a steady-state error as small as possible. In LMS algorithms, the smallest possible steady-state error corresponds, most likely, to a small step-size. As a result, it makes the convergence speed of the algorithms severely slow. Moreover, in MCG measurements for example, the dwelling time of data acquisition must be limited to a patient-acceptable length typically 30 periods of heart beating or so, in which the signal and reference sequences could be too short to match the needs of an algorithm processing with small step-size. Obviously, both good tracking and fast convergence are needed when a real time trace of cardiac signal is required.

Recently, the above problem has been partially solved by Kong.[3] The data-reusing technique works well if the real time trace of the MCG signal is not required. In this paper, we present a complementary LMS algorithm for SQUID-based MCG measurements, in which both tracking property and convergence are concerned.

2. Complementary normalized LMS algorithm

2.1. Standard normalized LMS algorithm

An adaptive filter has two input channels, signal

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and reference channels as depicted schematically in Fig.1.[4] The signal input $S(k)$ is composed of a vertical component of the magnetic field produced by patient cardiac activities and a background noise. The reference input $X(k)$ contains merely the background noise assumed to be correlated to that in the signal input. Here the independent intrinsic noises due to sensors themselves are supposed to be small and thus not taken into account. The number k denotes time index of the input data acquisition.

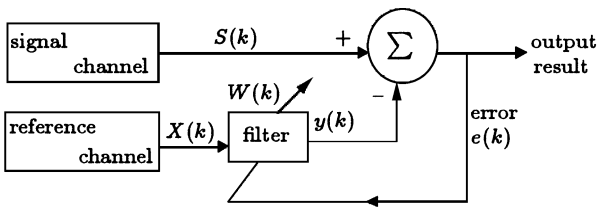


Fig.1. A schematic block diagram of an adaptive filter.

The filter output $y(k)$ is given by the inner product of vectors $X(k)$ and $W(k)$[5]

$$y(k) = W^{\mathrm{T}}(k)X(k), \tag{1}$$

where $X(k) = [x_k, x_{k-1}, ..., x_{k-M+1}]^{\mathrm{T}}$ is the reference vector and $W(k) = [w_k, w_{k-1}, ..., w_{k-M+1}]^{\mathrm{T}}$ the filter weight vector, in which the superscript T stands for transposition of vectors and M is the filter length. The error is obtained as

$$e(k) = S(k) - y(k), \tag{2}$$

which is uncorrelated with the input noise. The weight update equation is given by

$$W(k+1) = W(k) + \mu(k)X(k)e(k). \tag{3}$$

In the normalized LMS (NLMS) algorithm the step-size parameter of the adaptive filter at time k is defined by

$$\mu(k) = \frac{\alpha}{X(k)^{\mathrm{T}}X(k) + \gamma}, \tag{4}$$

where α is a normalized adaptive constant and γ is a very small positive number, by which the divergence of Eq.(4) can be avoided when $X^{\mathrm{T}}(k)X(k)$ =0. Equation (3) shows that the updated value of filter weight $W(k+1)$ is determined by $W(k)$, $\mu(k)$, $e(k)$ and $X(k)$. The filter will not work well or even diverge if α and γ are not properly chosen.

Before using the algorithm, special attention has to be paid to the choice of α and γ values for a suitable step-size μ, which controls the convergence speed of the algorithm in finding the optimal solution. Generally, the larger the μ, the faster the convergence. However, too large a value of μ makes a steady noise worse or even may result in an unstable algorithm. In addition, the filter length M, or the dimension of weight space $W(k)$, should be selected so that the filter models the regular features of a set of input data best.

2.2. Complementary LMS algorithm

In MCG measurement, we are faced with short sequences of data, which are extremely weak compared with ambient disturbances. Therefore, it is desirable to have an adaptive filtering with both fine tracking and rapid convergence in the shortest time for catching the real cardiac signals. Such an adaptive filter may be realized by simultaneously employing two filters (denoted by A and B respectively) operating in parallel and complementing each other with different step-size.

