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Another anticontrol method of chaos in the sense of Devaney from a Takagi–Sugeno fuzzy system via the overflow nonlinearity*

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Another simple and systematic decentralized feedback anticontrol algorithm of chaos in the sense of Devaney is proposed for arbitrarily given discrete-time Takagi-Sugeno (TS) fuzzy systems. Based on a given discrete-time TS fuzzy system, the new chaotification algorithm uses the decentralized state feedback control and the overflow nonlinear function of the 2's complement arithmetic to construct a chaotic system. Based on the Shi–Chen theorem, we prove mathematically that the constructed system is indeed chaotic in the sense of Devaney. In particular, an explicit formula for the computation of chaotification parameters is also obtained. A numerical example is used to visualize and illustrate the theoretic results.

Keywords: chaos, chaotification, Devaney chaos, TS fuzzy system, 2's complement arithmetic
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1. Introduction

In addition to the rapid development of chaos suppression and chaos synchronization in the last decade,[1,2] chaotification (also called "anticontrol of chaos") by means of making an originally non-chaotic dynamical system chaotic, or enhancing and retaining existing chaos, has attracted some special attention from the engineering and physics communities lately due to the great potential of chaos in nontraditional applications.[2,3] In the pursuit of chaotifying discrete dynamical systems, the first mathematically rigorous chaotification method was developed by Chen and Lai[4–6] from the engineering feedback control approach. Up to now, the chaotification problem of discrete dynamical systems can be divided into two kinds. The first kind is to consider how to directly design a nonlinear state feedback controller such that the controlled discrete dynamical systems become chaotic such as in Refs.[3, 7–11]. The second kind is to consider how to first design a linear state feedback controller to ensure that all the Lyapunov exponents of the controlled system become strictly positive so that the system trajectory is expanding in all directions, and then apply a nonlinear operation to fold the trajectories back into a compact region whenever the expansion takes them beyond it. A combination of these two expanding and folding effects is expected to create complex chaotic dynamics within the bounded region of system orbits, thereby leading to chaotic dynamics such as in Refs.[4–6, 12–16].

The chaotification problem of discrete-time Takagi–Sugeno (TS) fuzzy systems has been studied in Refs.[9–11, 13,16]. In the first kind, the authors of Refs.[9, 10] and [11] have studied how to make a stable discrete-time TS fuzzy system chaotic in the sense of Li–Yorke[17] and Devaney[18] respectively. In the second kind, Li *et al*[13] used the centralized control and mod-operation to chaotify a TS fuzzy system and applied the Marotto theorem[19] to prove their results. Moreover, the authors of Refs.[20–22] have pointed out that the Marotto theorem[19] suffers a problem since Marotto had used the following property $A' \Rightarrow B'$ in his theorem:

A': All eigenvalues of the real matrix A are larger than 1 in absolute value;

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B' :A satisfies $\|Ax\|_2 \geq \mu \|x\|_2$ for some $\mu > 1$, for all $x \in R^n$.

Later, in Ref.[16], we used the decentralized feedback controller and simple sawtooth functions to chaotify a discrete-time TS fuzzy system and applied the corrected Marotto theorem to mathematically prove that the controlled system is chaotic in the sense of Li and Yorke.[17]

In this paper, we are interested in the second kind of chaotification problem and use the mathematical definition of chaos introduced by Devaney[18] as the design criterion. More precisely, for an arbitrarily given discrete-time TS fuzzy system regardless of its stability, we propose another simple and systematic chaotification method belonging to the second kind problem mentioned above, which uses the decentralized state feedback control and the overflow nonlinear function of the 2's complement arithmetic[23] to construct a chaotic system. In each dimension of the system, the decentralized controller uses only one local state variable. We apply the Shi–Chen theorem[24] to mathematically prove that the proposed chaotification method can yield chaos in the sense of Devaney.[18] In particular, an explicit formula for the computation of chaotification parameters is also obtained. Finally, a numerical example is used to illustrate the theoretical analysis and the controller design.

Throughout this paper, all vector inequalities such as $x \geq 0$ are component-wise inequalities,$b = [1, \ 1, \ \ldots, \ 1]^{\mathrm{T}}$. For a vector $x \in R^n$, its infinity-norm is defined by $\|x\|_\infty = \max\limits_{1\leq i\leq n} |x_i|$, and its corresponding induced norm is defined by $\|A\|_\infty = \max\limits_{1\leq i\leq n} \left\{ \sum\limits_{j=1}^{n} |a_{ij}| \right\}$.

