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H_∞ tracking control of coupled spatiotemporal chaos with parametric uncertainties based on fuzzy observers*

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Due to the interactions among coupled spatiotemporal subsystems and the parametric uncertainties, it is difficult to achieve the tracking control of the coupled spatiotemporal chaos with parametric uncertainties. However, every subsystem of the coupled spatiotemporal chaos can be approximated by a set of fuzzy models. Considering that all states of the subsystem are not available, fuzzy observers are suggested to estimate these states. Based on these fuzzy models and observers, and the consideration of the parametric uncertainties, a new H_∞ fuzzy tracking control scheme is proposed, where a linear matrix inequality (LMI) is employed to represent the feedback controller. The parameters of the controller are obtained by using convex optimization techniques of LMI. The robust stability of the controlled system is guaranteed in this scheme. The tracking performances are tested by simulation as an example.

Keywords: spatiotemporal chaos, fuzzy observers, fuzzy tracking control, linear matrix inequalities (LMI)

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1. Introduction

Because of large-scale nonlinear chaotic systems rapidly emerging and studies of chaotic control problems continuously penetrating, more and more people are interested in more complicated chaotic control problems, such as the spatiotemporal chaotic control problem, and so on.[1,2] In general, the stabilization and the tracking are two typical control problems; the former is more difficult than the latter. For this reason, there has existed no satisfactory tracking control scheme for the coupled spatiotemporal chaos with parametric uncertainties so far.[3]

In the past few years, there has been a significant discovery that a nonlinear system can be well approximated by a Takagi–Sugeno (T–S) fuzzy linear model. Therefore the interest in fuzzy control for nonlinear chaos is rapidly growing. However, there exist very few studies concerning the fuzzy control for the coupled spatiotemporal chaos with parametric uncertainties based on the T–S model.[4–7]

In order to solve this kind of difficult problem, first, the coupled spatiotemporal chaos is approximated by T–S fuzzy models. Considering that all states of the subsystem are not all available, fuzzy observers are suggested to estimate these states. Second, a new H_∞ fuzzy tracking control scheme is designed for the coupled spatiotemporal chaos with parametric uncertainties. Finally, the problem about tracking control of output feedback is characterized by solving a linear matrix inequality (LMI). The convex optimization techniques of LMI are used to achieve the parameters of controller here in this paper.[8–19] In the scheme, the external disturbances can be overridden and the robust gradual stability of the coupled spatiotemporal chaos with parametric uncertainties can be also guaranteed. Therefore, the difficult problem about tracking control of the coupled spatiotemporal chaos with parametric uncertainties is solved. The tracking performances are well tested by simulation as an example.

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2. The fuzzy models of the coupled spatiotemporal chaos with parametric uncertainties

Consider a class of coupled spatiotemporal chaos with parametric uncertainties which is composed of

$$\begin{aligned}\dot{\boldsymbol{x}}_i(t) &= f_i(\boldsymbol{X}(t), \boldsymbol{u}_i(t)) + \boldsymbol{\omega}_i(t),\\ \boldsymbol{y}_i(t) &= g_i(\boldsymbol{x}_i(t)) + \boldsymbol{\nu}_i(t),\end{aligned} \tag{1}$$

where $\boldsymbol{x}_i(t) \in R^{n_i}$ is the state vector of the ith subsystem, $\boldsymbol{u}_i(t) \in R^{m_i}$ is the control vector of the ith subsystem, $\boldsymbol{\omega}_i(t)$, $\boldsymbol{y}_i(t)$ and $\boldsymbol{\nu}_i(t)$ are respectively the external disturbance vector, output vector and measurement noise vector of the ith subsystem, $i = 1, 2, \ldots, n$, $N = \sum_{i=1}^{n} n_i$, $f_i(\cdot)$ and $g_i(\cdot)$ are the smooth nonlinear functions of the ith subsystem, $\boldsymbol{X}(t) = (\boldsymbol{x}_1^{\mathrm{T}}(t), \boldsymbol{x}_2^{\mathrm{T}}(t), \ldots, \boldsymbol{x}_n^{\mathrm{T}}(t))^{\mathrm{T}} \in R^N$.

It is very difficult to design an overall controller for the coupled spatiotemporal chaos with parametric uncertainties based on Eq. (1). The T–S fuzzy models with parametric uncertainties can be employed to represent every subsystem of the coupled spatiotemporal chaos. And these models are described by fuzzy If–Then rules in the following form.[6]

T–S fuzzy model of the ith subsystem

R_{p}^k: If $z_1(t)$ is $F_{k1}, \ldots,$ and $z_g(t)$ is F_{kg}, then

$$\begin{cases}\dot{\boldsymbol{x}}_i(t) = (\boldsymbol{A}_{ik} + \Delta\boldsymbol{A}_{ik})\boldsymbol{X}(t)\\ \qquad\quad +(\boldsymbol{B}_{ik} + \Delta\boldsymbol{B}_{ik})\boldsymbol{u}_i(t) + \boldsymbol{\omega}_i(t),\\ \boldsymbol{y}_i(t) = \boldsymbol{C}_{ik}\boldsymbol{x}_i(t) + \boldsymbol{\nu}_i(t),\end{cases}$$

where $k = 1, 2, \ldots, l$ are fuzzy rule numbers, $F_{k1}, \ldots, F_{kg}$ is fuzzy set, and $z_1(t), \ldots, z_g(t)$ are premise variables. $\boldsymbol{A}_{ik}$, $\boldsymbol{B}_{ik}$ and $\boldsymbol{C}_{ik}$ are coefficient matrices, $\Delta\boldsymbol{A}_{ik}$ and $\Delta\boldsymbol{B}_{ik}$ are uncertain parameter matrices.

