Vol 14 No 5, May 2005
1009-1963/2005/14(05)/0980-05

Chinese Physics

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Evolution of field entropy and entanglement in the intensity-dependent two-mode Jaynes–Cummings model*

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(Received 31 August 2004; revised manuscript received 29 November 2004)

The evolution of the entanglement degree of two-mode fields and atom with the intensity-dependent coupling is investigated using von Neumann entropy. The results for the initial fields in both coherent states and two-mode squeezed vacuum state are calculated. The influence of the field intensity on the entropy is discussed. It is found that the field and atom are generally in maximum entanglement but subject to periodic pulsed disentanglement completely under the condition of strong initial field.

Keywords: degree of entanglement, von Neumann entropy, two-mode Jaynes–Cummings model, intensity-dependent coupling

PACC: 4250, 0365

1. Introduction

Quantum entanglement plays an essential role in quantum information theory. Entanglement is essential for quantum communication, quantum computation, quantum teleportation and quantum cryptography.[1−4] And disentanglement is also necessary for some aspects of quantum computation.[5] With the recent rapid developments in quantum information, the degree of entanglement has been the subject of considerable study.[6−8] It has been shown that entropy is a "good" measure of entanglement for two subsystems in a pure state.[8] Recently, much attention has been focused on the entanglement of the field and atom[9−14] in Jaynes–Cummings model (JCM) since a quantum logic gate was demonstrated successfully by using the entanglement state of the field and atom in an ideal cavity.[15,16] After the realization of a single-mode two-photon laser[17] based upon dressed atomic states, considerable attention has been payed to the two-photon JCM. It is well known that in two-photon processes, intensity-dependent effects must be taken into account, so Napoli and Messina generalized the intensity-dependent two-mode JCM.[18] In this paper, we study the degree of atom-field entanglement for the intensity-dependent two-mode two-photon JCM by using von Neumann entropy. The result shows that the time evolution of the field entropy has exact periodicity. Degree of atom-field entanglement is in maximum value with long period and goes to zero instantaneously for strong initial field(s).

2. Model and its quantum dynamics

We consider a two-level atom consisting of upper level $|+\rangle$ and lower level $|-\rangle$, with intensity- dependent coupling between two-mode quantized field through a two-photon process. Photons are either emitted or absorbed in pairs by the atom. The Hamiltonian under the rotating-wave approximation is given by[18,19]

$$H = H_0 + H_{\rm I}, \tag{1a}$$

$$H_0 = \sum_{i=1}^{2} \omega_i \left(a_i^+ a_i + \frac{1}{2} \right) + \frac{1}{2}\omega_0 \sigma_z, \tag{1b}$$

$$H_{\rm I} = g(\sqrt{a_1^+ a_1 a_2^+ a_2}\, a_1^+ a_2^+ \sigma + \sigma^+ a_1 a_2 \sqrt{a_1^+ a_1 a_2^+ a_2}), \tag{1c}$$

*Project supported by the Natural Science Foundation of Shandong Province of China (Grant No Y2004A09).
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where ω_0 and $\omega_i (i=1,2)$ are the frequencies of the atomic transition and the ith mode of the radiation field, respectively. a_i and a_i^+ are the annihilation and creation operators for the ith mode of the field, and σ_z, σ and σ^+ are the pseudospin operators for the atom. The constant g combined with the operator $\sqrt{a_1^+ a_1 a_2^+ a_2}$ plays the role of the intensity-dependent coupling constant between the atom and the two-mode cavity field.

Let

$$H|\Phi_j^N\rangle = E_j^n|\Phi_j^N\rangle. \quad (j=1,2) \tag{2}$$

The eigenvector of Hamiltonian (1) may be represented by

$$|\Phi_j^N\rangle = \sum_{m=1}^{2} C_{jm}^N|\Psi_m^N\rangle, \quad (j=1,2) \tag{3}$$

where

$$|\Psi_1^N\rangle = |n_1, n_2\rangle_{\rm F} \otimes |+\rangle,$$
$$|\Psi_2^N\rangle = |n_1+1, n_2+1\rangle_{\rm F} \otimes |-\rangle,$$

and $|n_i\rangle$ is the photon-number state of the ith mode field. Superscript N represents (n_1, n_2).