2. Description of the problem and the controller design

Consider a general discrete-time TS fuzzy system[25] described by

Rule i: IF $x_1(k)$ is $M_{i1}, \cdots$, and $x_n(k)$ is M_{in}, THEN

$$x_{k+1} = A_i x_k + u_k, \tag{1}$$

where $x_k = [x_k(1), \ \cdots, \ x_k(n)]^{\mathrm{T}}$, $u_k = [u_k(1), \ \cdots, \ u_k(n)]^{\mathrm{T}}$, $i = 1, 2, \cdots, r$, r is the number of IF-THEN rules, M_{ij} are fuzzy sets, and the equation $x_{k+1} = A_i x_k + u_k$ is the output from the ith IF-THEN rule. Now, given a pair of (x_k, u_k), the final output of the fuzzy system is inferred from

$$x_{k+1} = \frac{\sum\limits_{i=1}^{r} w_i(k) \{A_i x_k + u_k\}}{\sum\limits_{i=1}^{r} w_i(k)}, \tag{2}$$

where $w_i(k) = \prod\limits_{j=1}^{n} M_{ij}(x_k(j))$, $M_{ij}(x_k(j))$ is the degree of membership of $x_k(j)$ in M_{ij}, with

$$\begin{cases} \sum\limits_{i=1}^{r} w_i(k) > 0 \\ w_i(k) \geq 0, \quad i = 1, 2, \cdots, r. \end{cases}$$

By using $h_i(k) = w_i(k) / \sum_{i=1}^{r} w_i(k)$ instead of $w_i(k)$, Eq.(2) is rewritten as

$$x_{k+1} = \left[\sum_{i=1}^{r} h_i(k) A_i \right] x_k + u_k. \tag{3}$$

Note that

$$\begin{cases} \sum\limits_{i=1}^{r} h_i(k) = 1 \\ h_i(k) \geq 0, \quad i = 1, 2, \cdots, r, \end{cases} \tag{4}$$

in which $h_i(k)$ can be regarded as the normalized weight of the IF-THEN rules.

The problem investigated is to chaotify the arbitrarily given discrete-time TS fuzzy systems (3) regardless of their stability with $\|u_k\|_\infty \leq u_{\max}$, where $u_{\max}$ is an arbitrarily small control magnitude bound. For this purpose, we use the decentralized control and the overflow nonlinear function of the 2's complement arithmetic[23] to obtain the following closed-loop controlled system:

$$x_{k+1} = \Phi \left(\left[\sum_{i=1}^{r} h_i(k) A_i \right] x_k + u_k \right), \tag{5}$$

$$u_k = \sigma x_k, \tag{6}$$

where $\Phi(y) = [\varphi(y_1), \cdots, \varphi(y_n)]^{\mathrm{T}}$, for any $y = [y_1, \cdots, y_n]^{\mathrm{T}} \in R^n$; $\sigma > 0$ is a constant to be designed, and $\varphi : R \to R$ is the overflow nonlinear function of the 2's complement arithmetic defined by (see Fig.1)[23]

$$\varphi(y) = y - 2m\varepsilon \ \text{ for } \ (2m-1)\varepsilon \leq y \leq (2m+1)\varepsilon, \ m = 0, \ \pm 1, \ \pm 2, \ \cdots \tag{7}$$

Here $\varepsilon > 0$ is the maximum of the 2's complement arithmetic and can be designed such that $\|u_k\|_\infty = \|\sigma x_k\|_\infty \leq \sigma\varepsilon \leq u_{\max}$. Later, we will apply the Shi–Chen theorem[24] to mathematically prove that the

controlled system (5) with the decentralized controller (6) is chaotic in the sense of Devaney.[18]

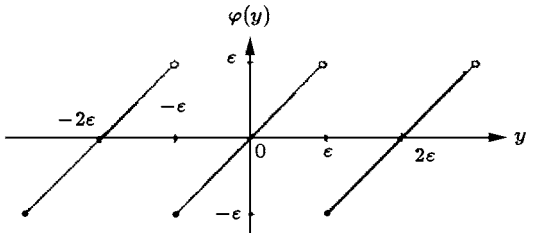


Fig.1. The overflow nonlinearity.

Finally, it should be noted that in employing the overflow nonlinearity of the 2's complement arithmetic, it is typical to identify ("paste") the extreme value ε to another extreme value $-\varepsilon$ in the range of the map, thereby obtaining a continuous differentiable map on a circle (the "circle map" from the unit circle to the unit circle).[3,12] Here and throughout, we follow this convention.

