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# Interaction of mesoscopic Josephson junction with the excited even and odd coherent states*

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The interaction of a mesoscopic Josephson junction (MJJ) with the excited even and odd coherent states is investigated. Some properties of the supercurrent in the MJJ in the presence of nonclassical light fields, such as collapse and revival phenomenon, the current–voltage step, dc component fluctuation and quantum coherence of supercurrent, are studied. It is shown that the expectation values and the fluctuation of supercurrent can exhibit dc components. It is also shown that the static supercurrent flows, dc components of the supercurrent fluctuation and correlation function of the dc supercurrent components, are related to the phase of the light field.

**Keywords:** mesoscopic Josephson junction, excited even and odd coherent states, collapse and revival, current–voltage step, second-order correlation function

**PACC:** 7450, 4250

## 1. Introduction

In recent years the mesoscopic Josephson junction (MJJ) has been widely studied.[1−3] There have been many new quantum phenomena observed in the system, such as Coulomb blockade of Cooper pair tunnelling, Bloch oscillations and Zener tunnelling. Recently, the interaction of a quantized electromagnetic field (QEMF) with the MJJ has been investigated.[4−9] The quantum effects of some nonclassical states, such as even and odd coherent states, squeezed states, etc, on the MJJ have been studied extensively. In Ref.[10], the interaction of MJJ with the second-order excited even coherent state is investigated and the current–voltage step (CVS) of the MJJ is given. The aim of this paper is to study the interaction of the MJJ with the excited even coherent states (EECS) and the excited odd states (EOCS).

The paper is organized as follows. In Section 2, the collapse and revival (CR) phenomenon is discussed. In Section 3, the CVS of the supercurrent in the MJJ is given. The dc components of the supercurrent fluctuation are investigated in Section 4. In Section 5, the correlation function of the dc supercurrent component is studied. Section 6 contains a brief discussion and conclusions.

## 2. Collapse and revival of supercurrent

In this paper a MJJ with a capacitance $C$ witch operates in a circular superconducting device[4,5] is considered. It is described by the following Hamiltonian ($\hbar = c = 1$):

$$H = -\frac{1}{2}E_{\rm c}\left(\partial\theta + {\rm i}\frac{Q}{2C}\right)^2 + E_{\rm J}\left(1-\cos\theta\right), \qquad (1)$$

where $Q$ is the external charge which can be controlled by the external quantized electromagnetic fields, $E_{\rm c} = (2e)^2/C$ is the charging energy associated with the Cooper pair charge $2e$, $E_{\rm J} = I_{\rm cr}/2e$ is the Josephson coupling constant, $\theta$ is the phase difference of the superconducting order parameters in the two electrodes.

From the Hamiltonian (1), we take the Josephson-current operator $\widehat{I}$ in the Heisenberg picture

$$\widehat{I} = I_{\rm cr}\sin\widehat{\theta}(t). \qquad (2)$$

We now impose the classical static electromotive force (voltage) $V_0$ and the quantum time-varying electromotive force on the MJJ through applying the classical magnetic flux $\phi_0 = V_0 t$ and the ac quantum single-mode magnetic field of angular frequency $\omega_1$, respectively. For nonclassical fields, magnetic flux $\phi$ and

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external voltage $V_{\rm EMF}$ are a pair of quantum mechanical conjugate variables $[\phi, V_{\rm EMF}] = {\rm i}\omega_1$, and can be written as (in the Heisenberg picture)

$$\phi_{(t)} = \frac{1}{\sqrt{2}}\left[\exp({\rm i}\omega_1 t)\, a^+ + \exp(-{\rm i}\omega_1 t)\, a\right], \tag{3}$$

$$V_{\rm EMF(t)} = \frac{\rm i}{\sqrt{2}}\omega_1\left[\exp({\rm i}\omega_1 t)\, a^+ - \exp(-{\rm i}\omega_1 t)\, a\right], \tag{4}$$

where $a$ and $a^+$ are the annihilation and creation operators of the quantized electromagnetic field, respectively.

