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Proposal of many-party controlled teleportation for multi-qubit entangled W state*

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A scheme of M-party controlled teleportation for one N-qubit entangled W state via (N-1) Einstein–Podolsky–Rosen (EPR) pairs and one (M+2)-qubit Greenberger–Horne–Zeilinger (GHZ) state is proposed. We achieve the teleportation in such a way that M agents can execute the Hadamard transformation, perform the measurement on their qubits and inform the receiver of their measurements. Then we discuss that the receiver cannot fully recover the state from the sender if one agent does not co-operate with him.

Keywords: many-party controlled teleportation, multi-qubit entangled W state, Hadamard transformation, EPR pair, GHZ state

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1. Introduction

Quantum teleportation has received much attention because it plays an important role in quantum information and communication. Since Bennett *et al*[1] teleported an unknown quantum state of a two-state particle from one place to another with the aid of long-range Einstein–Podolsky–Rosen (EPR) correlations, scientists have made dramatic progress in the field of quantum teleportation. Quantum teleportation has been extended from discrete-variable systems to continuous-variable systems[2−6] and also from single particle to multi-particles.[7−10] Experimental demonstration of quantum teleportation has been realized with photon polarized states,[11] optical coherent states,[12] and nuclear magnetic resonance[13] recently.

In 2000, Zhou *et al* presented a theoretical scheme for controlled quantum teleportation.[14] According to the scheme, a third side is included beside a sender and a receiver, and the quantum channel is supervised by this additional side. The signal state cannot be transmitted unless all three sides agree to co-operate with each other. Recently, Yang *et al* have presented a way to teleport multi-qubit quantum information via the control of many agents in a network.[15] de Sen Aditi *et al* have found that the N-qubit states of W class,[16] for $N > 10$, lead to more robust violations of local realism than the Greenberger–Horne–Zeilinger (GHZ) states.

In this paper, we present a simple scheme to teleport an unknown N-qubit entangled W state from a sender to a remote receiver via the control of M agents in a network by using (N-1) EPR pairs and one (M+2)-qubit GHZ state[17] as the quantum channels. We show that the receiver can successfully get access to the original W state if all the agents collaborate with each other through their local operation and classical communication with the receiver; however, if even one of agents does not co-operate with the others, the original W state cannot be fully recovered by the receiver. Finally, we give an example of the controlled teleportation for three-qubit entangled W states.

2. Many-party controlled teleportation for multi-qubit entangled W state

We suppose that the sender Alice wants to teleport an unknown multi-qubit entangled W state to the receiver Bob. The W state of message qubits labelled

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by $(C_1, C_2, \cdots, C_N)$ can be expressed as[16]

$$\begin{aligned}|\varphi_w\rangle =&(c_0|000\ldots0\rangle + c_1|100\ldots0\rangle + c_2|010\ldots0\rangle \\ &+ \cdots\cdots + c_N|000\ldots1\rangle)_{C_1C_2\ldots C_N} \\ =&\sum_{k=0}^{N} c_k \prod_{i=1}^{N} |\delta_{ki}\rangle_{C_i}, \end{aligned} \tag{1}$$

where $\sum_{k=0}^{N} c_k = 1$, $c_k \neq 0$, $\delta_{ki} = \begin{cases} 0, & \text{if } k \neq i, \\ 1, & \text{if } k = i. \end{cases}$

Firstly, Alice prepares one (N-1) EPR pairs $|\phi\rangle_{AB}$ and one (M+2)-qubit GHZ state $|\phi\rangle_{A_NB_ND}$:

$$|\phi\rangle_{AB} = \prod_{i=1}^{N-1} |\phi^+\rangle_{A_iB_i} = \prod_{i=1}^{N-1} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_iB_i}, \tag{2}$$

$$|\phi\rangle_{A_NB_ND} = \frac{1}{\sqrt{2}}(|000\ldots0\rangle + |111\ldots1\rangle)_{A_NB_ND_1\cdots D_M}. \tag{3}$$