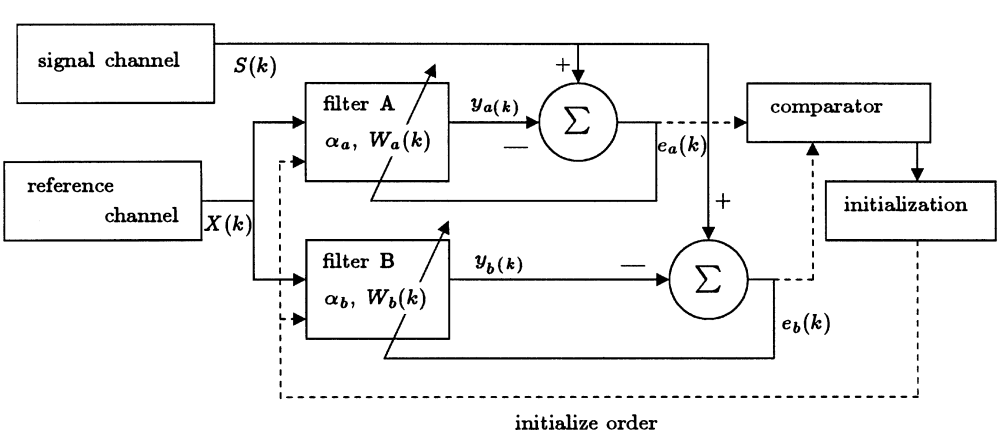


Fig.2. Block diagram of the complementary adaptive filter.

The algorithm in our complementary adaptive filter can be described with a block diagram as shown in Fig.2. There are two weights and two errors denoted by subscripts a and b with respect to the two filters A and B. The processing formulas for the error and update weights are the same as before:

$$e_{a,b}(k) = S_{a,b}(k) - W_{a,b}^{\mathrm{T}}(k)X(k), \tag{5}$$

$$\begin{aligned} &W_{a,b}(k+1) \\ =&W_{a,b}(k) + \frac{\alpha_{a,b}}{\gamma_{a,b} + X^{\mathrm{T}}(k)X(k)}X(k)e_{a,b}(k), \end{aligned} \tag{6}$$

when initializing command is not provided.

We carefully choose α_a and α_b ($\alpha_a < \alpha_b$), so that the filter A has a fine tracking ability, while filter B has high convergence speed. Their error outputs $e_a(k)$ and $e_b(k)$ are fed simultaneously to a comparator and will trigger an initialization device in a way according to the result of comparison between $e_a^2(k)$ and $e_b^2(k)$.

In order to describe how the initialization device works, we define two state quality factors $q_{a,b}(m)$ as

$$q_a(m) = \begin{cases} 1 & \text{if } \sum_{i=m}^{m+l} e_a^2(i) < \sum_{i=m}^{m+l} e_b^2(i), \\ 0 & \text{otherwise;} \end{cases} \tag{7}$$

and

$$q_a(m) = \begin{cases} 1 & \text{if } \sum_{i=m}^{m+l} e_b^2(i) < \sum_{i=m}^{m+l} e_a^2(i), \\ 0 & \text{otherwise,} \end{cases} \tag{8}$$

where l is the length of summation. The meaning of the above definition is clear: if $q_a(m)$=1, the state of filter A in the weight space W is better than that of filter B, and vice versa. Obviously $q_a(m) \times q_b(m) = 0$. In addition, we define two more factors—cumulative quality factors as

$$Q_{a,b}(k) = \prod_{j=1}^{J} q_{a,b}(k - jl), \tag{9}$$

where J is the comparison length. The definition of the factors (9) tells us that if one of the factors equals unity, say $Q_a(k)$ =1, we have $q_a(k - jl) = 1$ for all $j = 1, 2, 3, \ldots, J$, which implies that the states of filter A are continually better than those of filter B in successive comparisons with length J (J times). In this case, the comparator will initialize the weight vector W_b of filter B by update vector W_a of filter A and leave the value of W_a of filter A unchanged. A similar processing will be carried out for filter A in the case of $Q_b(k)$ =1. Otherwise, when $Q_a(k) = Q_b(k)$=0, the recognition for the better weight parameters cannot be made statistically by the comparator and there is nothing to happen in both filters A and B.

The above processing may be expressed by two modified update weight equations written as

$$\begin{aligned} W_{a,b}(k+1) =&[1 - Q_{b,a}(k)]W_{a,b}(k) \\ &+ Q_{b,a}(k)W_{b,a}(k) \\ &+ \frac{\alpha_{a,b}}{\gamma_{a,b} + X^{\mathrm{T}}(k)X(k)}X(k)e_{a,b}(k), \end{aligned} \tag{10}$$

when initializing command is provided.