The overall fuzzy model of the subsystem can be rearranged in the following form

$$\begin{cases}\dot{\boldsymbol{x}}_i(t) = \sum\limits_{k=1}^{l} h_k(z(t))[(\boldsymbol{A}_{ik} + \Delta\boldsymbol{A}_{ik})\boldsymbol{X}(t)\\ \qquad\quad +(\boldsymbol{B}_{ik} + \Delta\boldsymbol{B}_{ik})\boldsymbol{u}(t) + \boldsymbol{\omega}_i(t)],\\ \boldsymbol{y}_i(t) = \sum\limits_{k=1}^{l} h_k(z(t))[\boldsymbol{C}_{ik}\boldsymbol{x}_i(t) + \boldsymbol{\nu}_i(t)],\end{cases}$$

where $\mu_k(z(t)) = \prod_{j=1}^{g} F_{kj}(z_j(t))$,

$$h_k(z(t)) = \mu_k(z(t)) \Big/ \sum_{k=1}^{l} \mu_k(z(t)).$$

Assume $\mu_k(z(t)) \geq 0$; therefore, $h_k(z(t)) \geq 0$, and $\sum_{k=1}^{l} h_k(z(t)) = 1$.

Considering that all states of the subsystem are not available, the fuzzy observer of the ith subsystem is assumed to be in the following form R_p^k: if $z_1(t)$ is $F_{k1}, \ldots,$ and $z_g(t)$ is F_{kg}, then

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}}_i(t) =&(\boldsymbol{A}_{ik} + \Delta\boldsymbol{A}_{ik})\hat{\boldsymbol{X}}(t) + (\boldsymbol{B}_{ik} + \Delta\boldsymbol{B}_{ik})\boldsymbol{u}_i(t)\\ &+ \boldsymbol{L}_{ik}(\boldsymbol{y}_i(t) - \hat{\boldsymbol{y}}_i(t)),\end{aligned}$$

where $\hat{\boldsymbol{y}}_i(t) = \boldsymbol{C}_{ik}\hat{\boldsymbol{x}}_i(t)$; $\hat{\boldsymbol{x}}_i(t)$, $\hat{\boldsymbol{y}}_i(t)$ and $\boldsymbol{L}_{ik}$ are respectively the state estimation vector, output vector and gain matrix of the ith subsystem observer.

The fuzzy observer of the subsystem can be rearranged in the following form:

$$\begin{aligned}\dot{\hat{\boldsymbol{x}}}_i(t) =& \sum_{k=1}^{l} h_k(z(t))[(\boldsymbol{A}_{ik} + \Delta\boldsymbol{A}_{ik})\hat{\boldsymbol{X}}(t) + (\boldsymbol{B}_{ik}\\ &+ \Delta\boldsymbol{B}_{ik})\boldsymbol{u}_i(t) + \boldsymbol{L}_{ik}(\boldsymbol{y}_i(t) - \hat{\boldsymbol{y}}_i(t))].\end{aligned}$$

The overall coupled spatiotemporal chaos with parametric uncertainties can be arranged as follows:

$$\begin{cases}\dot{\boldsymbol{X}}(t) = \sum\limits_{k=1}^{l} h_k(z(t))[(\boldsymbol{A}_k + \Delta\boldsymbol{A}_k)\boldsymbol{X}(t)\\ \qquad\quad +(\boldsymbol{B}_k + \Delta\boldsymbol{B}_k)\boldsymbol{u}(t) + \boldsymbol{\omega}(t)],\\ \boldsymbol{y}(t) = \sum\limits_{k=1}^{l} h_k(z(t))[\boldsymbol{C}_k\boldsymbol{X}(t) + \boldsymbol{\nu}(t)].\end{cases} \tag{2}$$

The overall fuzzy observer can be arranged in the following form:

$$\begin{aligned}\dot{\hat{\boldsymbol{X}}}(t) =& \sum_{k=1}^{l} h_k(z(t))[(\boldsymbol{A}_k + \Delta\boldsymbol{A}_k)\hat{\boldsymbol{X}}(t)\\ &+ (\boldsymbol{B}_k + \Delta\boldsymbol{B}_k)\boldsymbol{u}(t) + \boldsymbol{L}_k(\boldsymbol{y}(t) - \hat{\boldsymbol{y}}(t))],\end{aligned} \tag{3}$$

where $\boldsymbol{A}_k = (\boldsymbol{A}_{1k}^{\mathrm{T}}, \ldots, \boldsymbol{A}_{nk}^{\mathrm{T}})^{\mathrm{T}}$,