And then she sends the first (N-1) EPR qubits $(B_1, B_2, \ldots, B_{N-1})$ and one GHZ qubit B_N to Bob and M GHZ qubits to the M agents while keeps the GHZ qubit A_N and the other (N-1) EPR qubits $(A_1, A_2, \ldots, A_{N-1})$ by herself. The state of the whole system is given by

$$\begin{aligned}|\psi_{\rm T}\rangle =&|\varphi_w\rangle \otimes |\phi\rangle_{AB} \otimes |\phi\rangle_{A_NB_ND} \\ =&\sum_{k=0}^{N} c_k \prod_{i=1}^{N} |\delta_{ki}\rangle_{C_i} \otimes \prod_{i=1}^{N-1} \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)_{A_iB_i} \\ &\otimes \frac{1}{\sqrt{2}}(|000\ldots0\rangle + |111\ldots1\rangle)_{A_NB_ND_1\ldots D_M}. \end{aligned} \tag{4}$$

Secondly, Alice performs a series of joint Bell-state measurements on one EPR qubit and one message qubit $(A_i, C_i), (i = 1, 2, \cdots, N-1)$ and informs Bob of her measurements through a classical channel. According to Alice's measurements, Bob performs a series of single-qubit unitary transformations $U_i (i = 1, 2, \cdots, N-1)$ on his corresponding EPR qubits $B_i (i = 1, 2, \cdots, N-1)$.

Thirdly, Alice performs a Bell-state measurement on one message qubit and one GHZ qubit (A_N, C_N) and broadcasts her measurements. Then according to Alice's measurements, Bob performs a single-qubit unitary transformation U_N on his qubit B_N and all of M agents perform a series of single-qubit unitary transformations $(U_{D_1}, U_{D_2}, \cdots, U_{D_M})$ on their qubits $(D_1, D_2, \cdots, D_M)$.

After the preceding manipulations, the state becomes

$$\begin{aligned}|\psi_{\rm T}\rangle_1 =&U_1U_2\cdots U_{N-1}U_NU_{D_1}\cdots U_{D_M} \\ &\times {}_{A_1C_1}\langle\psi|_{A_2C_2}\langle\psi|\cdots{}_{A_NC_N}\langle\psi|\psi_{\rm T}\rangle \\ =&\sum_{k=0}^{N} c_k \Bigg[\Bigg(\prod_{i=1}^{N-1} U_{iA_iC_i}\langle\psi|\delta_{ki}\rangle_{C_i}|\phi\rangle_{A_iB_i}\Bigg) \\ &\otimes \Bigg(U_NU_{D_1}\cdots U_{D_M} \\ &\times {}_{A_NC_N}\langle\psi|\delta_{kN}\rangle_{C_N}|\phi\rangle_{A_NB_ND}\Bigg)\Bigg], \end{aligned} \tag{5}$$

where $|\psi\rangle_{A_iC_i}$ are four Bell states involved in Alice's Bell-state measurements on her qubits (A_i, C_i), which are expressed as

$$|\phi^{\pm}\rangle_{A_iC_i} = \frac{1}{\sqrt{2}}(|00\rangle \pm |11\rangle)_{A_iC_i}, \tag{6}$$

$$|\varphi^{\pm}\rangle_{A_iC_i} = \frac{1}{\sqrt{2}}(|01\rangle \pm |10\rangle)_{A_iC_i}. \tag{7}$$

The U_i and $U_{D_j} (j = 1, 2, \cdots, M)$ performed by Bob and one of M agents are Pauli matrices, which are shown in the following equation.

For simplicity, we define

$$|\eta\rangle_{B_i} = U_{iA_iC_i}\langle\psi|\delta_{ki}\rangle_{C_i}|\phi\rangle_{A_iB_i}, (N > i \geq 1), \tag{8}$$

$$|\eta\rangle_{B_ND} = U_NU_{D_1}\cdots U_{D_MA_NC_N}\langle\psi|\delta_{kN}\rangle_{C_N}|\phi\rangle_{A_NB_ND}. \tag{9}$$

