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Quantum logic networks for controlled teleportation of a single particle via W state

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We discuss the scheme for probabilistic and controlled teleportation of an unknown state of one particle using the general three-particle W state as the quantum channel. The feature of this scheme is that teleportation between two sides depends on the agreement of the third side (Charlie), who may participate the process of quantum teleportation as a supervisor. In addition, we also construct efficient quantum logic networks for implementing the new scheme by means of the primitive operations.

Keywords: controlled teleportation, W state, quantum circuit
PACC: 0365, 0367

1. Introduction

In the field of quantum computation and communication, quantum channel plays a crucial role. Quantum teleportation, originally proposed by Bennett *et al*[1] in 1993, is one of the most striking progresses of quantum information theory. It allows the transmission of an unknown qubit state from a sender "Alice" to a spatially distant receiver "Bob" via a quantum channel with the aid of some classical communication. Based on the quantum teleportation scheme, quantum teleportation has been demonstrated with the polarization photon[2] and a single coherent mode of field[3] in the experiments. The new methods of teleportation have interested many research groups, and research work on quantum teleportation has been greatly developed. Many papers have discussed all kinds of teleportation of an unknown state via the different quantum channels.[4−12]

As an obvious extension of the entanglement of assistant, recently Zhou *et al*[13] and Yan and Wang[14] proposed protocols for controlled teleporting of an unknown quantum state of one particle. In their protocols, a GHZ state ($\varphi_{\rm GHZ} = a|000\rangle + b|111\rangle, a^2 + b^2 = 1$), shared by a sender "Alice", a receiver "Bob" and a supervisor "Charlie", will serve as the quantum channel, and without the cooperation of Charlie, the teleportation between Alice and Bob cannot be realized, namely, the teleportation is supervised by Charlie.

In this paper, we utilize another kind of peculiar genuine multipartite entangled state, which is called the W state ($|W\rangle = \frac{1}{\sqrt{3}}(a|001\rangle + b|010\rangle + c|100\rangle)$, $a^2 + b^2 + c^2 = 1$) in Ref.[15], in order to control teleportation of an unknown state different from Refs.[13] and [14]. We can directly use the W state and do not need to distill the GHZ state from the W state.[16] The W state is a special kind of entangled state in the tripartite system. There is a more robust entanglement in it than in the GHZ state when one of the three particles is traced out.[15] The family of W states leads to a stronger nonclassicality than the family of GHZ states.[17] Because of the special property, when the W state is used in the quantum communication, there will be some novel results.[18−21]

This paper is organized as follows. In Section 2, we briefly present our schemes for probabilistic and controlled teleportation of an unknown state of one particle via the general three-particle W state. Then, we construct the efficient quantum logic networks for implementing the new scheme by means of the primitive operations. In addition, we give the quantum logic networks for controlled teleportation of a single particle via W state as well. Finally, a brief summary

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of our results is given in Section 3.

2. Quantum logic networks for controlled teleportation of a single particle via the general three-particle W state

Suppose the sender "Alice", the receiver "Bob" and the supervisor "Charlie" share the three-particle W state in the form

$$|W\rangle_{234} = [a|001\rangle + b|010\rangle + c|100\rangle]_{234}, \qquad (1)$$

where particles 2, 3, 4 belong to Alice, Bob, Charlie, respectively. Without loss of generality, we suppose that the coefficients a, b and c are real numbers,$a^2 + b^2 + c^2 = 1$, $a > b > c > 0$ are all known. Alice wants to teleport an unknown state of particle 1, $|\Phi\rangle_1 = x|0\rangle_1 + y|1\rangle_1$ ($|x|^2 + |y|^2 = 1$) to Bob via a quantum channel of their shared $|W\rangle_{234}$ state.

The state for the whole system (four particles) can be expressed as

$$\begin{aligned}
&|\psi\rangle_{1234} = |\Phi\rangle_1 \otimes |W\rangle_{234} \\
=&\frac{1}{\sqrt{2}}|\phi\rangle_{12}^{+} \otimes [xa|0\rangle_3|1\rangle_4 + (xb|1\rangle_3 + yc|0\rangle_3)|0\rangle_4] \\
&+ \frac{1}{\sqrt{2}}|\phi\rangle_{12}^{-} \otimes [xa|0\rangle_3|1\rangle_4 + (xb|1\rangle_3 - yc|0\rangle_3)|0\rangle_4] \\
&+ \frac{1}{\sqrt{2}}|\psi\rangle_{12}^{+} \otimes [ya|0\rangle_3|1\rangle_4 + (xc|0\rangle_3 + yb|1\rangle_3)|0\rangle_4] \\
&+ \frac{1}{\sqrt{2}}|\psi\rangle_{12}^{-} \otimes [-ya|0\rangle_3|1\rangle_4 + (xc|0\rangle_3 - yb|1\rangle_3)|0\rangle_4],
\end{aligned} \qquad (2)$$

where

$$|\phi\rangle_{12}^{\pm} = \frac{1}{\sqrt{2}}[|00\rangle \pm |11\rangle]_{12}$$

and

$$|\psi\rangle_{12}^{\pm} = \frac{1}{\sqrt{2}}[|01\rangle \pm |10\rangle]_{12}$$

are the Bell states of particles 1 and 2.