$$\begin{aligned}&\Delta\boldsymbol{A}_k = (\Delta\boldsymbol{A}_{1k}^{\mathrm{T}}, \ldots, \Delta\boldsymbol{A}_{nk}^{\mathrm{T}})^{\mathrm{T}}, \quad \boldsymbol{B}_k = \mathrm{diag}(\boldsymbol{B}_{ik}),\\ &\Delta\boldsymbol{B}_k = \mathrm{diag}(\Delta\boldsymbol{B}_{ik}), \quad \boldsymbol{C}_k = \mathrm{diag}(\boldsymbol{C}_{ik}),\\ &\boldsymbol{u}(t) = [\boldsymbol{u}_1^{\mathrm{T}}(t), \ldots, \boldsymbol{u}_n^{\mathrm{T}}(t)]^{\mathrm{T}},\\ &\boldsymbol{\omega}(t) = [\boldsymbol{\omega}_1^{\mathrm{T}}(t), \ldots, \boldsymbol{\omega}_n^{\mathrm{T}}(t)]^{\mathrm{T}},\\ &\boldsymbol{y}(t) = [\boldsymbol{y}_1^{\mathrm{T}}(t), \ldots, \boldsymbol{y}_n^{\mathrm{T}}(t)]^{\mathrm{T}},\\ &\hat{\boldsymbol{y}}(t) = [\hat{\boldsymbol{y}}_1^{\mathrm{T}}(t), \ldots, \hat{\boldsymbol{y}}_n^{\mathrm{T}}(t)]^{\mathrm{T}},\\ &\boldsymbol{\nu}(t) = [\boldsymbol{\nu}_1^{\mathrm{T}}(t), \ldots, \boldsymbol{\nu}_n^{\mathrm{T}}(t)]^{\mathrm{T}}, \quad \boldsymbol{L}_k = \mathrm{diag}(\boldsymbol{L}_{ik}).\end{aligned}$$

3. H_∞ tracking control for the coupled spatiotemporal chaos with parametric uncertainties

Consider a reference model for the chaotic system in the form

$$\dot{\boldsymbol{X}}_{\mathrm{r}}(t) = \boldsymbol{A}_{\mathrm{r}}\boldsymbol{X}_{\mathrm{r}}(t) + \boldsymbol{r}(t), \tag{4}$$

where $\boldsymbol{X}_{\mathrm{r}}(t) \in R^N$ is the reference state, $\boldsymbol{A}_{\mathrm{r}}$ is a specific asymptotically stable matrix, and $\boldsymbol{r}(t) \in R^N$ is the bounded reference input.

The fuzzy controller is designed to be in the following form

R_p^k: If $z_1(t)$ is $F_{k1}, \ldots$, and $z_g(t)$ is F_{kg}, then

$$\boldsymbol{u}(t) = \boldsymbol{K}_k(\hat{\boldsymbol{X}}(t) - \boldsymbol{X}_{\mathrm{r}}(t)).$$

And the overall controller has the form

$$\boldsymbol{u}(t) = \sum_{k=1}^{l} h_k(z(t))[\boldsymbol{K}_k(\hat{\boldsymbol{X}}(t) - \boldsymbol{X}_{\mathrm{r}}(t)]. \tag{5}$$

Let the estimative error be denoted by

$$\boldsymbol{e}(t) = \boldsymbol{X}(t) - \hat{\boldsymbol{X}}(t). \tag{6}$$

Substituting Eqs.(2) and (3) into Eq.(6) yields

$$\begin{aligned}\dot{\boldsymbol{e}}(t) =& \dot{\boldsymbol{X}}(t) - \dot{\hat{\boldsymbol{X}}}(t)\\ =& \sum_{k=1}^{l}\sum_{m=1}^{l} h_k(z(t))h_m(z(t))\\ & \times [(\boldsymbol{A}_k - \boldsymbol{L}_k\boldsymbol{C}_m)\boldsymbol{e}(t) + \Delta\boldsymbol{A}_k\boldsymbol{e}(t)\\ & + \boldsymbol{\omega}(t) - \boldsymbol{L}_k\boldsymbol{\nu}(t)].\end{aligned} \tag{7}$$

Substituting Eqs.(5) and (6) into Eq.(2) gives

$$\begin{aligned}\dot{\boldsymbol{X}}(t) =& \sum_{k=1}^{l}\sum_{m-1}^{l} h_k(z(t))h_m(z(t))\\ & \times [(\boldsymbol{A}_k + \boldsymbol{B}_k\boldsymbol{K}_m)\boldsymbol{X}(t)\\ & + (\Delta\boldsymbol{A}_k + \Delta\boldsymbol{B}_k\boldsymbol{K}_m)\boldsymbol{X}(t)\\ & - \boldsymbol{B}_k\boldsymbol{K}_m\boldsymbol{X}_{\mathrm{r}}(t) - \Delta\boldsymbol{B}_k\boldsymbol{K}_m\boldsymbol{X}_{\mathrm{r}}(t)\\ & - \boldsymbol{B}_k\boldsymbol{K}_m\boldsymbol{e}(t) - \Delta\boldsymbol{B}_k\boldsymbol{K}_m\boldsymbol{e}(t) + \boldsymbol{\omega}(t)].\end{aligned} \tag{8}$$

H_∞ tracking performance is related to the tracking error in the following form:[5,6]

$$\frac{\displaystyle\int_0^{t_f} [(\boldsymbol{X}(t) - \boldsymbol{X}_{\mathrm{r}}(t))^{\mathrm{T}}\boldsymbol{Q}(\boldsymbol{X}(t) - \boldsymbol{X}_{\mathrm{r}}(t))]\mathrm{d}t}{\displaystyle\int_0^{t_f} \hat{\boldsymbol{\omega}}(t)^{\mathrm{T}}\hat{\boldsymbol{\omega}}(t)\mathrm{d}t} \le \rho^2,$$

where $\hat{\boldsymbol{\omega}}(t) = [\boldsymbol{\omega}^{\mathrm{T}}(t), \boldsymbol{r}^{\mathrm{T}}(t)]^{\mathrm{T}}$, and $\boldsymbol{Q} = \boldsymbol{Q}^{\mathrm{T}} > 0$.