From Eq.(8), we have

$$|\eta\rangle_{B_i} = \begin{cases} I_{B_i}({}_{C_i}\langle0|\delta_{ki}\rangle_{C_i}|0\rangle_{B_i} + {}_{C_i}\langle1|\delta_{ki}\rangle_{C_i}|1\rangle_{B_i}), & \text{for} \quad |\psi\rangle_{A_iC_i} = |\phi^+\rangle_{A_iC_i}, U_i = I_{B_i}, \\ (\sigma_z)_{B_i}({}_{C_i}\langle0|\delta_{ki}\rangle_{C_i}|0\rangle_{B_i} - {}_{C_i}\langle1|\delta_{ki}\rangle_{C_i}|1\rangle_{B_i}, & \text{for} \quad |\psi\rangle_{A_iC_i} = |\phi^-\rangle_{A_iC_i}, U_i = (\sigma_z)_{B_i}, \\ (\sigma_x)_{B_i}({}_{C_i}\langle1|\delta_{ki}\rangle_{C_i}|0\rangle_{B_i} + {}_{C_i}\langle0|\delta_{ki}\rangle_{C_i}|1\rangle_{B_i}), & \text{for} \quad |\psi\rangle_{A_iC_i} = |\varphi^+\rangle_{A_iC_i}, U_i = (\sigma_x)_{B_i}, \\ (\mathrm{i}\sigma_y)_{B_i}({}_{C_i}\langle1|\delta_{ki}\rangle_{C_i}|0\rangle_{B_i} - {}_{C_i}\langle0|\delta_{ki}\rangle_{C_i}|1\rangle_{B_i}), & \text{for} \quad |\psi\rangle_{A_iC_i} = |\varphi^-\rangle_{A_iC_i}, U_i = (\mathrm{i}\sigma_y)_{B_i}. \end{cases} \tag{10}$$

By calculation, we find that Eq.(10) can be reduced to the following expression:

$$|\eta\rangle_{B_i} = {}_{C_i}\langle0|\delta_{ki}\rangle_{C_i}|0\rangle_{B_i} + {}_{C_i}\langle1|\delta_{ki}\rangle_{C_i}|1\rangle_{B_i} = \begin{cases} |0\rangle_{B_i}, & k \neq i, \quad \delta_{ki} = 0, \\ |1\rangle_{B_i}, & k = i, \quad \delta_{ki} = 1, \end{cases} = |\delta_{ki}\rangle_{B_i} \quad (N > i \geq 1). \tag{11}$$

From Eq.(9), we have

$$|\eta\rangle_{B_N D}=\begin{cases} I_{B_N}\left(\prod_{j=1}^{M} I_{D_j}\right)({}_{C_N}\langle 0|\delta_{kN}\rangle_{CN}|00\cdots 0\rangle_{B_N D_1\cdots D_M C_N}+{}_{C_N}\langle 1|\delta_{kN}\rangle_{C_N}|11\cdots 1\rangle_{B_N D_1\cdots D_M}), \\ \quad \text{for} \quad |\psi\rangle_{A_N C_N}(|\phi^+\rangle_{A_N C_N}, \quad U_N=I_{B_N}, U_{D_1}=I_{D_1},\cdots,U_{D_M}=I_{D_M}, \\ (\sigma_z)_{B_N}\left(\prod_{j=1}^{M} I_{D_j}\right)({}_{C_N}\langle 0|\delta_{kN}\rangle_{CN}|00\cdots 0\rangle_{B_N D_1\cdots D_M}-{}_{C_N}\langle 1|\delta_{kN}\rangle_{C_N}|11\cdots 1\rangle_{B_N D_1\cdots D_M}), \\ \quad \text{for} \quad |\psi\rangle_{A_N C_N}=|\phi^-\rangle_{A_N C_N}, \quad U_N=(\sigma_z)_{B_N}, U_{D_1}=I_{D_1},\cdots,U_{D_M}=I_{D_M}, \\ (\sigma_x)_{B_N}\left[\prod_{j=1}^{M}(\sigma_x)_{D_j}\right]({}_{C_N}\langle 1|\delta_{kN}\rangle_{CN}|00\cdots 0\rangle_{B_N D_1\cdots D_M C_N}+\langle 0|\delta_{kN}\rangle_{C_N}|11\cdots 1\rangle_{B_N D_1\cdots D_M}), \\ \quad \text{for} \quad |\psi\rangle_{A_N C_N}=|\varphi^+\rangle_{A_N C_N}, \quad U_N=(\sigma_x)_{B_N}, U_{D_1}=(\sigma_x)_{D_1},\cdots,U_{D_M}=(\sigma_x)_{D_M}, \\ (\mathrm{i}\sigma_y)_{B_N}\left[\prod_{j=1}^{M}(\sigma_x)_{D_j}\right]({}_{C_N}\langle 1|\delta_{kN}\rangle_{CN}|00\cdots 0\rangle_{B_N D_1\cdots D_M}-{}_{C_N}\langle 0|\delta_{kN}\rangle_{C_N}|11\cdots 1\rangle_{B_N D_1\cdots D_M}), \\ \quad \text{for} \quad |\psi\rangle_{A_N C_N}=|\varphi^-\rangle_{A_N C_N}, \quad U_N=(\mathrm{i}\sigma_y)_{B_N}, U_{D_1}=(\sigma_x)_{D_1},\cdots,U_{D_M}=(\sigma_x)_{D_M}. \end{cases} \tag{12}$$