3. Verification of chaos in the sense of Devaney

In this section, we apply the Shi–Chen theorem[24] to prove that the controlled system (5) with the decentralized controller (6) is chaotic in the sense of Devaney.

3.1. Chaos in the sense of Devaney: the Shi–Chen theorem[24]

Since Li and Yorke[17] first introduced a precise mathematical definition of chaos, there have appeared several different definitions of chaos, some are stronger and some are weaker, depending on the requirements in different problems. In 1987, Devaney[18] presented an explicit definition of chaos for the following systems:

$$x_{n+1} = F(x_n), \quad n \geq 0 \tag{8}$$

where $F : V \to V$ is a map and V is a subset of R^m, which is described as follows. A continuous map F is said to be chaotic on V (in the sense of Devaney) if

(i) F is transitive;

(ii) the periodic points of F are dense in V;

(iii) F has a sensitive dependence on initial conditions.

By a relevant result given in Ref.[26], Properties (i) and (ii) together imply Property (iii). So, Property (iii) is redundant in the above definition.

Since results obtained in this paper are related to a symbolic dynamical system, for convenience, some results of symbolic dynamical system[18] are first reviewed.

Let

$$\sum\nolimits_2^+ := \{s = (s_0 s_1 s_2 \ldots) : s_j = 0 \text{ or } 1\}$$

and define a distance between two points $s = (s_0 s_1 s_2 \ldots)$ and $t = (t_0 t_1 t_2 \ldots)$ in $\sum_2^+$ by

$$\rho(s,t) = \sum_{i=0}^{\infty} \frac{|s_i - t_i|}{2^i}.$$

Lemma 1[24] $\left(\sum_2^+, \rho\right)$ is a complete, compact, totally disconnected, and perfect metric space.

The shift map $\alpha : \sum_2^+ \to \sum_2^+$ defined by $\alpha(s_0 s_1 s_2 \ldots) = (s_1 s_2 \ldots)$ is continuous. The dynamical system governed by α is called a symbolic dynamical system and it has the following properties:

Lemma 2[18]

(i) Card $Per_n(\alpha) = 2^n$,

(ii) Per(α) is dense in $\sum_2^+$,

(iii) there exists a dense orbit of α in $\sum_2^+$,

where Card $\mathrm{Per}_n(\alpha)$ denotes the number of periodic points of period n for α.

It is clear that Property (iii) implies that α is transitive. Hence, this symbolic dynamical system is chaotic in the sense of Devaney.

The following criterion of chaos for system (8) was established in Ref.[24], which plays an important role in the rest of this paper.

Theorem (Shi–Chen[24]) Let (X, d) be a metric space and V_1, V_2 be two disjoint compact subsets of X. If the continuous map $F : V_1 \cup V_2 \to X$ satisfies

(1) $F(V_j) \supset V_1 \cup V_2$ for $j = 1, 2$;

(2) there exists a constant $\lambda_0 > 1$ such that

$$d(F(x), F(y)) \geq \lambda_0 d(x, y),$$
$$\forall x, y \in V_1 \quad \text{and} \quad \forall x, y \in V_2;$$

then there exists a Cantor set $\Lambda \in V_1 \cup V_2$ such that $F : \Lambda \to \Lambda$ is topologically conjugate to the symbolic dynamical system $\alpha : \sum_2^+ \to \sum_2^+$. Consequently, F is chaotic on Λ in the sense of Devaney.

3.2. The proposed chaotification algorithm yielding Devaney chaos

Theorem 1 Suppose that $h_i(k)$, $i = 1, \cdots, r$, are continuously differentiable in the neighbourhood of the fixed point, $x^* = 0$ of the controlled system (5) with the decentralized controller (6). If

$$\sigma > \sigma^* = \max\left\{\frac{(N+1)(2+\theta)}{\theta}, \frac{(2+\theta)N}{1-\theta}\right\}, \tag{9}$$

where $N = \max\limits_{1\le i\le r}\{\|A_i\|_\infty\}$ and $\theta \in (0,\ 1)$, then the controlled system (5) with the decentralized controller (6) is chaotic in the sense of Devaney.