By use of Eqs.(8), (4) and (7) the tracking system can be represented as follows:

$$\begin{aligned}\dot{\bar{\boldsymbol{X}}}(t) =& \sum_{k=1}^{l}\sum_{m=1}^{l} h_k(z(t))h_m(z(t))\{\boldsymbol{A}_{km}\bar{\boldsymbol{X}}(t)\\ & + (\Delta\hat{\boldsymbol{A}}_k + \Delta\hat{\boldsymbol{B}}_k\hat{\boldsymbol{K}}_m)\bar{\boldsymbol{X}}(t) + \boldsymbol{D}_k\bar{\boldsymbol{\omega}}(t)\},\end{aligned} \tag{9}$$

where $\bar{\boldsymbol{X}}(t) = [\boldsymbol{X}^{\mathrm{T}}(t), \boldsymbol{X}_{\mathrm{r}}^{\mathrm{T}}(t), \boldsymbol{e}^{\mathrm{T}}(t)]^{\mathrm{T}}$,

$$\boldsymbol{A}_{km} = \begin{bmatrix} \boldsymbol{A}_k + \boldsymbol{B}_k\boldsymbol{K}_m & -\boldsymbol{B}_k\boldsymbol{K}_m & -\boldsymbol{B}_k\boldsymbol{K}_m \\ \boldsymbol{0} & \boldsymbol{A}_{\mathrm{r}} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{A}_k - \boldsymbol{L}_k\boldsymbol{C}_m \end{bmatrix},$$

$$\Delta\hat{\boldsymbol{A}}_k = \begin{bmatrix} \Delta\boldsymbol{A}_k & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \Delta\boldsymbol{A}_k \end{bmatrix},$$

$$\Delta\hat{\boldsymbol{B}}_k = \begin{bmatrix} \Delta\boldsymbol{B}_k & -\Delta\boldsymbol{B}_k & -\Delta\boldsymbol{B}_k \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix},$$

$$\boldsymbol{D}_k = \begin{bmatrix} \boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{I} & \boldsymbol{0} & -\boldsymbol{L}_k \end{bmatrix},$$

$$\hat{\boldsymbol{K}}_m = \begin{bmatrix} \boldsymbol{K}_m & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{K}_m & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{K}_m \end{bmatrix},$$

$\bar{\boldsymbol{\omega}}(t) = [\boldsymbol{\omega}^{\mathrm{T}}(t), \boldsymbol{r}^{\mathrm{T}}(t), \boldsymbol{\nu}^{\mathrm{T}}(t)]^{\mathrm{T}}$.

Considering the initial condition, the H_∞ tracking performance can be rewritten as follows:

$$\begin{aligned}\int_0^{t_f} \bar{\boldsymbol{X}}^{\mathrm{T}}(t)\bar{\boldsymbol{Q}}\bar{\boldsymbol{X}}(t)\mathrm{d}t \le& \rho^2\int_0^{t_f} \bar{\boldsymbol{\omega}}^{\mathrm{T}}(t)\bar{\boldsymbol{\omega}}(t)\mathrm{d}t\\ & + \bar{\boldsymbol{X}}^{\mathrm{T}}(0)\boldsymbol{P}\bar{\boldsymbol{X}}(0),\end{aligned} \tag{10}$$

where $\bar{\boldsymbol{Q}} = \begin{bmatrix} \boldsymbol{Q} & -\boldsymbol{Q} & \boldsymbol{0} \\ -\boldsymbol{Q} & \boldsymbol{Q} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \end{bmatrix}$, and $\boldsymbol{P} = \boldsymbol{P}^{\mathrm{T}} > 0$.

Assumption The parameter uncertainties considered here are assumed to be norm-bounded in the form

$$[\Delta\hat{\boldsymbol{A}}_k \quad \Delta\hat{\boldsymbol{B}}_k] = \boldsymbol{H}_k\boldsymbol{F}_k(t)[\boldsymbol{E}_{1k} \quad \boldsymbol{E}_{2k}],$$

where $\boldsymbol{H}_k$, $\boldsymbol{E}_{1k}$ and $\boldsymbol{E}_{2k}$ are known real constant matrices with appropriate dimensions, and $\boldsymbol{F}_k(t)$ is unknown matrix function with Lebesgue-measurable elements and satisfies $\boldsymbol{F}_k^{\mathrm{T}}(t)\boldsymbol{F}_k(t) \le \boldsymbol{I}$, in which $\boldsymbol{I}$ is the identity matrix with an appropriate dimension.

Lemma 1[17] For matrices (or vectors) $\boldsymbol{X}$ and $\boldsymbol{Y}$ with appropriate dimensions, we have

$$\boldsymbol{X}^{\mathrm{T}}\boldsymbol{Y} + \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{X} \le \boldsymbol{X}^{\mathrm{T}}\boldsymbol{J}X + \boldsymbol{Y}^{\mathrm{T}}\boldsymbol{J}^{-1}\boldsymbol{Y},$$

where $\boldsymbol{J}$ is any positive-definite symmetric matrix. In this paper, we let $\boldsymbol{J}$ be an identity matrix.