By calculation, we find that Eq.(12) can be expressed as

$$\begin{aligned} |\eta\rangle_{B_N D} &= {}_{C_N}\langle 0|\delta_{kN}\rangle_{CN}|00\cdots 0\rangle_{B_N D_1\cdots D_M}+{}_{C_N}\langle 1|\delta_{kN}\rangle_{C_N}|11\cdots 1\rangle_{B_N D_1\cdots D_M} \\ &= \begin{cases} |00\cdots 0\rangle_{B_N D_1\cdots D_M}, & k=1,2,\cdots N-1, \quad \delta_{kN}=0, \\ |11\cdots 1\rangle_{B_N D_1\cdots D_M}, & k=N, \qquad\qquad \delta_{kN}=1. \end{cases} \end{aligned} \tag{13}$$

Then, Eq.(5) can be rewritten as

$$|\psi_{\mathrm{T}}\rangle_1=\left(c_N|11\cdots 1\rangle_{B_N D_1\cdots D_M}+\sum_{k=0}^{N-1} c_k|00\cdots 0\rangle_{B_N D_1\cdots D_M}\right)\otimes\prod_{i=1}^{N-1}|\delta_{ki}\rangle_{B_i}. \tag{14}$$

Lastly, all of the M agents execute the Hadamard transformation on their qubits in the following forms: $|0\rangle_{D_j}\to\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_{D_j}$ and $|1\rangle_{D_j}\to\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)_{D_j}(j=1,2,\cdots,M)$. Then the state of Eq.(14) becomes

$$\begin{aligned} |\psi_{\mathrm{T}}\rangle_2 &= \left[(c_N|1\rangle_{B_N}\prod_{j=1}^{M}(|0\rangle-|1\rangle)_{D_j}+\sum_{k=0}^{N-1}c_k|0\rangle_{B_N}\otimes\prod_{j=1}^{M}(|0\rangle+|1\rangle)_{D_j}\right]\otimes\prod_{i=1}^{N-1}|\delta_{ki}\rangle_{B_i} \\ &= \Bigg[c_N|1\rangle_{B_N}\otimes\left(\sum|D_{\mathrm{e}}\rangle_{D_1\cdots D_M}-\sum|D_{\mathrm{o}}\rangle_{D_1\cdots D_M}\right) \\ &\quad +\sum_{k=0}^{N-1}c_k|0\rangle_{B_N}\otimes\left(\sum|D_{\mathrm{e}}\rangle_{D_1\cdots D_M}+\sum|D_{\mathrm{o}}\rangle_{D_1\cdots D_M}\right)\Bigg]\otimes\prod_{i=1}^{N-1}|\delta_{ki}\rangle_{B_i} \\ &= \sum|D_{\mathrm{e}}\rangle_{D_1\cdots D_M}\otimes\left[c_N|1\rangle_{B_N}+\sum_{k=0}^{N-1}c_k|0\rangle_{B_N}\right]\otimes\prod_{i=1}^{N-1}|\delta_{ki}\rangle_{B_i} \\ &\quad +\sum|D_{\mathrm{o}}\rangle_{D_1\cdots D_M}\otimes\left[-c_N|1\rangle_{B_N}+\sum_{k=0}^{N-1}c_k|0\rangle_{B_N}\right]\otimes\prod_{i=1}^{N-1}|\delta_{ki}\rangle_{B_i}, \end{aligned} \tag{15}$$

where $\sum|D_{\mathrm{e}}\rangle_{D_1\cdots D_M}$ is a sum over all possible binary-computational bases of M GHZ qubits (D_1, $D_2,\cdots,D_M$), for which each basis $|D_{\mathrm{e}}\rangle$ contains even number 1 (i.e. the number of $|1\rangle$ contained in each $|D_{\mathrm{e}}\rangle$ is even), while $\sum|D_{\mathrm{o}}\rangle_{D_1\cdots D_M}$ represents a sum over all possible binary-computational bases of qubits $(D_1,D_2,\cdots,D_M)$, for which each basis $|D_{\mathrm{o}}\rangle$ contains odd number 1 (i.e. the number of $|1\rangle$ contained in each $|D_{\mathrm{o}}\rangle$ is odd).