For simplicity, we consider the resonant case (i.e. $\omega_0 = \omega_1 + \omega_2$). With the help of Eqs.(2) and (3) the Hamiltonian (1) may be diagonalized and the result is

$$E_j^N = E_0^N \pm \Omega_N g \quad (j=1,2) \tag{4a}$$

with

$$E_0^N = \omega_1(n_1+1) + \omega_2(n_2+1), \tag{4b}$$
$$\Omega_N = (n_1+1)(n_2+1), \tag{4c}$$

and

$$C_{11} = C_{12} = C_{21} = -C_{22} = \frac{1}{\sqrt{2}}. \tag{4d}$$

Since C_{jm}^N is not related to n_i, we can omit its superscript N.

Let us consider the atom in its excited state $|+\rangle$ and the two fields are assumed to be initially in superposition states, i.e.

$$|\Psi(0)\rangle = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} q_1(n_1)q_2(n_2)|n_1, n_2\rangle \otimes |+\rangle, \tag{5}$$

where $q_j(n_j)$ describes the amplitude of state $|n_j\rangle$ of the jth mode. For the initial condition Eq.(5), the solution of the Schrödinger equation is given by

$$\begin{aligned}|\Psi(t)\rangle =& {\rm e}^{-{\rm i}Ht}|\Psi(0)\rangle \\ =& |C(t)\rangle \otimes |+\rangle + |D(t)\rangle \otimes |-\rangle\end{aligned} \tag{6}$$

with

$$|C(t)\rangle = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} {\rm e}^{-{\rm i}E_0^N t} q_1(n_1)q_2(n_2) \times A_N(t)|n_1, n_2\rangle, \tag{7a}$$

$$|D(t)\rangle = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} {\rm e}^{-{\rm i}E_0^N t} q_1(n_1)q_2(n_2) \times B_N(t)|n_1+1, n_2+1\rangle, \tag{7b}$$

and

$$A_N(t) = \cos(\Omega_N g t), \tag{8a}$$
$$B_N(t) = -{\rm i}\sin(\Omega_N g t). \tag{8b}$$

With the wavefunction $|\Psi(t)\rangle$ calculated, any property related to the atom or field can be ascertained.

3. Degree of entanglement

In the case of pure quantum states for two subsystems, a number of physically intuitive measures of entanglement have been known; however, for general mixed states of an arbitrary number of subsystems, entanglement measures are still under development. In this paper, we use the field entropy as a measurement of the degree of entanglement between the field and atom of the system under consideration. The field entropy is defined through reduced field density operator by

$$S_{\rm F}(t) = -{\rm Tr}(\rho_{\rm F}(t)\ln\rho_{\rm F}(t)). \tag{9}$$

By using equation (6) the reduced field density operator can be derived in the following form:

$$\begin{aligned}\rho_{\rm F}(t) \equiv& {\rm Tr}_{\rm atom}\rho(t) = {\rm Tr}_{\rm atom}|\Psi(t)\rangle\langle\Psi(t)| \\ =& |C(t)\rangle\langle C(t)| + |D(t)\rangle\langle D(t)|.\end{aligned} \tag{10}$$

To calculate the eigenvalues of the reduced field density operator, we write the eigen equation as

$$\rho_{\rm f}|\Psi_{\rm f}(t)\rangle = \pi_{\rm f}(t)|\Psi_{\rm f}(t)\rangle,$$

and let

$$|\Psi_{\rm f}(t)\rangle = \xi|C(t)\rangle + \eta|D(t)\rangle.$$

The eigenvalues of $\rho_{\rm F}(t)$ are

$$\begin{aligned}\pi_{\rm f}^{\pm}(t) =& \frac{1}{2}(\langle C|C\rangle + \langle D|D\rangle \\ & \pm\sqrt{(\langle C|C\rangle - \langle D|D\rangle)^2 + 4|\langle C|D\rangle|^2}).\end{aligned} \tag{11}$$

Obviously, it follows from Eq.(6), that

$$\langle\Psi(t)|\Psi(t)\rangle = \langle C(t)|C(t)\rangle + \langle D(t)|D(t)\rangle = 1. \tag{12}$$

Substitution of Eq.(12) into Eq.(11) gives

$$\pi_{\rm f}^{\pm}(t)=\frac{1}{2}(1\pm\sqrt{1-4(\langle C|C\rangle\langle D|D\rangle-|\langle C|D\rangle|^2)}) \tag{13a}$$

with

$$\langle C(t)|C(t)\rangle=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}p_1(n_1)p_2(n_2)A_N^2(t), \tag{13b}$$

$$\langle D(t)|D(t)\rangle=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}p_1(n_1)p_2(n_2)B_N^2(t), \tag{13c}$$

$$\langle C(t)|D(t)\rangle=\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}{\rm e}^{-{\rm i}\omega_0 t}q_1^*(n_1+1)q_1(n_1)\times q_2^*(n_2+1)q_2(n_2)A_{N'}^*(t)B_N(t), \tag{13d}$$

where $p_j(n_j)=|q_j(n_j)|^2,(j$=1,2). Subscript N' represents (n_1+1,n_2+1).