For simplicity, we consider the case $\omega_1 \gg \sqrt{E_{\rm c}E_{\rm J}}$, where $\sqrt{E_{\rm c}E_{\rm J}}$ is the angular frequency of the MJJ when it is approximated to a harmonic oscillator. In this case the Hamiltonian of the MJJ itself produces an evolution which is much slower than the Hamiltonian describing the effect of external fields, and the phase evolves approximately in the following form:

$$\exp({\rm i}\widehat{\theta}(t)) = \exp({\rm i}\omega_0 t) D(\xi\exp({\rm i}\omega_1 t)), \tag{5}$$

where $\omega_0 = 2eV_0$, $\xi = \sqrt{2}e$, $e$ is the charge of the electron and $D_{(z)}$ is the displacement operator defined by

$$D_{(z)} = \exp(za^+ - z^* a). \tag{6}$$

Then the expectation value of the supercurrent can be given by tracing the current operator $\widehat{I}$ over the density operator of the external QEMF,

$$\langle \hat{I} \rangle = I_{\rm cr} I_{\rm m}\{\exp({\rm i}\omega_0 t){\rm Tr}[\rho D(\xi\exp({\rm i}\omega_1 t))]\}. \tag{7}$$

We now consider the excited even and odd coherent states to act with MJJ. The EECS $|\alpha, m\rangle_{\rm e}$ and EOCS $|\alpha, m\rangle_{\rm o}$ are defined by[11]

$$|\alpha, m\rangle_{\rm e} = N_{{\rm e},m}^{-1/2} a^{+m}|\alpha\rangle_{\rm e}, \tag{8}$$

$$|\alpha, m\rangle_{\rm o} = N_{{\rm o},m}^{-1/2} a^{+m}|\alpha\rangle_{\rm o}, \tag{9}$$

$$N_{{\rm e},m} = \frac{m!}{2\cosh|\alpha|^2}[L_{\rm m}(-|\alpha|^2){\rm e}^{|\alpha|^2} + L_{\rm m}(|\alpha|^2){\rm e}^{-|\alpha|^2}], \tag{10}$$

$$N_{{\rm o},m} = \frac{m!}{2\sinh|\alpha|^2}[L_{\rm m}(-|\alpha|^2){\rm e}^{|\alpha|^2} - L_{\rm m}(|\alpha|^2){\rm e}^{-|\alpha|^2}], \tag{11}$$

where $m$ is a non-negative integer and $L_{\rm m}(x)$ is the Laguerre polynomial of order $m$:

$$L_{\rm m}(x) = \sum_{l=0}^{m} \frac{(-1)^l x^l m!}{(l!)^2(m-l)!}; \tag{12}$$

$|\alpha\rangle_{\rm e}$ and $|\alpha\rangle_{\rm o}$ are, respectively, the even coherent state and odd coherent state defined by Hillery[12]

$$|\alpha\rangle_{\rm e} = N_{\rm e}^{-1/2}(|\alpha\rangle + |-\alpha\rangle), \tag{13}$$

$$|\alpha\rangle_{\rm o} = N_{\rm o}^{-1/2}(|\alpha\rangle - |-\alpha\rangle), \tag{14}$$

$$N_{\rm e} = 2(1 + {\rm e}^{-2|\alpha|^2}) \tag{15}$$

$$N_{\rm o} = 2(1 - {\rm e}^{-2|\alpha|^2}) \tag{16}$$

where $|\alpha\rangle$ is the usual coherent state

$$|\alpha\rangle = \exp(\alpha a^+ - \alpha^* a)|0\rangle, \quad \alpha = |\alpha|{\rm e}^{{\rm i}\phi}. \tag{17}$$

From Eqs.(7)–(9), we have for EECS and EOCS

$$W_{\rm e} = {}_{\rm e}\langle\alpha, m|D[\xi\exp({\rm i}\omega_1 t)]|\alpha, m\rangle_{\rm e} = 2N_+^{-1}{\rm e}^{-\xi^2/2}\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\left\{\begin{pmatrix} m \\ l \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(m-l)![(A_1+A_2) + {\rm e}^{-2|\alpha|^2}(B_1+B_2)]\right\}, \tag{18}$$

$$W_{\rm o} = {}_{\rm o}\langle\alpha, m|D[\xi\exp({\rm i}\omega_1 t)]|\alpha, m\rangle_{\rm o} = 2N_-^{-1}{\rm e}^{-\xi^2/2}\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\left\{\begin{pmatrix} m \\ l \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(m-l)![(A_1+A_2) - {\rm e}^{-2|\alpha|^2}(B_1+B_2)]\right\}, \tag{19}$$