And then each agent makes a measurement on his GHZ qubit in the single-qubit computational basis $\{|0\rangle,|1\rangle\}$ and sends their measurements to receiver

Bob. From Eq.(15) one can see that if Bob knows the M agents' measurements for their qubits contain even number 1 (i.e. $|D_{\rm e}\rangle$), he can predicate that his N qubits must be in the state

$$|\psi_{\rm T}\rangle_3 = \left[c_N|1\rangle_{B_N} + \sum_{k=0}^{N-1} c_k|0\rangle_{B_N}\right] \otimes \prod_{i=1}^{N-1} |\delta_{ki}\rangle_{B_i} = \sum_{k=0}^{N} c_k \prod_{i=1}^{N} |\delta_{ki}\rangle_{B_i}. \quad (16)$$

On the other hand, if Bob knows that the M agents' measurements include odd number 1 (i.e. $|D_{\rm o}\rangle$), he knows that his N qubits must be in the state

$$|\psi_{\rm T}\rangle_4 = \left[-c_N|1\rangle_{B_N} + \sum_{k=0}^{N-1} c_k|0\rangle_{B_N}\right] \otimes \prod_{i=1}^{N-1} |\delta_{ki}\rangle_{B_i}. \quad (17)$$

Then Bob executes the unitary transformation $(\sigma_z)_{B_N}$ on the qubit B_N

$$(\sigma_z)_{B_N}|\psi_{\rm T}\rangle_4 = \sum_{k=0}^{N} c_k \prod_{i=1}^{N} |\delta_{ki}\rangle_{B_i} = |\psi_{\rm T}\rangle_3. \quad (18)$$

From Eq(16) and Eq(18), it is evident that Bob fully recovers the original state of Alice. So the many-party controlled teleportation of multi-qubit entangled W state is successfully achieved.

3. The many-party controlled teleportation is a failure if even one of agents does not co-operate with the others

In this section, our purpose is to show that in the case when even one of agents does not collaborate with the others, it is impossible for Bob to gain the full quantum message.

Assume that one agent, say agent D_M, does not co-operate with other agents. In this case, after other agents $(D_1, D_2, \cdots, D_{M-1})$ have performed a Hadamard transformation on their GHZ qubits in the following forms: $|0\rangle_{D_j} \to \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)_{D_j}$ and $|1\rangle_{D_j} \to \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)_{D_j} (j = 1, 2, \ldots, M-1)$, one can see that the state of Eq.(14) will be transformed into

$$\begin{aligned}
|\psi'_{\rm T}\rangle_2 =& \left(c_N|11\rangle_{B_N D_M} \prod_{j=1}^{M-1} (|0\rangle - |1\rangle)_{D_j} + \sum_{k=0}^{N-1} c_k|00\rangle_{B_N D_M} \otimes \prod_{j=1}^{M-1} (|0\rangle + |1\rangle)_{D_j}\right) \otimes \prod_{i=1}^{N-1} |\delta_{ki}\rangle_{B_i} \\
=& \left[c_N|11\rangle_{B_N D_M} \otimes \left(\sum |D_{\rm e}\rangle_{D_1 \cdots D_{M-1}} - \sum |D_{\rm o}\rangle_{D_1 \cdots D_{M-1}}\right)\right. \\
&\left. + \sum_{k=0}^{N-1} c_k|00\rangle_{B_N D_M} \otimes \left(\sum |D_{\rm e}\rangle_{D_1 \cdots D_{M-1}} + \sum |D_{\rm o}\rangle_{D_1 \cdots D_{M-1}}\right)\right] \otimes \prod_{i=1}^{N-1} |\delta_{ki}\rangle_{B_i} \\
=& \sum |D'_{\rm e}\rangle_{D_1 \cdots D_{M-1}} \otimes \left[c_N|11\rangle_{B_N D_M} + \sum_{k=0}^{N-1} c_k|00\rangle_{B_N D_M}\right] \otimes \prod_{i=1}^{N-1} |\delta_{ki}\rangle_{B_i} \\
&+ \sum |D'_{\rm o}\rangle_{D_1 \cdots D_{M-1}} \otimes \left[-c_N|11\rangle_{B_N D_M} + \sum_{k=0}^{N-1} c_k|00\rangle_{B_N D_M}\right] \otimes \prod_{i=1}^{N-1} |\delta_{ki}\rangle_{B_i}, \quad (19)
\end{aligned}$$