By use of the assumption and **Lemma 1**, we obtain

$$\begin{aligned}&\boldsymbol{P}(\Delta\hat{\boldsymbol{A}}_k+\Delta\hat{\boldsymbol{B}}_k\hat{\boldsymbol{K}}_m)+(\Delta\hat{\boldsymbol{A}}_k+\Delta\hat{\boldsymbol{B}}_k\hat{\boldsymbol{K}}_m)^{\mathrm{T}}\boldsymbol{P}\\ \le&(\boldsymbol{E}_{1k}+\boldsymbol{E}_{2k}\hat{\boldsymbol{K}}_m)^{\mathrm{T}}(\boldsymbol{E}_{1k}+\boldsymbol{E}_{2k}\hat{\boldsymbol{K}}_m)\\&+\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{P}\boldsymbol{P}\boldsymbol{H}_k.\end{aligned}$$

Let

$$\begin{aligned}&(\boldsymbol{E}_{1k}+\boldsymbol{E}_{2k}\hat{\boldsymbol{K}}_m)^{\mathrm{T}}(\boldsymbol{E}_{1k}+\boldsymbol{E}_{2k}\hat{\boldsymbol{K}}_m)\\&+\boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{P}\boldsymbol{P}\boldsymbol{H}_k=\boldsymbol{\psi}_{km}.\end{aligned}$$

Theorem 1 In the chaotic tracking system (9), if $\boldsymbol{P}=\boldsymbol{P}^{\mathrm{T}}>0$ and matrix inequality

$$\begin{bmatrix}\boldsymbol{A}_{km}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_{km}+\bar{\boldsymbol{Q}}+\boldsymbol{\psi}_{km} & \boldsymbol{P}D_K\\ \boldsymbol{D}_K^{\mathrm{T}}\boldsymbol{P} & -\rho^2\boldsymbol{I}\end{bmatrix}\le 0 \quad (11)$$

holds, then the H_∞ tracing performance in Eq.(10) of the whole chaotic system when $\bar{\boldsymbol{\omega}}(t)\neq 0$ is guaranteed for a prescribed ρ^2.

Proof:

$$\begin{aligned}&\int_0^{t_f}[(\boldsymbol{X}(t)-\boldsymbol{X}_{\mathrm{r}}(t))^{\mathrm{T}}\boldsymbol{Q}(\boldsymbol{X}(t)-\boldsymbol{X}_{\mathrm{r}}(t))]\mathrm{d}t\\ =&\int_0^{t_f}\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\bar{\boldsymbol{Q}}\bar{\boldsymbol{X}}(t)\mathrm{d}t\\ =&\bar{\boldsymbol{X}}^{\mathrm{T}}(0)\boldsymbol{P}\bar{\boldsymbol{X}}(0)-\bar{\boldsymbol{X}}^{\mathrm{T}}(t_f)\boldsymbol{P}\bar{\boldsymbol{X}}(t_f)\\&+\int_0^{t_f}\{\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\bar{\boldsymbol{Q}}\bar{\boldsymbol{X}}(t)+\frac{\mathrm{d}}{\mathrm{d}t}(\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{X}}(t))\}\mathrm{d}t\\ \le&\bar{\boldsymbol{X}}^{\mathrm{T}}(0)\boldsymbol{P}\bar{\boldsymbol{X}}(0)+\int_0^{t_f}\{\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\bar{\boldsymbol{Q}}\bar{\boldsymbol{X}}(t)\\&+\dot{\bar{\boldsymbol{X}}}^{\mathrm{T}}(t)\boldsymbol{P}\bar{\boldsymbol{X}}(t)+\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\boldsymbol{P}\dot{\bar{\boldsymbol{X}}}(t)\}\mathrm{d}t\\ =&\bar{\boldsymbol{X}}^{\mathrm{T}}(0)\boldsymbol{P}\bar{\boldsymbol{X}}(0)+\int_0^{t_f}\{\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\bar{\boldsymbol{Q}}\bar{\boldsymbol{X}}(t)\\&+\Big[\sum_{k=1}^{l}\sum_{m=1}^{l}h_k(z(t))h_m(z(t))\times(\boldsymbol{A}_{km}\bar{\boldsymbol{X}}(t)\\&+(\Delta\hat{\boldsymbol{A}}_k+\Delta\hat{\boldsymbol{B}}_k\hat{\boldsymbol{K}}_m)\bar{\boldsymbol{X}}(t)+\boldsymbol{D}_k\bar{\boldsymbol{\omega}}(t))\Big]^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{X}}(t)\\&+\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\boldsymbol{P}\Big[\sum_{k=1}^{l}\sum_{m=1}^{l}h_k(z(t))h_m(z(t))\\&\times(\boldsymbol{A}_{km}\bar{\boldsymbol{X}}(t)+(\Delta\hat{\boldsymbol{A}}_k+\Delta\hat{\boldsymbol{B}}_k\hat{\boldsymbol{K}}_m)\bar{\boldsymbol{X}}(t)\\&+\boldsymbol{D}_k\bar{\boldsymbol{\omega}}(t))\Big]\\&-\rho^2\bar{\boldsymbol{\omega}}^{\mathrm{T}}(t)\bar{\boldsymbol{\omega}}(t)+\rho^2\bar{\boldsymbol{\omega}}^{\mathrm{T}}(t)\bar{\boldsymbol{\omega}}(t)\}\mathrm{d}t\\ =&\bar{\boldsymbol{X}}^{\mathrm{T}}(0)\boldsymbol{P}\bar{\boldsymbol{X}}(0)\\&+\sum_{k=1}^{l}\sum_{m=1}^{l}h_k(z(t))h_m(z(t))\\&\times\int_0^{t_f}\left\{\begin{bmatrix}\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\\ \bar{\boldsymbol{\omega}}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}\right.\\&\begin{bmatrix}\boldsymbol{A}_{km}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_{km}+\bar{\boldsymbol{Q}} & \boldsymbol{P}D_k\\ \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P} & -\rho^2\boldsymbol{I}\end{bmatrix}\\&\begin{bmatrix}\bar{\boldsymbol{X}}(t)\\ \bar{\boldsymbol{\omega}}(t)\end{bmatrix}+\bar{\boldsymbol{X}}^{\mathrm{T}}(t)[\boldsymbol{P}(\Delta\hat{\boldsymbol{A}}_k+\Delta\hat{\boldsymbol{B}}_k\hat{\boldsymbol{K}}_m)\\&\left.+(\Delta\hat{\boldsymbol{A}}_k+\Delta\hat{\boldsymbol{B}}_k\hat{\boldsymbol{K}}_m)^{\mathrm{T}}\boldsymbol{P}]\bar{\boldsymbol{X}}(t)+\rho^2\bar{\boldsymbol{\omega}}^{\mathrm{T}}(t)\bar{\boldsymbol{\omega}}(t)\right\}\mathrm{d}t\\ \le&\bar{\boldsymbol{X}}^{\mathrm{T}}(0)\boldsymbol{P}\bar{\boldsymbol{X}}(0)+\sum_{k=1}^{l}\sum_{m=1}^{l}h_k(z(t))h_m(z(t))\\&\times\int_0^{t_f}\left\{\begin{bmatrix}\bar{\boldsymbol{X}}^{\mathrm{T}}(t)\\ \bar{\boldsymbol{\omega}}^{\mathrm{T}}(t)\end{bmatrix}^{\mathrm{T}}\right.\\&\begin{bmatrix}\boldsymbol{A}_{km}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_{km}+\bar{\boldsymbol{Q}}+\boldsymbol{\psi}_{km} & \boldsymbol{P}D_k\\ \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P} & -\rho^2\boldsymbol{I}\end{bmatrix}\\&\left.\begin{bmatrix}\bar{\boldsymbol{X}}(t)\\ \bar{\boldsymbol{\omega}}(t)\end{bmatrix}+\rho^2\bar{\boldsymbol{\omega}}^{\mathrm{T}}(t)\bar{\boldsymbol{\omega}}(t)\right\}\mathrm{d}t.\end{aligned}$$