We may elucidate the above equations most succinctly in W-space as shown in Fig.3. In the figure there are several circles representing subspaces of W. Each of the circles has the same mean square error $e^2(i)$, and at the central point C has its minimum value. Assuming that the both filters A and B are initialized at the same point $W(0)$, as time elapses, we obtain two random paths $W_a(i)$ and $W_b(i)$, towards the centre C in the statistical sense, developed by filters A and B respectively. Meanwhile, comparisons between $e_a^2(i)$ and $e_b^2(i)$ are kept taking time after time. As soon as a recognition of either Q_a or Q_b, say $Q_b(k)$ =1, works out at time k, the filter A will skip from $W_a(k)$ to $W_b(k)$. Each time, when such an initialization takes place, one of the two filters will get the position of the other filter to make its weight parameters closer to the central point C. Therefore, Eq.(10) gives a much higher adaptability.

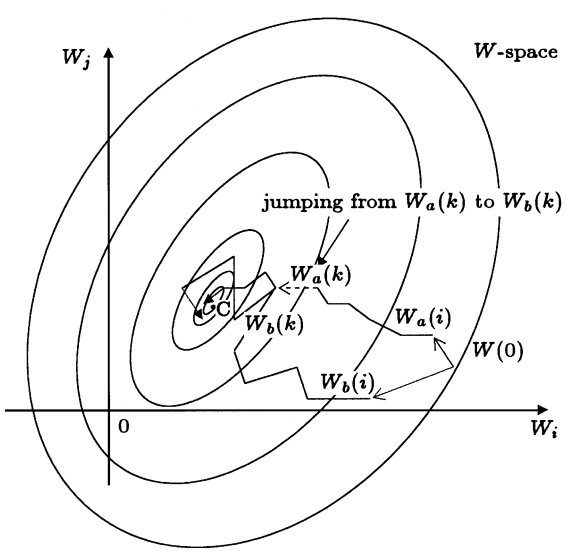


Fig.3. Jumping of the weight parameters back and forth between two filters A and B in the multi-dimensional W-space according to the least mean square principle.

3. Experiment and result

3.1. Numerical simulation

Numerical simulation tests of the algorithm specified by Eq.(10) are carried out in order to demonstrate its feasibility in magnetocardiography. In the tests, we choose $\alpha_a = 1 \times 10^{-3}$, $\alpha_b = 1 \times 10^{-2}$ and $\gamma_a = \gamma_b = 10^{-10}$. The reference input sequence $X(k)$ generated by a computer is a random white noise with a suitable normalized amplitude, the signal input sequence $S(k)$ is formed by a combination of several noises with different amplitudes and time delays in $X(k)$ and a sinusoidal test function with an amplitude of unity. The frequency band of the test function from 0.5Hz up to 0.2kHz covers almost all frequencies spent by the MCG signal. The sampling frequency is 2kHz and the filter length M is chosen to be 10. We choose the summation length l=50 and the comparison length J=20.