where $\sum |D'_{\rm e}\rangle_{D_1 \cdots D_{M-1}}$ represents a sum over all possible binary-computational bases of the $(M-1)$ GHZ qubits belonging to the agents $(D_1, D_2, \ldots, D_{M-1})$, for which each binary-computational bases $|D'_{\rm e}\rangle$ contains even number 1; while $\sum |D'_{\rm o}\rangle_{D_1 \cdots D_{M-1}}$ represents a sum over all possible binary-computational basis of the $(M-1)$ GHZ qubits $(D_1, D_2, \cdots, D_{M-1})$, for which each binary-computational basis $|D'_{\rm o}\rangle$ contains odd number 1.

From Eq.(19) one can see that for every measurement on the $(M-1)$ GHZ qubits $(D_1, D_2, \ldots, D_{M-1})$, the density operator ρ_B of the N qubits $(B_1, B_2, \cdots, B_N)$ belonging to Bob, after having traced over the qubit D_M, can be written as

$$\rho_B = {\rm Tr}_{D_M}(|\psi'_{\rm T}\rangle_{33}\langle\psi'_{\rm T}|), \quad (20)$$

where

$$|\psi'_{\rm T}\rangle_3 = \left[c_N|11\rangle_{B_N D_M} + \sum_{k=0}^{N-1} c_k|00\rangle_{B_N D_M}\right] \otimes \prod_{i=1}^{N-1} |\delta_{ki}\rangle_{B_i}. \quad (21)$$

The trace of ρ_B^2 is

$$\begin{aligned}\mathrm{Tr}\rho_B^2 =&\mathrm{Tr}[\mathrm{Tr}_{D_M}(|\psi_{\mathrm{T}}'\rangle_3\langle\psi_{\mathrm{T}}'|)]^2\\ =&1-2\sum_{k=0}^{N-1}c_k^2\times c_N^2<1.\end{aligned}\tag{22}$$

From Eqs.(19), (20), (21) and (22), one can see that Bob cannot fully recover the original state of Alice if even one of agents does not co-operate with the others.

4. The case of three-qubit entangled W state via three agents

For example, Alice wants to teleport three-qubit entangled W state $|\varphi_W\rangle$ to Bob via three agents. The state $|\varphi_W\rangle$ is

$$\begin{aligned}|\varphi_W\rangle =&(c_0|000\rangle+c_1|100\rangle+c_2|010\rangle\\ &+c_3|001\rangle)_{C_1C_2C_3}.\end{aligned}\tag{23}$$

Firstly Alice prepares two EPR pairs,

$$|\phi\rangle_{A_iB_i}=\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{A_iB_i}\quad(i=1,2)\tag{24}$$

and a five-qubit GHZ state.

$$|\phi\rangle_{A_3B_3D}=\frac{1}{\sqrt{2}}(|00000\rangle+|11111\rangle)_{A_3B_3D_1D_2D_3}.\tag{25}$$

Then Alice sends the first two EPR qubits (B_1, B_2) and one GHZ qubit B_3 to Bob and 3 qubits (D_1, D_2, D_3) to the three agents while keeps qubits (A_1, A_2, A_3) by herself.

The state of the whole system is given by

$$\begin{aligned}|\psi_{\mathrm{T}}\rangle =&|\varphi_W\rangle\otimes|\phi\rangle_{A_1B_1}\otimes|\phi\rangle_{A_2B_2}\otimes|\phi\rangle_{A_3B_3D}\\ =&(c_0|000\rangle+c_1|100\rangle+c_0|010\rangle+c_0|001\rangle)_{C_1C_2C_3}\\ &\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{A_1B_1}\\ &\otimes\frac{1}{\sqrt{2}}(|00\rangle+|11\rangle)_{A_2B_2}\\ &\otimes\frac{1}{\sqrt{2}}(|00000\rangle+|11111\rangle)_{A_3B_3D_1D_2D_3}.\end{aligned}\tag{26}$$