By use of matrix inequality (11), the H_∞ tracing performance in Eq.(10) is achieved with a prescribed ρ^2.

To obtain a better robust tracking performance, the robust tracking performance can be treated as a minimization problem: in $\min_P\rho^2$, choosing such a $\boldsymbol{P}=\boldsymbol{P}^{\mathrm{T}}>0$ that the matrix inequality (11) holds true.

Let $\boldsymbol{W}=\begin{bmatrix}\bar{\boldsymbol{W}} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}=\begin{bmatrix}\boldsymbol{P}^{-1} & \boldsymbol{0}\\ \boldsymbol{0} & \boldsymbol{I}\end{bmatrix}$ and $\boldsymbol{P}=\boldsymbol{P}^{\mathrm{T}}>0$, therefore $\boldsymbol{W}=\boldsymbol{W}^{\mathrm{T}}>0$,

$$\boldsymbol{W}\begin{bmatrix}\boldsymbol{A}_{km}^{\mathrm{T}}\boldsymbol{P}+\boldsymbol{P}A_{km}+\bar{\boldsymbol{Q}}+\boldsymbol{\psi}_{km} & \boldsymbol{P}D_k\\ \boldsymbol{D}_k^{\mathrm{T}}\boldsymbol{P} & -\rho^2\boldsymbol{I}\end{bmatrix}\boldsymbol{W}\le 0$$

and

$$\begin{bmatrix}\bar{\boldsymbol{W}}\boldsymbol{A}_{km}^{\mathrm{T}}+\boldsymbol{A}_{km}\bar{\boldsymbol{W}}+\bar{\boldsymbol{W}}\bar{\boldsymbol{Q}}\bar{\boldsymbol{W}}+\bar{\boldsymbol{W}}\boldsymbol{\psi}_{km}\bar{\boldsymbol{W}} & \boldsymbol{D}_k\\ \boldsymbol{D}_k^{\mathrm{T}} & -\rho^2\boldsymbol{I}\end{bmatrix}\le 0$$

yield

$$\begin{bmatrix} \bar{\boldsymbol{W}}\boldsymbol{A}_{km}^{\mathrm{T}} + \boldsymbol{A}_{km}\bar{\boldsymbol{W}} & \bar{\boldsymbol{W}}\bar{\boldsymbol{Q}}^{\frac{1}{2}} & \bar{\boldsymbol{W}}(\boldsymbol{E}_{1k} + \boldsymbol{E}_{2k}\hat{\boldsymbol{K}}_m) & \bar{\boldsymbol{W}}\boldsymbol{P}\boldsymbol{H}_k & \boldsymbol{D}_k \\ (\bar{\boldsymbol{Q}}^{\frac{1}{2}})^{\mathrm{T}}\bar{\boldsymbol{W}} & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ (\boldsymbol{E}_{1k} + \boldsymbol{E}_{2k}\hat{\boldsymbol{K}}_m)^{\mathrm{T}}\bar{\boldsymbol{W}} & \boldsymbol{0} & -\boldsymbol{I} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{H}_k^{\mathrm{T}}\boldsymbol{P}\bar{\boldsymbol{W}} & \boldsymbol{0} & \boldsymbol{0} & -\boldsymbol{I} & \boldsymbol{0} \\ \boldsymbol{D}_k^{\mathrm{T}} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & -\rho^2\boldsymbol{I} \end{bmatrix} \leq 0.$$