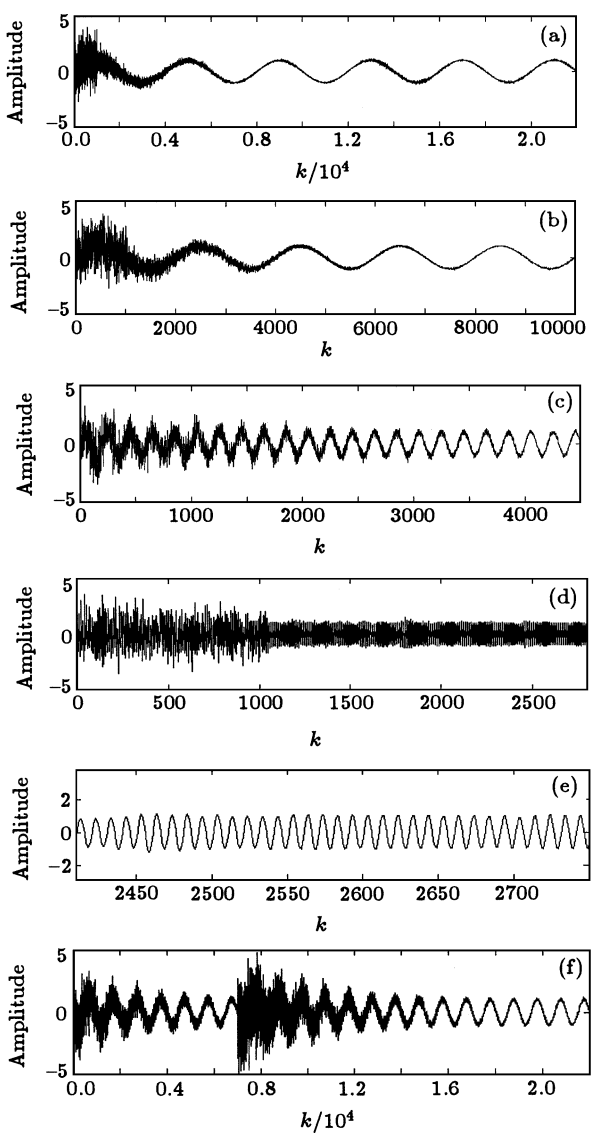


Fig.4. Filter response to a sinusoidal test function at (a) 0.5Hz, (b) 1Hz, (c) 10Hz and (d) 200Hz. In (e) the horizontal scale is enlarged. The background noise changes suddenly at k=7×10^3 in (f).

The response of the filtering is constantly appropriate to the test functions with signal to noise ratio $\eta \geq 1$ in the above frequency band as shown in Fig.4. In the figure, we can see that the noise level is reduced by 20dB in only a few seconds for higher frequency and a value of η as good as 10 is achieved in less than 10 seconds after the adaptation gets started, showing both high convergence speed and fine tracking of the filtering. The adaptability for the change of environment noise is also tested by suddenly switching the central point C from w_1=1 to w_3=3 in W space at t=3.5s (k=7×10^3) during operation as shown in Fig.4(f). Every time as changing takes place, the filter makes the test signal tracked again with a time constant of t=4s (see Fig.4(f)), which is quick enough for clinical MCG applications in most cases.

3.2. Magnetocardiography experiments and results

In the numerical simulation the noise parts in the two inputs generated artificially by the computer are completely correlated. Unfortunately, in practical measurements they may be only partly correlated due to the intrinsic noises in the two sensors and nonsimilarity of the background noises taken by different sensors in measurements. In our MCG experiments, we use two high-T_c SQUID sensors, a pickup SQUID for the signal channel and a compensatory one for the reference channel. The former is positioned as close as possible ($\leq$1cm) to the subject's chest, and the latter is placed 10cm apart (baseline) above the chest. Both of them are immersed in liquid nitrogen at 77K.

The MCG experiments are taken inside a magnetically shielded room that attenuates the earth's steady field by a factor of 10^{-4}, i.e. to nT level, and the environment interference down to 20pT, mainly from the power line. In comparison with the environmental variation, the intrinsic noise due to sensors themselves with a typical value of 2pT is still negligible in this situation. In order to make the noise components in the two channels correlated to a great extent, we fix the two SQUIDs in parallel to form a vertical axial gradiometer. With this configuration the SQUIDs may respond only the z-components of the signal and noise fields. When the distance between the noise source

and the sensors is much larger than the baseline of the SQUID gradiometer, the source can be regarded as a remote one and the major parts of the noises in the two channels are correlated.

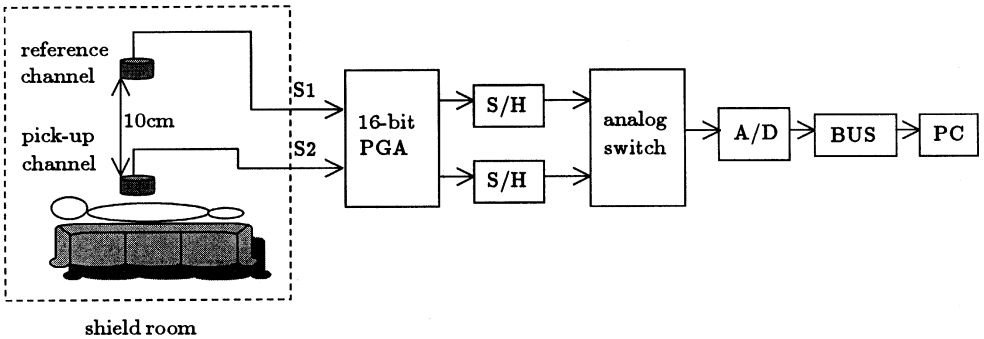


Fig.5. A block diagram of the data acquisition and processing.