Secondly, Alice performs a joint Bell-state measurement on her qubits (A_1, C_1), (A_2, C_2), (A_3, C_3), and informs Bob and the three agents of her measurements through a classical channel. According to Alice's measurements, Bob performs three corresponding unitary transformations on his qubits (B_1, B_2, B_3), and the three agents perform a corresponding unitary transformation on their qubits (D_1, D_2, D_3). After the above manipulations, the state becomes

$$\begin{aligned}|\psi_{\mathrm{T}}\rangle_1 =&(c_0|000\rangle+c_1|100\rangle+c_2|010\rangle)_{B_1B_2B_3}\\ &\otimes|000\rangle_{D_1D_2D_3}+c_3|001\rangle_{B_1B_2B_3}\\ &\otimes|111\rangle_{D_1D_2D_3}.\end{aligned}\tag{27}$$

Lastly the three agents execute the Hadamard transformation on their qubits in the following forms $|0\rangle_{D_j}\to\frac{1}{\sqrt{2}}(|0\rangle+|1\rangle)_{D_j}$ and $|1\rangle_{D_j}\to\frac{1}{\sqrt{2}}(|0\rangle-|1\rangle)_{D_j}$ (j=1, 2, 3). The Eq.(25) transfers to

$$\begin{aligned}|\psi_{\mathrm{T}}\rangle_2 =&(c_0|000\rangle+c_1|100\rangle+c_2|010\rangle+c_3|001\rangle)_{B_1B_2B_3}\\ &\otimes(|000\rangle+|011\rangle+|101\rangle+|110\rangle)_{D_1D_2D_3}\\ &+(c_0|000\rangle+c_1|100\rangle\\ &+c_2|010\rangle-c_3|001\rangle)_{B_1B_2B_3}\\ &\otimes(|001\rangle+|010\rangle+|100\rangle+|111\rangle)_{D_1D_2D_3}.\end{aligned}\tag{28}$$

Then, the agents perform a measurement on their qubits in the single-qubit computational basis $\{|0\rangle, |1\rangle\}$ and send their measurements to Bob. If their GHZ qubits contain even number $|1\rangle$ (i.e. $|000\rangle$, $|011\rangle$, $|101\rangle$, $|110\rangle$), Bob can predicate that his three qubits must be in the state,

$$\begin{aligned}|\psi_{\mathrm{T}}\rangle_3 =&(c_0|000\rangle+c_1|100\rangle+c_2|010\rangle\\ &+c_3|001\rangle)_{B_1B_2B_3}.\end{aligned}\tag{29}$$

If their GHZ qubits contains odd number $|1\rangle$ (i.e. $|001\rangle$, $|010\rangle$, $|100\rangle$, $|111\rangle$), Bob can predicate that his three qubits must be in the state,

$$\begin{aligned}|\psi_{\mathrm{T}}\rangle_4 =&(c_0|000\rangle+c_1|100\rangle+c_2|010\rangle\\ &+c_3|001\rangle)_{B_1B_2B_3}.\end{aligned}\tag{30}$$

Then Bob executes the unitary transformation $(\sigma_z)_{B_3}$ on the qubit B_3,

$$(\sigma_z)_{B_3}|\psi_{\mathrm{T}}\rangle_4=|\psi_{\mathrm{T}}\rangle_3.\tag{31}$$

From Eq.(27) and Eq.(29), it is evident that Bob can fully recover the original three-qubit entangled W state of Alice under the co-operation with three agents.

5. Conclusion

We have proposed a simple scheme for the M-party controlled teleportation of N-particle entangled W state. First, the sender (Alice) and the receiver (Bob) need to share (N-1)-EPR pairs, and Alice, Bob and M controlled-sides need to share a (M+2)-qubit GHZ state. In order to implement such a teleportation, it requires that Alice perform N-times two-qubit

Bell-state measurement and Bob perform N-times a single-qubit unitary transformation and all agents perform a single-qubit unitary transformation. But Bob cannot acquire the message from Alice because the qubits of Bob and M agents are in an entangled state. If all agents agree to co-operate with each other, they execute the Hadamard transformation and measurement on their qubits and inform Bob of their measurements, Bob can achieve the teleportation. Then we discuss that Bob cannot fully recover the original state of Alice if one of agents does not co-operate with the others because the qubits of Bob are in the mixed state. So the teleportation is a failure. Lastly, we have achieved the controlled teleportation as an example of three-qubit entangled W state.

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