where

$$N_+ = N_{{\rm e},m}N_{\rm e}, \tag{20}$$

$$N_- = N_{{\rm o},m}N_{\rm o}, \tag{21}$$

$$A_1 = \begin{pmatrix} l \\ 2k \end{pmatrix}(-1)^k a_1^{l-2k}(2u)^{2k}\cos 2u, \qquad A_2 = -\begin{pmatrix} l \\ 2k+1 \end{pmatrix}(-1)^k a_1^{l-2k-1}(2u)^{2k+1}\sin 2u, \tag{22}$$

$$B_1 = \begin{pmatrix} l \\ l2k \end{pmatrix} a_2^{l-2k}(2v)^{2k}\cosh 2v, \qquad B_2 = \begin{pmatrix} l \\ 2k+1 \end{pmatrix} a_2^{l-2k-1}(2v)^{2k+1}\sinh 2v. \tag{23}$$

Here

$$u = \xi|\alpha|\sin(\omega_1 t - \phi),$$
$$v = \xi|\alpha|\cos(\omega_1 t - \phi), \tag{24}$$

$$a_1 = |\alpha|^2 - \xi^2,$$
$$a_2 = -|\alpha|^2 - \xi^2. \tag{25}$$

Combining Eq.(7) with Eqs.(18) and (19), the expectation values of supercurrent can be written as

$$\langle \hat{I}(t)\rangle_{\rm e} = I_{\rm cr} W_{\rm e} \sin\omega_0 t, \tag{26a}$$
$$\langle \hat{I}(t)\rangle_{\rm o} = I_{\rm cr} W_{\rm o} \sin\omega_0 t, \tag{26b}$$

for EECS or EOCS, respectively.

Now we are going to investigate CR of the supercurrents in the MJJ. In Ref.[6] the case of EECS for $m = 0$ was considered in which we have found that the supercurrent can exhibit the CR phenomenon (when $\omega_{\rm o} = 12\omega_1$ and $|\alpha| = 1.83$) and the CR is very sensitive to $|\alpha|$. From Eq.(26a), we plot the time evolution of CR in the MJJ when $\omega_{\rm o} = 12\omega_1$, $m = 2$ for EECS in Fig.1. We can see that the CR is also sensitive to $|\alpha|$. We notice that, when $|\alpha|$ is relatively small, the supercurrent exhibits incomplete collapse and complete revivals (Fig.1(a)); as $|\alpha|$ increases, the CR becomes more and more pronounced, and when $|\alpha|$ is around 0.98, the supercurrent exhibits almost complete CR (Fig.1(b)); and after $|\alpha|$ increases above this value, the CR gradually disappears and the oscillation of the supercurrent becomes irregular (Fig.1(c)). By calculation we find that, when $m = 1$ and 3, the dynamical behaviours of the supercurrent are similar to that at $m = 2$. However, for $m = 1$ and 3, the supercurrents exhibit almost complete CR when $\omega_{\rm o} = 12\omega_1$, $|\alpha| = 1.36$ and 0.69, respectively. The dynamical behaviours of the supercurrent for EOCS when $\omega_{\rm o} = 12\omega_1, m = 2$ is shown in Fig.2. We see from Fig.2 that, similarly to the case of the EECS, the supercurrents can exhibit almost complete CR phenomenon under an appropriate value of $|\alpha|$. From numerical calculation, one can find that, for $m = 1, 2$ and 3, the supercurrents for EOCS exhibit almost complete CR when $\omega_{\rm o} = 12\omega_1$, $|\alpha| = 1.38$, 1.04 and 0.79, respectively. From the above discussion we can conclude that the CR behaviours of supercurrent for EECS and EOCS are dependent on not only the average photon number ($|\alpha|^2$ or $|\alpha|$ ) but also the degree of excitation ($m$).

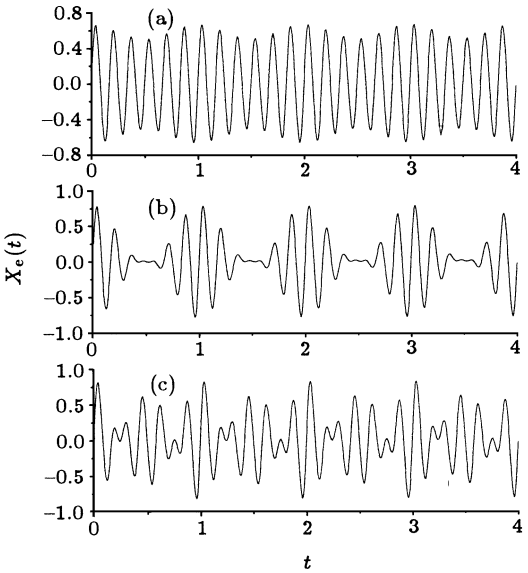



**Fig.1.** Time evolution of supercurrent $X_{\rm e}(t)(= \langle I(t)\rangle_{\rm e}/I_{\rm cr})$ for EECS with $\phi = 0, \omega_0/\omega_1 = 12, m = 2$, (a) $|\alpha| = 0.3$; (b) $|\alpha| = 0.98$; (c) $|\alpha| = 2$.