For the convenience of design, assuming $\boldsymbol{P} = \begin{bmatrix} \boldsymbol{P}_{11} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_{11} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{P}_{11} \end{bmatrix}$ and $\boldsymbol{P}_{11} = \boldsymbol{P}_{11}^{\mathrm{T}} > 0$ gives

$$\bar{\boldsymbol{W}} = \begin{bmatrix} \boldsymbol{W}_{11} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{W}_{11} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{W}_{11} \end{bmatrix} = \begin{bmatrix} \boldsymbol{P}_{11}^{-1} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{P}_{11}^{-1} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{P}_{11}^{-1} \end{bmatrix}.$$

And let $\boldsymbol{Y}_m = \boldsymbol{K}_m\boldsymbol{W}_{11}$, $\boldsymbol{Y}_m$ and $\boldsymbol{W}_{11}$ can be achieved by MATLAB tool boxes of convex optimization techniques of LMI,[17,18] thus parameters of controller $\boldsymbol{K}_m$ are achieved.

4. Simulation as an example

Describing a nonlinear propagating wave $\phi(\lambda, t)$ (a coupled Duffing system) as

$$\frac{\partial^2\phi}{\partial t^2} = -\alpha\frac{\partial\phi}{\partial t} - \delta_1\phi^3 + \delta_2\phi + \delta\frac{\partial^2\phi}{\partial\lambda^2} + \eta\sin(\beta t) + d(t)$$

where, δ_1, δ_2 are the spreading coefficients in infinitely extended space, α is the medium damping coefficient, δ is a physical quantity related to the wave velocity , η and β are the gain and frequency of extend force, $d(t)$ is the extend disturbance; assuming that $\eta\sin(\beta t) + d(t) = \hat{d}(t)$, $\phi(\lambda, t) \to x_{i1}(t)$, the number of the subsystems is 2, and $\partial^2\phi/\partial\lambda^2 = x_{22} - x_{12}$, the Duffing system can be represented by two coupled subsystems as follows:

$$\begin{cases} \dot{x}_{i1} = x_{i2}, \\ \dot{x}_{i2} = -\alpha x_{i2} - \delta_1 x_{i1}^3 + \delta_2 x_{i1} + \delta(x_{22} - x_{12}) + \hat{d}(t), \\ y_i(t) = x_{i1}(t) + \nu_i(t), \end{cases}$$

where $\boldsymbol{x}_i = [x_{i1}, x_{i2}]^{\mathrm{T}} \in R^2$ is the state vector of the subsystem, $y_i(t)$ is the output vector of the subsystem, δ is the coupled strength between the subsystem. Let $\alpha = 1$, $\delta_1 = 100$, $\delta_2 = 10$, $\eta = 1$, $\beta = 3.5$, along with δ strengthening in the range 0 to 10, the Duffing system will show the characteristics of nonlinear coupled spatiotemporal chaotic system. Here we let $\delta = 8$ and reference input

$$\boldsymbol{r}(t) = [12.5, (12.5 + 3.5\cos 1.5\pi t, 15, (7.5 + 2.5\cos 1.5\pi t)]^{\mathrm{T}},$$

and assume that all system parameters are uncertain but bounded within 30% of their nominal values. The fuzzy model of the coupled Duffing system is achieved in four fuzzy rules, then $\boldsymbol{H}_k = 0.3\boldsymbol{I}$, where $\boldsymbol{I} \in R^{12\times 12}$ is an identity matrix. Using MATLAB tool boxes of convex optimization techniques of LMI gives

$$\boldsymbol{W}_{11} = \begin{bmatrix} 64.2453 & -31.3896 & 43.5786 & 2.4059 \\ -9.3456 & 18.6895 & -5.8237 & 15.8672 \\ 9.6886 & -5.9780 & 20.6789 & -7.4683 \\ 12.4569 & 6.4656 & -10.6453 & 42.7903 \end{bmatrix},$$

and the parameters $\boldsymbol{K}_m$ of the controller

$$\boldsymbol{K}_1 = \begin{bmatrix} 8.3656 & -68.8965 & -27.7636 & 4.2540 \\ 16.9562 & -7.1221 & -53.8974 & 43.7213 \end{bmatrix},$$

$$\boldsymbol{K}_2 = \begin{bmatrix} 8.5435 & -67.5543 & -25.3567 & 5.9087 \\ 17.9654 & -7.7865 & -52.7423 & 44.4377 \end{bmatrix},$$

$$\boldsymbol{K}_3 = \begin{bmatrix} 9.6543 & -68.0432 & -26.9342 & 4.9080 \\ 16.3894 & -6.7865 & -54.5642 & 42.8782 \end{bmatrix},$$

$$\boldsymbol{K}_4 = \begin{bmatrix} 8.5674 & -66.4523 & -24.98751 & 4.8895 \\ 17.8969 & -5.9069 & -53.7895 & 43.0782 \end{bmatrix}.$$

The initial position is chosen to be at [12.5, 10, 10, 7.5], and the control is applied at $t = 1.6$s. The tracking performance of the system is shown in Fig.1. From the simulation results, the performance of the suggested H_∞fuzzy tracking control scheme is obviously efficacious.