Proof The controlled system (5) with the decentralized controller (6) is

$$\begin{aligned} x_{k+1} =& \Phi\left(\left[\sum_{i=1}^{r} h_i(k)A_i\right]x_k + u_k\right) \\ =& \Phi\left(\left[\sum_{i=1}^{r} h_i(k)A_i\right]x_k + \sigma x_k\right) \equiv F(x_k). \end{aligned} \tag{10}$$

Let

$$F(x) = [F_1(x),\ F_2(x),\ \cdots,\ F_n(x)]^{\mathrm{T}},$$

$$\left[\sum_{i=1}^{r} h_i(k)A_i\right]x = f(x) = [f_1(x),\ \ \cdots,\ f_n(x)]^{\mathrm{T}},$$

$X = R^n$, $d(\cdot) = \|\cdot\|_\infty$, and choose two disjoint closed rectangular regions $V_1 = [-\theta\sigma^{-1}\varepsilon b, \theta\sigma^{-1}\varepsilon b]$ and $V_2 = [(2-\theta)\sigma^{-1}\varepsilon b, (2+\theta)\sigma^{-1}\varepsilon b]$. Apparently, $(R^n, \|\cdot\|_\infty)$ is a complete metric space. Since $\left\|\sum\limits_{i=1}^{r} h_i(k)A_i\right\|_\infty \le N$, we have

$$|f_i(x)| \le N\|x\|_\infty. \tag{11}$$

Using Eqs.(11) and (9), we can obtain

$$-\varepsilon \le f_j(x) - \theta\varepsilon \le \varepsilon,$$

$$\text{for any } x \in V_1 \text{ with } x_j = -\theta\sigma^{-1}\varepsilon; \tag{12}$$

$$-\varepsilon \le f_j(x) + \theta\varepsilon \le \varepsilon,$$

$$\text{for any } x \in V_1 \text{ with } x_j = \theta\sigma^{-1}\varepsilon; \tag{13}$$

$$\varepsilon \le f_j(x) + (2-\theta)\varepsilon \le 3\varepsilon,$$

$$\text{for any } x \in V_2 \text{ with } x_j = (2-\theta)\sigma^{-1}\varepsilon; \tag{14}$$

$$\varepsilon \le f_j(x) + (2+\theta)\varepsilon \le 3\varepsilon,$$

$$\text{for any } x \in V_2 \text{ with } x_j = (2+\theta)\sigma^{-1}\varepsilon. \tag{15}$$

Further, it follows from the definition of the overflow nonlinear function of the 2's complement arithmetic (7), (9) and (11)–(15) that
for any $x \in V_1$ with $x_j = -\theta\sigma^{-1}\varepsilon$,

$$\begin{aligned} F_j(x) =& \varphi(f_j(x) - \theta\varepsilon) \\ =& f_j(x) - \theta\varepsilon \le \theta\sigma^{-1}\varepsilon\cdot N - \theta\varepsilon \le -\theta\sigma^{-1}\varepsilon; \end{aligned} \tag{16}$$

for any $x \in V_1$ with $x_j = \theta\sigma^{-1}\varepsilon$,

$$\begin{aligned} F_j(x) =& \varphi(f_j(x) + \theta\varepsilon) \\ =& f(x) + \theta\varepsilon \ge -\theta\sigma^{-1}\varepsilon\cdot N + \theta\varepsilon \ge (2+\theta)\sigma^{-1}\varepsilon; \end{aligned} \tag{17}$$

for any $x \in V_2$ with $x_j = (2-\theta)\sigma^{-1}\varepsilon$,

$$\begin{aligned} F_j(x) =& \varphi(f_j(x) + (2-\theta)\varepsilon) \\ =& f_j(x) + (2-\theta)\varepsilon - 2\varepsilon \\ \le& (2+\theta)\sigma^{-1}\varepsilon\cdot N - \theta\varepsilon \le -\theta\sigma^{-1}\varepsilon; \end{aligned} \tag{18}$$

for any $\forall \in V_2$ with $x_j = (2+\theta)\sigma^{-1}\varepsilon$,

$$\begin{aligned} F_j(x) =& \varphi(f_j(x) + (2+\theta)\varepsilon) \\ =& f_j(x) + (2+\theta)\varepsilon - 2\varepsilon \\ \ge& -(2+\theta)\sigma^{-1}\varepsilon\cdot N + \theta\varepsilon \ge (2+\theta)\sigma^{-1}\varepsilon. \end{aligned} \tag{19}$$