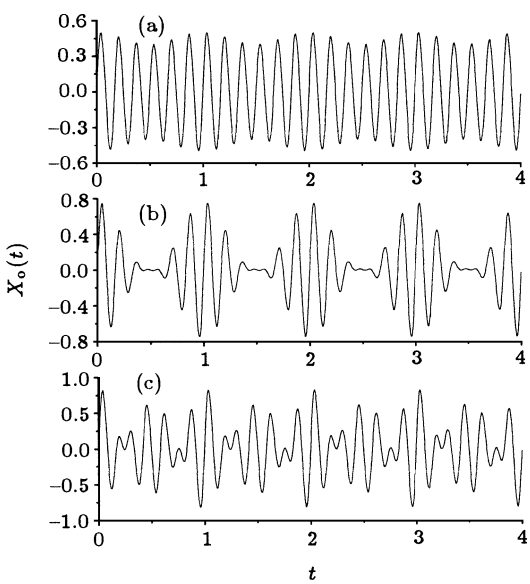



**Fig.2.** Time evolution of supercurrent $X_{\rm o}(t)(= \langle I(t)\rangle_{\rm o}/I_{\rm cr})$ for EOCS with $\phi = 0, \omega_0/\omega_1 = 12, m = 2$, (a) $|\alpha| = 0.3$; (b) $|\alpha| = 1.04$; (c) $|\alpha| = 2$.

## 3. CVS of supercurrent

The dc component of the supercurrent can be obtained by taking the time mean values of the expressions (18) and (19) in the interval $(0, t)$. It is easy to find that when the dc voltage $V_0$ satisfies the following condition:

$$\omega_0 - 2n\omega_1 = 0, \text{ i.e., } V_0 = n\omega_1/e \quad (n = 1, 2, 3, \cdots), \tag{27}$$

the direct supercurrent is expressed as

$$I_{\mathrm{e},m}^{\mathrm{dc}} = 2I_{\mathrm{cr}}N_{+}^{-1}\mathrm{e}^{-\xi^2/2}\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\left[\left(\begin{array}{c} m \\ l \end{array}\right)\left(\begin{array}{c} m \\ l \end{array}\right)(m-l)!(\xi|\alpha|)^{2k}(D+F+G+K)\right], \tag{28}$$

$$I_{\mathrm{o},m}^{dc} = 2I_{\mathrm{cr}}N_{-}^{-1}\mathrm{e}^{-\xi^2/2}\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\left[\left(\begin{array}{c} m \\ l \end{array}\right)\left(\begin{array}{c} m \\ l \end{array}\right)(m-l)!(\xi|\alpha|)^{2k}(D+F-G-K)\right]. \tag{29}$$

Here

$$D = \left(\begin{array}{c} l \\ 2k \end{array}\right)a_1^{l-2k}(D_1+D_2), \tag{30}$$

$$\begin{aligned} D_1 =& \sum_{j=0}^{k-1}(-1)^{2k+j}C_{2k}^{j}[J_{2n+2k-2j}(2\xi|\alpha|) \\ &+ J_{2n-2k+2j}(2\xi|\alpha|)]\sin(2n\phi), \end{aligned} \tag{31}$$

$$D_2 = (-1)^k C_{2k}^{k} J_{2n}(2\xi|\alpha|)\sin(2n\phi), \tag{32}$$

$$\begin{aligned} F =& -\xi|\alpha|\left(\begin{array}{c} l \\ 2k+1 \end{array}\right)a_1^{l-2k-1} \\ &\times\sum_{j=0}^{k}(-1)^{2k+j}C_{2k+1}^{j} \\ &\times[J_{2n+2k-2j+1}(2\xi|\alpha|) \\ &- J_{2n-2k+2j-1}(2\xi|\alpha|)]\sin(2n\phi), \end{aligned} \tag{33}$$