The field entropy $S_{\rm f}(t)$ may be expressed in terms of the eigenvalues $\pi_{\rm f}^{\pm}(t)$ for the reduced field density operator as[9]

$$S_{\rm f}(t)=-[\pi_{\rm f}^{+}(t)\ln\pi_{\rm f}^{+}(t)+\pi_{\rm f}^{-}(t)\ln\pi_{\rm f}^{-}(t)]. \tag{14}$$

It does not appear possible to express the sums in Eq.(13) in closed form, but for not too large $\bar{n}_j$, direct numerical evaluation can be performed. In the case of disentangled pure joint state $S_{\rm f}(t)$ is zero, and for maximal entangled states it is ln2.

4. Results and discussion

To investigate the evolution of $S_{\rm f}(t)$, we firstly discuss the periodicity of the eigenvalues $\pi_{\rm f}^{\pm}(t)$ with the help of Eqs.(13) , (8) and (4c). We can see $A_N(t)$ and $B_N(t)$ have a common periodicity $2\pi/g$ for all of the pairs (n_1, n_2) because n_1 and n_2 are both integers, then the field entropy $S_{\rm f}(t)$ has an exact periodicity 2π for scaled time gt. We display the numerical results with time from zero to $2\pi/g$.

4.1. Initial fields both in coherent state

For a coherent field, the amplitude of state $|n_j\rangle$ is described as

$$q_j(n_j)=\exp\left(-\frac{\bar{n}_j}{2}\right)\bar{n}_j^{\frac{n_j}{2}}(n_j!)^{-\frac{1}{2}}, \tag{15}$$

where $\bar{n}_j$ is the jth mode average photon number. The influence of the initial field intensity on the field entropy under the condition of $\bar{n}_1=\bar{n}_2(\equiv\bar{n})$ is shown in Fig.1. When the two fields are both in vacuum ($\bar{n}=0$), the field entropy as a function of scaled time gt evolves like a sine function with a periodicity $\pi/2$. The degree of the atom-field entanglement reaches maximum at $gt=\left(k+\frac{1}{2}\right)\frac{\pi}{2}$, ($k$=0,1,2,...). And disentanglement takes place at $gt=k\frac{\pi}{2}$, $(k=0,1,2,\ldots)$ (Fig.1(a)). When the average photon number $\bar{n}>0$, the periodicity of the entropy becomes 2π (see Fig.1).

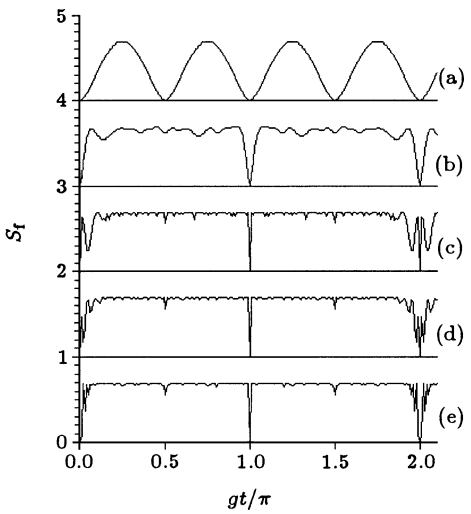


Fig.1. The evolution of the field entropy $S_{\rm f}$ as a function of the scaled time gt with the two mode fields both in coherent state; (a) $\bar{n}_1=\bar{n}_2$=0 with $S_{\rm f}$+4, (b) $\bar{n}_1=\bar{n}_2$=1 with$S_{\rm f}$+3, (c) $\bar{n}_1=\bar{n}_2$=4 with $S_{\rm f}$+2, (d) $\bar{n}_1=\bar{n}_2$=10 with $S_{\rm f}$+1, and (e) $\bar{n}_1=\bar{n}_2$=25.