The above relations (16)–(19) and the continuity of F on V_1 and V_2, imply that for each $\sigma > \sigma^*$,

$$\begin{aligned} &[-\varepsilon b, \varepsilon b] \supset F(V_i) \supset [-\theta\sigma^{-1}\varepsilon b, (2+\theta)\sigma^{-1}\varepsilon b] \\ &\supset V_1 \cup V_2, \quad i = 1,\ 2. \end{aligned} \tag{20}$$

On the other hand, it follows from Eq.(9) that for each $\sigma > \sigma^*$,

$$\|f(x) + \sigma x\|_\infty < \varepsilon, \quad \forall x \in V_1;$$

$$\|f(x) + \sigma x - 2\varepsilon\, b\|_\infty < \varepsilon, \quad \forall x \in V_2.$$

Therefore, for all $x, y \in V_1$ and for all $x, y \in V_2$

$$\begin{aligned} &\|F(x) - F(y)\|_\infty \\ =& \|\ \Phi(f(x) + \sigma x) - \Phi(f(y) + \sigma y)\|_\infty \\ =& \left\|\left[\sum_{i=1}^{r} h_i(k)A_i + \sigma I_{n\times n}\right](x-y)\right\|_\infty \\ \ge& \left\|\left[\sum_{i=1}^{r} h_i(k)A_i + \sigma I_{n\times n}\right]^{-1}\right\|_\infty^{-1}\|x-y\|_\infty, \end{aligned} \tag{21}$$

and

$$\begin{aligned} &\left\|\left[\sum_{i=1}^{r} h_i(k)A_i + \sigma I_{n\times n}\right]^{-1}\right\|_\infty \\ =& (\sigma)^{-1}\left\|\left((\sigma)^{-1}\left(\sum_{i=1}^{r} h_i(k)A_i\right) + I\right)^{-1}\right\|_\infty \\ \le& \frac{1}{\sigma\left(1 - \left\|(\sigma)^{-1}\left(\sum\limits_{i=1}^{r} h_i(k)A_i\right)\right\|_\infty\right)} \\ \le& \frac{1}{\sigma - N}. \end{aligned} \tag{22}$$

Here $\sigma - N \ge \lambda_0 > 1$ for $\sigma > \sigma^*$. Since V_1 and V_2 are two disjoint compact subsets, for any $\sigma > \sigma^*$, all the conditions in Shi–Chen theorem[24] are satisfied

and consequently the theorem follows from Shi–Chen theorem.[24] This completes the proof.

We can easily obtain the following Corollary 1 from Theorem 1.

Corollary 1 Suppose that $h_i(k),\ i = 1, \cdots, r$, are continuously differentiable in the neighbourhood of the fixed point, $x^* = 0$ of the controlled system (5) with the decentralized controller (6). If

$$\sigma > \sigma^* = \min_{\theta \in (0,1)} \max \left\{ \frac{(N+1)(2+\theta)}{\theta}, \frac{(2+\theta)N}{1-\theta} \right\}, \tag{23}$$

where $N = \max\limits_{1 \le i \le r} \{ \|A_i\|_\infty \}$, then the controlled system (5) with the decentralized controller (6) is chaotic in the sense of Devaney.

Remark 1 It is surprising to note that σ^* in Theorem 1 and Corollary 1 is independent of ε.

Remark 2 Obviously, $\sigma > N + e^c > N + 1$ for some $c > 0$. Thus, all the Lyapunov exponents of the controlled system (5) with the controller (6) are strictly positive.[4]

4. A simulation example

Consider a nonchaotic discrete-time TS fuzzy model, given by

Rule i: IF x_k is M_i, THEN $X_{k+1} = \begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = A_i \begin{bmatrix} x_k \\ y_k \end{bmatrix} + u_k,\ i = 1,\ 2,$

where $A_1 = \begin{bmatrix} d & 0.1 \\ -5 & 0 \end{bmatrix}, A_2 = \begin{bmatrix} -d & 0.1 \\ -5 & 0 \end{bmatrix}$, $x_k \in [-d, d]$, $d > 0$, and the ership functions

$$M_1 = 0.5\,(1 - x_k/d), \quad M_2 = 0.5\,(1 + x_k/d).$$

The controlled TS fuzzy system is constructed as follows:

$$\begin{aligned} X_{k+1} &= \Phi \left(\left[\sum_{i=1}^{2} h_i(k) A_i \right] X_k + u_k \right) \\ &= \Phi \left(\left[\sum_{i=1}^{2} h_i(k) A_i \right] X_k + \sigma X_k \right). \end{aligned} \tag{24}$$