$$G = \mathrm{e}^{-2|\alpha|^2}\left(\begin{array}{c} l \\ 2k \end{array}\right)a_2^{l-2k}(G_1+G_2), \tag{34}$$

$$\begin{aligned} G_1 =& \sum_{j=0}^{k-1}C_{2k}^{j}[I_{2n+2k-2j}(2\xi|\alpha|) \\ &+ I_{2n-2k+2j}(2\xi|\alpha|)]\sin(2n\phi), \end{aligned} \tag{35}$$

$$G_2 = C_{2k}^{k}I_{2n}(2\xi|\alpha|)\sin(2n\phi), \tag{36}$$

$$\begin{aligned} K =& \mathrm{e}^{-2|\alpha|^2}\xi|\alpha|\left(\begin{array}{c} l \\ 2k+1 \end{array}\right)a_2^{l-2k-1}\sum_{j=0}^{k}C_{2k+1}^{j} \\ &\times[I_{2n+2k-2j+1}(2\xi|\alpha|) \\ &+I_{2n-2k+2j-1}(2\xi|\alpha|)]\sin(2n\phi), \end{aligned} \tag{37}$$

where $J_n(x)$ and $I_n(x)$ are Bessel and modified Bessel function, respectively, and we have used the formulae

$$\begin{aligned} \mathrm{e}^{\mathrm{i}x\sin\theta} &= \sum_{-\infty}^{\infty}J_n(x)\mathrm{e}^{\mathrm{i}n\theta}, \\ \mathrm{e}^{\mathrm{i}x\cos\theta} &= \sum_{-\infty}^{\infty}I_n(x)\mathrm{e}^{\mathrm{i}n\theta}. \end{aligned} \tag{38}$$

It can be seen from Eqs.(27)–(29) that EECS and EOCS have similar CVS. Obviously, when $\phi = k\pi/n$ $(n \neq 0, k = 0, \pm1, \pm2, \cdots)$, the dc component disappears, i.e., $I_{\mathrm{e},m}^{\mathrm{dc}} = I_{\mathrm{o},m}^{\mathrm{dc}} = 0$ ; and when $|\alpha| \gg 1$, it can be found that EECS and EOCS have the same CVS. It is easy to verify that the formulae of CVS of the supercurrent in Ref.[10] are special cases of the above results (28) and (29) when $m = 2$.

## 4. The dc component of fluctuation of the supercurrent

The fluctuation of the supercurrent is defined as $\langle(\Delta\widehat{I})^2\rangle = \langle\widehat{I}^2\rangle - \langle\widehat{I}\rangle^2$, which can be explicitly expressed as[10]

$$\langle(\Delta\widehat{I})^2\rangle = I_{\mathrm{cr}}^2[\langle\sin^2\widehat{\theta}\rangle - \langle\sin\widehat{\theta}\rangle^2]. \tag{39}$$

From Eq.(5) we have the following relations:

$$\begin{aligned} &{}_{\mathrm{e}}\langle\alpha,m|\sin\widehat{\theta}|\alpha,m\rangle_{\mathrm{e}} \\ =&{}_{\mathrm{e}}\langle\alpha,m|D[\xi\exp(\mathrm{i}\omega_1 t)]|\alpha,m\rangle_{\mathrm{e}}\sin\omega_0 t, \end{aligned} \tag{40}$$

$$\begin{aligned} &{}_{\mathrm{o}}\langle\alpha,m|\sin\widehat{\theta}|\alpha,m\rangle_{\mathrm{o}} \\ =&{}_{\mathrm{o}}\langle\alpha,m|D[\xi\exp(\mathrm{i}\omega_1 t)]|\alpha,m\rangle_{\mathrm{o}}\sin\omega_0 t, \end{aligned} \tag{41}$$

where

$${}_{\mathrm{e}}\langle\alpha,m|D[\xi\exp(\mathrm{i}\omega_1 t)]|\alpha,m\rangle_{\mathrm{e}}$$

and

$${}_{\mathrm{o}}\langle\alpha,m|D[\xi\exp(\mathrm{i}\omega_1 t)]|\alpha,m\rangle_{\mathrm{o}}$$