The entanglement degree function oscillates near the maximum values when the time goes through almost the whole period but tends to zero at $gt=k\pi,(k=0,1,2,\ldots)$ instantaneously. The amplitude of the entanglement degree oscillations is decreased with the increasing of $\bar{n}$. Figure 2 shows the influence of the initial field intensity of one mode on the field entropy with the other mode in vacuum state. The curves of field entropy versus scaled time was similar to those in Fig.1 but the periodicity becomes π (except the case of $\bar{n}$=0). These results are quite different from the entropy of single-mode intensity-dependent JCM with large $\bar{n}$, which also oscillates from zero to ln2 with a periodicity π but reaches its maximum values instantaneously at only two times $gt\approx k\pi$.[11]

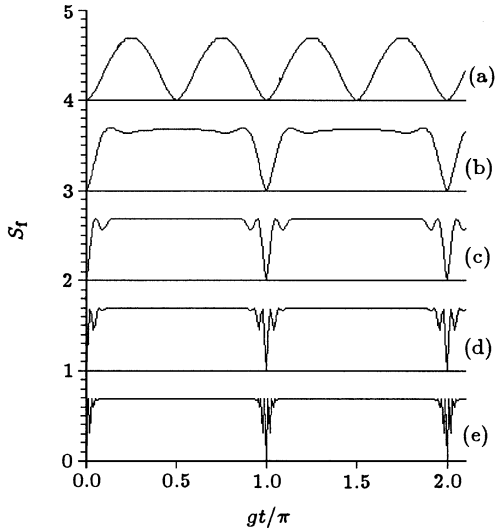


Fig.2. The same as Fig.1 but with $\bar{n}_2$=0.

4.2. Initial field in two-mode squeezed vacuum state

Two-mode squeezed vacuum field is a strong correlative state. Its photon number distribution can be given by

$$q_1(n_1)q_2(n_2) = \begin{cases} \dfrac{1}{\sqrt{1+\bar{n}}}\left(\dfrac{\bar{n}}{1+\bar{n}}\right)^{\frac{n}{2}}, & n_1 = n_2 \\ 0, & n_1 \neq n_2 \end{cases}$$

where $\bar{n}$ is the average photon number in each mode.

The evolution of field entropy for initial two-mode squeezed vacuum field is shown in Fig.3. Under the condition of weak field, the variation of entanglement is more acute than that in Fig.1. Under the condition of strong field, the entropy is stable at maximum values and the field is strongly entangled with the atom for most of the time; the entropy equals to zero and the field is completely disentangled from the atom instantaneously at $gt = k\pi, (k = 0, 1, 2, \ldots)$. This characteristic property appears also in the intensity-dependent single mode JCM[12] but their period is $\dfrac{\pi}{2}$. It would be usable for a quantum information process since the state of system is in stable strong entanglement but goes with pulsed complete disentanglement, because entanglement and disentanglement are both necessary for quantum computation.

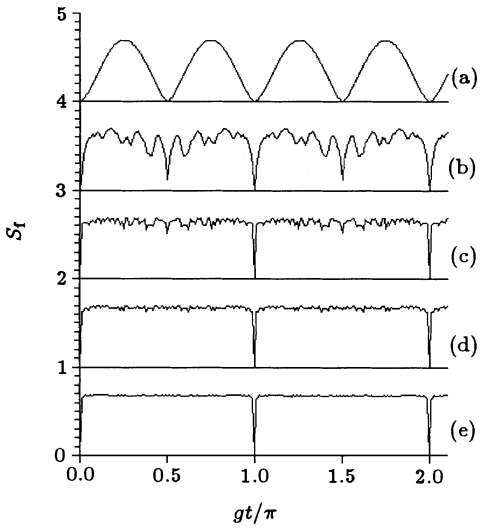


Fig.3. Evolution of the field entropy S_f as a function of the scaled time gt with two-mode squeezed vacuum state fields ; (a) $\bar{n}$=0 with S_f+4, (b) $\bar{n}$=1 with S_f+3, (c) $\bar{n}$=4 with S_f+2, (d) $\bar{n}$=10 withS_f+1, and (e) $\bar{n}$=25.

5. Conclusion

We have investigated the properties of entanglement between an atom and two-mode field with the intensity-dependent coupling in an ideal cavity by using reduced field entropy. It is found that the entropy has exact periodicity. Under the condition of weak initial field, the entanglement degree is affected strongly by superposition of field states. Under the condition of strong field, the influence of the initial field superposition on the evolution of entanglement degree is very weak. The entropy is stable at maximum values for almost all the time but goes to zero at $gt = k\pi, (k = 0, 1, 2, \ldots)$ instantaneously. The state of the system is in stable strong entanglement accompanied by periodically pulsed complete disentanglement.

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