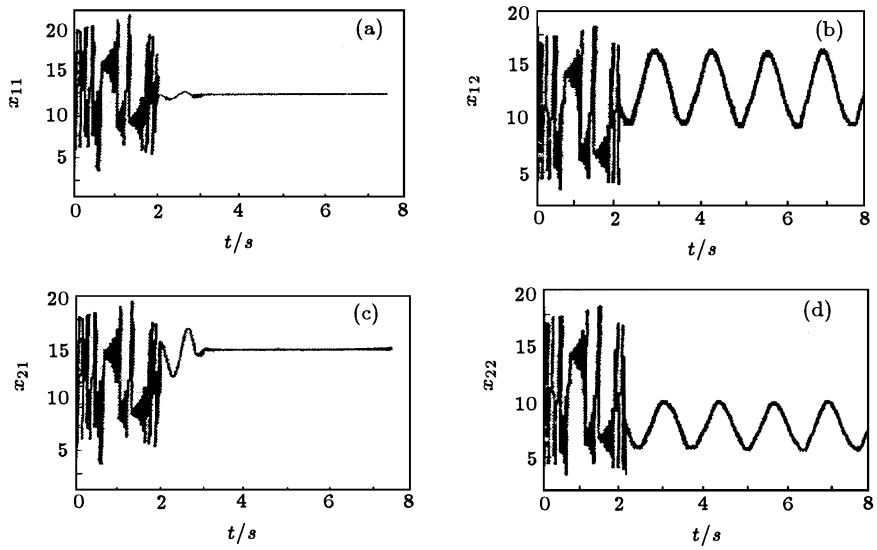


Fig.1. The state trajectories of the chaotic system (a) x_{11} state trajectory, (b) x_{12} state trajectory, (c) x_{21} state trajectory, (d) x_{22} state trajectory.

5. Conclusion

In this paper, a kind of T–S fuzzy model with parametric uncertainties is adopted to approximate the coupled spatiotemporal chaos with parametric uncertainties. Considering that all states of the subsystem are not available, the fuzzy observers are suggested to estimate these states. Based on the fuzzy models and the fuzzy observers, a new H_∞ fuzzy tracking control scheme for the coupled spatiotemporal chaos with parametric uncertainties is suggested and formulated in the LMI format. A simple and systematic algorithm based on LMI optimization techniques is developed to solve the H_∞ fuzzy tracking control problem. The simulation is given as an example to test the tracking performance of the scheme. By use of the scheme, the difficult problem for the tracking control to the coupled spatiotemporal chaos with parametric uncertainties is well solved. The main contributions of this paper are the suggestions for a new H_∞ fuzzy tracking control design scheme and a new sufficient condition in the LMI format of the controller for the coupled spatiotemporal chaos with parametric uncertainties as well as the solution to the robust stabilization problem for the coupled spatiotemporal chaos with parametric uncertainties.

References

[1] Yu Y N and Vong S K 1997 *IEEE Trans. Fuzzy System* **89** 52

[2] Zhang X and Shen K 2002 *Acta Phys. Sin.* **51** 2702 (in Chinese)

[3] Singh S N 1980 *IEE Proc. Gener. Trans. Distrib.* **127** 131

[4] Ren H P and Liu D 2002 *Acta Phys. Sin.* **51** 982 (in Chinese)

[5] Lu Q, Hu W and Song Y H 2000 *IEE Proc. Gener. Trans. Distrib.* **147** 245

[6] Tseng C S, Chen B S and Uang H J 2001 *IEEE Trans. Fuzzy System* **9** 381

[7] Chen B S, Tseng C S and Uang H J 1999 *IEEE Trans. Fuzzy System* **7** 571

[8] Yue D 2003 *Acta Phys. Sin.* **52** 292 (in Chinese)

[9] Isidori A and Asolfi A 1992 *IEEE Trans. Automat. Contr.* **37** 1283

[10] Wu Z Q, Ao D and Liu K 2004 *Acta Phys. Sin.* **53** 21 (in Chinese)

[11] Chen S H, Liu L M and Liu J 2002 *Chin. Phys.* **11** 543

[12] Tseng C S and Chen B S 2001 *IEEE Trans. Fuzzy System* **9** 795

[13] Boyd S and Ghaoui E 1994 *Linear Matrix Inequalities in System and Control Theory* (Philadelphia, PA: SIAM) p78

[14] Wu Z Q 2002 *Acta Phys. Sin.* **51** 1193 (in Chinese)

[15] Mu J, Tao C and Du G H 2003 *Chin. Phys.* **12** 381

[16] Lu Q, Sun Y and Wei S 2001 *Nonlinear Control System and Power System Dynamics* (Dordrecht: Kluwer) p50

[17] Chen B S and Chen Y Y 2002 *IEEE Trans. Contr. Syst.* **10** 769

[18] Scherer C and Gahinet P 1997 *IEEE Trans. Automat. Contr.* **42** 896

[19] Dou C X and Zhang S Q 2004 *Acta Phys. Sin.* **53** 12 (in Chinese)