have been given in Eqs.(18) and (19), and we also have

$$\begin{aligned} {}_{\mathrm{e}}\langle\alpha,m|\sin^2\widehat{\theta}|\alpha,m\rangle_{\mathrm{e}} =& \frac{1}{2} - N_{+}^{-1}\mathrm{e}^{-2\xi^2}\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\left\{\left(\begin{array}{c} m \\ l \end{array}\right)\left(\begin{array}{c} m \\ l \end{array}\right)(m-l)! \\ &\times[(L_1+L_2)+\exp(-2|\alpha|^2)(M_1+M_2)]\right\}\cos 2\omega_0 t, \end{aligned} \tag{42}$$

$$ {}_{\rm o}\langle\alpha,m|\sin^2\widehat{\theta}|\alpha,m\rangle_{\rm o}=\frac{1}{2}-N_-^{-1}{\rm e}^{-2\xi^2}\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\Big\{\begin{pmatrix} m \\ l \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(m-l)! $$
$$ \times[(L_1+L_2)-\exp(-2|\alpha|^2)(M_1+M_2)]\Big\}\cos 2\omega_0 t. \tag{43} $$

Here

$$ L_1=\begin{pmatrix} l \\ 2k \end{pmatrix}(-1)^k b_1^{l-2k}(4u)^{2k}\cos 4u, \tag{44} $$

$$ L_2=-\begin{pmatrix} l \\ 2k+1 \end{pmatrix}(-1)^k b_1^{l-2k-1}(4u)^{2k+1}\sin 4u, \tag{45} $$

$$ M_1=\begin{pmatrix} l \\ 2k \end{pmatrix}b_2^{l-2k}(4v)^{2k}\cosh 4v, \tag{46} $$

$$ M_2=\begin{pmatrix} l \\ 2k+1 \end{pmatrix}b_2^{l-2k-1}(4v)^{2k+1}\sinh 4v, \tag{47} $$

where $u$ and $v$ have been given in Eq.(24) and we have let

$$ b_1=|\alpha|^2-4\xi^2, b_2=-|\alpha|^2-4\xi^2. \tag{48} $$

From Eqs.(39)–(43), we obtain some special values of dc voltage $V_0$ at which the supercurrent fluctuation has the following dc components:

(1) For $\omega_0-2n\omega_1=0$, i.e., $V_0=n\omega_1/e$ $(n=1,2,3,\cdots)$

$$ \langle(\Delta\widehat{I})^2\rangle_{{\rm e},m}^{\rm dc}=\frac{1}{2}I_{\rm cr}^2-I_{\rm cr}^2N_+^{-2}{\rm e}^{-\xi^2}\sin^2 2n\phi $$
$$ \times\Big\{\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\Big[\begin{pmatrix} m \\ l \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(m-l)!(\xi|\alpha|)^{2k}(D+F+G+K)\Big]\Big\}^2, \tag{49} $$

$$ \langle(\Delta\widehat{I})^2\rangle_{{\rm o},m}^{\rm dc}=\frac{1}{2}I_{\rm cr}^2-I_{\rm cr}^2N_-^{-2}{\rm e}^{-\xi^2}\sin^2 2n\phi $$
$$ \times\Big\{\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\Big[\begin{pmatrix} m \\ l \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(m-l)!(\xi|\alpha|)^{2k}(D+F-G-K)\Big]\Big\}^2, \tag{50} $$

where $D$, $F$, $G$ and $K$ have been given in Eqs.(30)–(37).

(2) For $2\omega_0-2n\omega_1=0$, *i.e.*, $V_0=n\omega_1/2e$ $(n=1,2,3,\cdots)$

$$ \langle(\Delta\widehat{I})^2\rangle_{{\rm e},m}^{\rm dc}=\frac{1}{2}I_{\rm cr}^2-I_{\rm cr}^2N_+^{-1}{\rm e}^{-2\xi^2} $$
$$ \times\Big\{\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\Big[\begin{pmatrix} m \\ l \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(m-l)!(2\xi|\alpha|)^{2k}(P+R+S+T)\Big]\Big\}, \tag{51} $$

$$ \langle(\Delta\widehat{I})^2\rangle_{{\rm o},m}^{\rm dc}=\frac{1}{2}I_{\rm cr}^2-I_{\rm cr}^2N_-^{-1}{\rm e}^{-2\xi^2} $$
$$ \times\Big\{\sum_{l=0}^{m}\sum_{k=0}^{[\frac{l}{2}]}\Big[\begin{pmatrix} m \\ l \end{pmatrix}\begin{pmatrix} m \\ l \end{pmatrix}(m-l)!(2\xi|\alpha|)^{2k}(P+R-S-T)\Big]\Big\}, \tag{52} $$