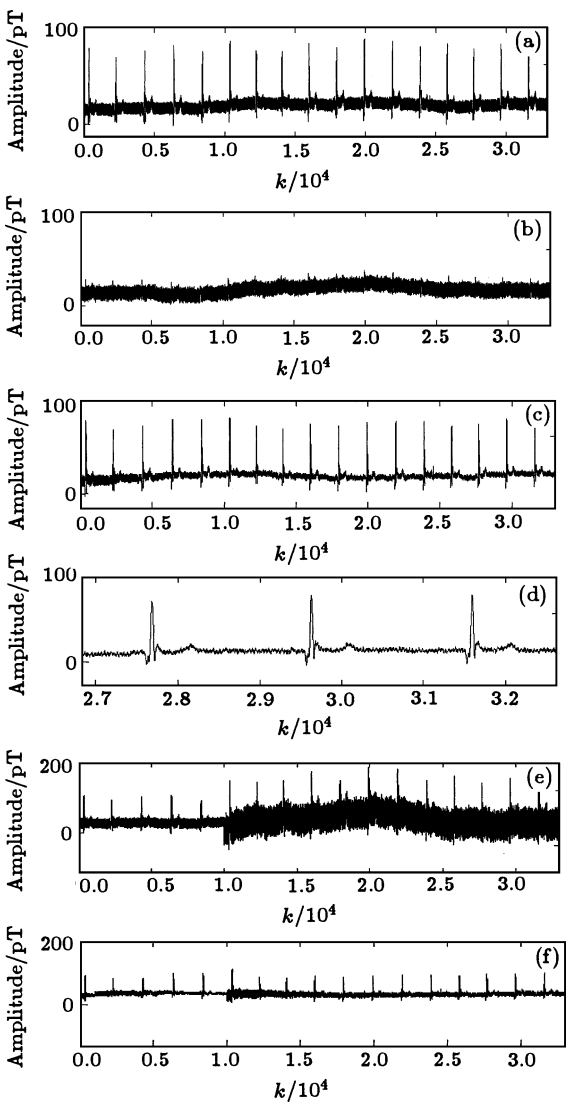


Fig.6. MCG experimental results: magnetic field signals in (a) the input channel, (b) the reference channel and the filtered signal at (c) the output of the filter. The horizontal scale of Fig.(c) has been enlarged in (d). Figures (e) and (f) are input and output signals respectively, showing a sudden change of the background noise at k=10^4.

The block diagram of the high-T_c SQUID-based MCG and data acquisition system in our experiments is shown in Fig.5. Both the MCG signal taken by the SQUID near the chest and the reference noise by the SQUID far from the chest are recorded by a 16-bit data-acquiring card with a programmable gate array (PGA), then fed into two parallel sampling/holding circuits (S/H) and an analogue to digital (A/D) converter, finally transmitted to a computer (PC) via a data BUS. The digital adaptive filtering is carried out in the PC.

The MCG experimental results are given in Fig.6. It can be seen from the input and output sequences of the adaptive filter that the MCG signal is well retrieved with signal-to-noise ratio improved by 3 times in a convergence time of 4s. The convergence of the filtering with time constant of 4s is also demonstrated by a sudden switch of the central point C from one place to the other, indicated at t=5s (k=10^4) in Fig.6 (e) and (f). From the figure we also notice that the amplitude of MCG signal in the output is slightly reduced due to some unwanted weak MCG signal involved in the reference channel. There is a residual noise in the output MCG signal, showing the uncorrelated part, which comes from SQUID sensors, electronics and the environment.

4. Discussion and conclusion

It has been demonstrated that the supplementary adaptive filtering with properly selected parameters may have both high-speed convergence and fine tracking in reducing correlated noises of the environment. With an algorithm, in which jumping from one path to the other in W-space is frequently takes place, the filtering provides much better static and dynamic

behaviours feasible for rapid and sensitive measurements, such as MCG. By filtering an MCG signal buried in an correlated environmental noise with a typical signal-to-noise ratio of 1 can be retrieved in less than 10 cardiac cycles; nevertheless, in the situations where uncorrelated noise is predominant, the quality of the signal cannot be improved much just as in the standard adaptive filtering.

The idea of supplement in this paper may be further developed to a multi-path processing, in which the jumping of weight parameters takes place among many paths in W-space, or to a programmable path-selection processing, where the optimum paths for specific time intervals in the processing are experimentally predetermined for a certain environment.

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