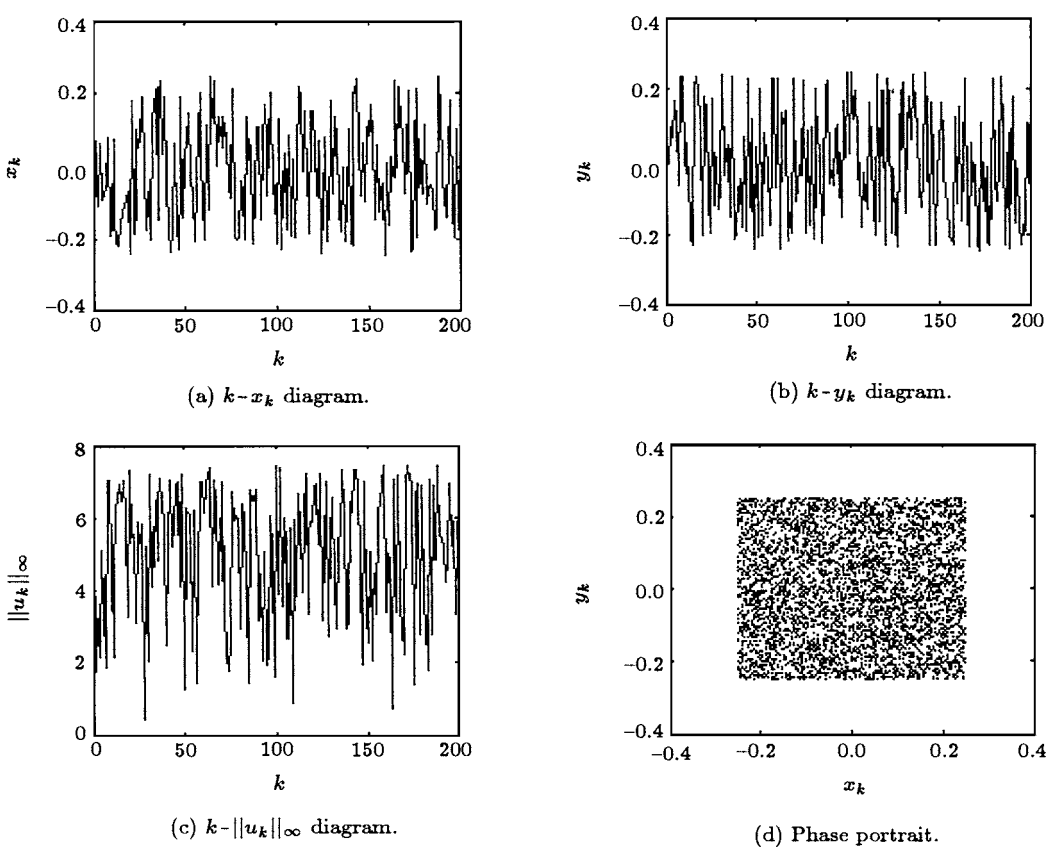


Fig.2. Simulation results.

The magnitude of the control input is arbitrarily chosen to be $\bar{u} = 8$. We choose $d = 0.7$ and know that the controlled TS fuzzy system (24) is chaotic if $\sigma > 28.0183$ from Corollary 1 with $N = \max_{1 \le i \le r} \{ \|A_i\|_\infty \} = 5$. In the simulation, we choose $X_0 = [0.2,\ 0.1]^{\mathrm{T}}$, $\sigma = 30$ and $\varepsilon = 0.25$. The phase portrait and time wave diagrams are shown in Fig.2(d) and Figs.2(a), (b), (c), respectively. From Fig.2, we can see that $\|u_k\|_\infty \le 8$ and the controlled TS fuzzy system (20) displays complex chaotic behaviours. The largest Lyapunov exponent of this controlled system (15) is calculated to be $\lambda_{\max} = 4.1805$. These simulation results verify the theoretical analysis in this paper.

5. Conclusions

In this paper, we have proposed another simple and systematic chaotification method to make arbitrarily given discrete TS fuzzy systems (regardless of their stability) chaotic in the sense of Devaney, in which we employ the decentralized control and the overflow nonlinear function of the 2's complement arithmetic. By applying the Shi–Chen theorem, we have proven mathematically that the proposed algorithm for chaotification leads to chaos in the mathematical sense of Devaney. In particular, an explicit formula for the computation of control parameters has also been given. The simulation results illustrate the effectiveness of the proposed method.

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