where

$$ P=\begin{pmatrix} l \\ 2k \end{pmatrix}b_1^{l-2k}(P_1+P_2), \tag{53} $$

$$ P_1=\sum_{j=0}^{k-1}(-1)^{2k+j}C_{2k}^j[J_{2n+2k-2j}(4\xi|\alpha|)+J_{2n-2k+2j}(4\xi|\alpha|)]\cos(2n\phi), \tag{54} $$

$$ P_2=(-1)^kC_{2k}^kJ_{2n}(4\xi|\alpha|)\cos(2n\phi), \tag{55} $$

$$ R=-2\xi|\alpha|\begin{pmatrix} l \\ 2k+1 \end{pmatrix}b_1^{l-2k-1}\sum_{j=0}^{k}(-1)^{2k+j}C_{2k+1}^j[J_{2n+2k-2j+1}(4\xi|\alpha|)-J_{2n-2k+2j-1}(4\xi|\alpha|)]\cos(2n\phi), \tag{56} $$

$$S = \mathrm{e}^{-2|\alpha|^2} \begin{pmatrix} l \\ 2k \end{pmatrix} b_2^{l-2k}(S_1 + S_2), \tag{57}$$

$$S_1 = \sum_{j=0}^{k-1} C_{2k}^j [I_{2n+2k-2j}(4\xi|\alpha|) + I_{2n-2k+2j}(4\xi|\alpha|)] \cos(2n\phi), \tag{58}$$

$$S_2 = C_{2k}^k I_{2n}(4\xi|\alpha|) \cos(2n\phi), \tag{59}$$

$$T = 2\mathrm{e}^{-2|\alpha|^2} \xi|\alpha| \begin{pmatrix} l \\ 2k+1 \end{pmatrix} b_2^{l-2k-1} \sum_{j=0}^{k} C_{2k}^j [I_{2n+2k-2j+1}(4\xi|\alpha|) + I_{2n-2k+2j-1}(4\xi|\alpha|)] \cos(2n\phi), \tag{60}$$

where $b_1$ and $b_2$ have been given in Eq.(48).

It can be seen from Eqs.(49)–(52) that for different CVSs the dc component fluctuations are quite different, and they also depend on the phase of the QEMF. It is easy to verify also that the dc component of fluctuation of the supercurrent in Ref.[10] are special cases of our results (49)–(52) when $m = 2$.

## 5. Correlation function of dc supercurrent components

The general expression of the second-order correlation function of the dc supercurrent component is written as

$$\langle g_{(t)}^{(2)} \rangle^{\mathrm{dc}} = \frac{\langle 1 - \mathrm{e}^{2\xi^2} \mathrm{Re}[\mathrm{e}^{\mathrm{i}2\omega_0 t} \langle\psi| D[2\xi \exp(\mathrm{i}\omega_1 t)] |\psi\rangle]\rangle_t}{2\langle (I_{\mathrm{m}}[\mathrm{e}^{\mathrm{i}\omega_0 t} \langle\psi| D[\xi \exp(\mathrm{i}\omega_1 t)] |\psi\rangle])^2 \rangle_t}, \tag{61}$$

where $\langle \cdots \rangle_t$ denotes the average over time. For the EECS and EOCS, respectively, we have

$$\langle g_{(t)}^{(2)} \rangle_{\mathrm{e},m}^{\mathrm{dc}} = \frac{1 - X_1}{Y_1}, \tag{62}$$

$$\langle g_{(t)}^{(2)} \rangle_{\mathrm{o},m}^{\mathrm{dc}} = \frac{1 - X_2}{Y_2}, \tag{63}$$

where

$$X_1 = 2N_+^{-1} \mathrm{e}^{-2\xi^2} \sum_{l=0}^{m} \sum_{k=0}^{[\frac{l}{2}]} \left\{ \begin{pmatrix} m \\ l \end{pmatrix} \begin{pmatrix} m \\ l \end{pmatrix} (m-l)! [(L_1 + L_2) + \mathrm{e}^{-2|\alpha|^2} (M_1 + M_2)] \right\} \cos 2\omega_0 t, \tag{64}$$

$$X_2 = 2N_-^{-1} \mathrm{e}^{-2\xi^2} \sum_{l=0}^{m} \sum_{k=0}^{[\frac{l}{2}]} \left\{ \begin{pmatrix} m \\ l \end{pmatrix} \begin{pmatrix} m \\ l \end{pmatrix} (m-l)! [(L_1 + L_2) - \mathrm{e}^{-2|\alpha|^2} (M_1 + M_2)] \right\} \cos 2\omega_0 t, \tag{65}$$

$$Y_1 = 2\left( 2N_+^{-1} \mathrm{e}^{-\xi^2/2} \sin\omega_0 t \sum_{l=0}^{m} \sum_{k=0}^{[\frac{l}{2}]} \left\{ \begin{pmatrix} m \\ l \end{pmatrix} \begin{pmatrix} m \\ l \end{pmatrix} (m-l)! [(A_1 + A_2) + \mathrm{e}^{-2|\alpha|^2} (B_1 + B_2)] \right\} \right)^2, \tag{66}$$

$$Y_2 = 2\left( 2N_-^{-1} \mathrm{e}^{-\xi^2/2} \sin\omega_0 t \sum_{l=0}^{m} \sum_{k=0}^{[\frac{l}{2}]} \left\{ \begin{pmatrix} m \\ l \end{pmatrix} \begin{pmatrix} m \\ l \end{pmatrix} (m-l)! [(A_1 + A_2) - \mathrm{e}^{-2|\alpha|^2} (B_1 + B_2)] \right\} \right)^2, \tag{67}$$

where $A_1$, $A_2$, $B_1$, $B_2$, $L_1$, $L_2$, $M_1$ and $M_2$ have been given in Eqs.(22), (23), (44), (45), (46) and (47).

From Eqs.(62)–(67), we find that the current fluctuations have the following dc components:

(1) For $\omega_0 - 2n\omega_1 = 0$, i.e., $V_0 = n\omega_1/e$ $(n = 1, 2, 3, \cdots)$,

$$\langle g_{(t)}^{(2)} \rangle_{\mathrm{e},m}^{\mathrm{dc}} = \frac{1}{2} N_+^2 \mathrm{e}^{\xi^2} \left\{ 2 \sum_{l=0}^{m} \sum_{k=0}^{[\frac{l}{2}]} \left[ \begin{pmatrix} m \\ l \end{pmatrix} \begin{pmatrix} m \\ l \end{pmatrix} (m-l)! (\xi|\alpha|)^{2k} (D + F + G + K) \sin 2n\phi \right] \right\}^{-2}, \tag{68}$$

$$\langle g_{(t)}^{(2)} \rangle_{\mathrm{o},m}^{\mathrm{dc}} = \frac{1}{2} N_-^2 \mathrm{e}^{\xi^2} \left\{ 2 \sum_{l=0}^{m} \sum_{k=0}^{[\frac{l}{2}]} \left[ \begin{pmatrix} m \\ l \end{pmatrix} \begin{pmatrix} m \\ l \end{pmatrix} (m-l)! (\xi|\alpha|)^{2k} (D + F - G - K) \sin 2n\phi \right] \right\}^{-2}, \tag{69}$$

where $D$, $F$, $G$ and $K$ have been given in Eqs.(30)–(37).

(2) For $2\omega_0 - 2n\omega_1 = 0$, i.e., $V_0 = n\omega_1/2e$ ($n = 1, 2, 3, \cdots$),

$$\langle g_{(t)}^{(2)} \rangle_{e,m}^{dc} = \langle g_{(t)}^{(2)} \rangle_{0,m}^{dc} = \infty. \tag{70}$$

It is shown that in the time evolution the quantum coherence of the Josephson current disappears at the singular points.

## 6. Conclusion

In this paper the interaction of MJJ with the excited even and odd coherent states as the external QEMF is studied. We have investigated the supercurrent in a MJJ and its quantum statistical properties. It is shown that the CR effects of the supercurrent, for EECS and EOCS, are very sensitive to the average photon number ($|\alpha|^2$) and the degree of excitation ($m$). The influence of external QEMF on CVS structures of the MJJ is discussed. It is found that the expectation values and the fluctuation of supercurrent can exhibit dc component and the conditions for the existence of the dc component are quite different. It is also shown that the static supercurrent flows, dc components of the supercurrent fluctuation and correlation function of the dc supercurrent components are related to the phase of the light field. Moreover, it is pointed out also that the results given in this paper are a supplement of Ref.[10], i.e. the effects of a QEMF acting on the MJJ discussed in Ref.[10] are only a special case of our general conclusion when $m = 2$.

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