In order to realize the teleportation, Alice must perform the Bell state measurement on particles 1 and 2, and particles 3 and 4 will collapse into the relevant state at the same time. If Charlie would like to help Bob with the teleportation, she should do the projection measurement on particle 4 in the basis $\{|0\rangle_4, |1\rangle_4\}$ and transfer the information to Bob via a classical channel. If and only if Charlie gains the result of $|0\rangle_4$, the teleportation will go on. As soon as Bob receives the classical information about the result of measurement from Alice and Charlie, he should introduce an auxiliary particle $|0\rangle_a$ in his side and perform a collective unitary transformation

$$U = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & c/b & -\sqrt{1-c^2/b^2} \\ 0 & 0 & \sqrt{1-c^2/b^2} & c/b \end{pmatrix} \qquad (3)$$

on the basis of $\{|00\rangle_{3a}, |01\rangle_{3a}, |10\rangle_{3a}, |11\rangle_{3a}\}$. After that, Bob performs a measurement on the auxiliary particle. If the result is $|1\rangle_a$, the teleportation fails. However, if the measurement outcome is $|0\rangle_a$, Bob will choose appropriate Pauli operations (X, Y, Z, I) in order to recover the original state he wants in his side. For example, under the result of measurement from Alice and Charlie, the state for the whole system will collapse into $xb|1\rangle_3 - yc|0\rangle_3$ and after U transformation, Bob will obtain

$$c(-y|0\rangle_3 + x|1\rangle_3)|0\rangle_a + xb\sqrt{1-c^2/b^2}|1\rangle_3|1\rangle_a, \qquad (4)$$

and transform $-y|0\rangle_3 + x|1\rangle_3$ into the original state by using first an X gate and then a Z gate on the particle 3. Equations (2) and (4) show that the probability of successful teleportation is $c^2/2$. Synthesizing all the cases, the teleportation is successfully realized with the probability $4 \times c^2/2 = 2c^2$. Evidently, if $a = b = c = 1/\sqrt{3}$, then the auxiliary particle is not needed, and as pointed out in Ref.[22], the probability of a successful teleportation is 2/3. Table 1 gives Alice's all different measurement results and Bob's relevant Pauli operations after U transformation on condition that Charlie obtains the result of $|0\rangle_4$.

Table 1. Bob's relevant unitary transformations corresponding to Alice's all different measurement results.

Alice's result	Bob's transformation
$\|\phi\rangle_{12}^{+}$	X
$\|\phi\rangle_{12}^{-}$	XZ
$\|\psi\rangle_{12}^{+}$	I
$\|\psi\rangle_{12}^{-}$	Z

Here X, Z are Pauli operations and I is a unity operation.

Increasing attention has been paid to quantum logic networks [23–26] since 1998, when Brassard *et al*[27] proposed quantum logic networks to carry out teleportation. Next, based on the above description, we construct the efficient quantum logic networks for implementing the new scheme by means of the primitive operations consisting of single-qubit gates, two-qubit controlled-NOT gates, Von Neumann measurement and classically controlled operations.[28] The

quantum circuit for implementing this new scheme is shown in Fig.1.

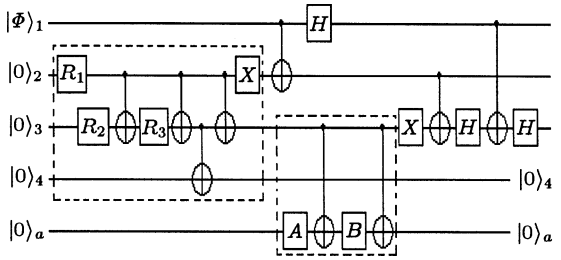


Fig.1. Quantum circuit for controlled teleportation of an unknown qubit via the general three-particle W state.

In Fig.1

$$R_1 = \begin{pmatrix} \cos\theta_1 & -\sin\theta_1 \\ \sin\theta_1 & \cos\theta_1 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} \cos(\theta_2/2) & -\sin(\theta_2/2) \\ \sin(\theta_2/2) & \cos(\theta_2/2) \end{pmatrix},$$

$$R_3 = \begin{pmatrix} \cos(\theta_2/2) & \sin(\theta_2/2) \\ -\sin(\theta_2/2) & \cos(\theta_2/2) \end{pmatrix}$$

are single-qubit rotation transformations, H is a Hadamard gate defined as $H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $X = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ denotes a Pauli operation, and the dot $\bullet$ is the control qubit and the cross $\oplus$ is the target qubit. In Fig.1, the two top lines represent Alice's system, the third line is Bob's system, where the fifth line is the auxiliary qubit introduced by Bob, while the fourth line is Charlie's system. By the first dashed rectangle, the state of particles 2, 3 and 4 is the quantum channel of the W state in Eq.(1) when $\cos\theta_1 = c$, $\sin\theta_1 = \sqrt{1-c^2}$, $\cos\theta_2 = b/\sqrt{1-c^2}$ and $\sin\theta_2 = a/\sqrt{1-c^2}$. In the second dashed rectangle, the unitary transformation U is the combination of single-bit gates and two-bit CNOT gates. Here, $A = R_y(\theta/2)$ and $B = R_y(-\theta/2)$ are rotations of θ and $-\theta$ around y, respectively, where letting $\cos(\theta/2) = c/b$, then we have

$$\begin{pmatrix} c/b & -\sqrt{1-c^2/b^2} \\ \sqrt{1-c^2/b^2} & c/b \end{pmatrix}$$

$$= R_y(\theta) = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix}. \quad (5)$$

Teleportation of classical message is not denoted in Fig.1. We notice that controlled unitary operation acting on any number of qubits, followed by the measurement of the control qubit, can be replaced by the measurement of the control qubit preceding the controlled operation.[29,30] Therefore, Fig.1 can be re-expressed as Fig.2.

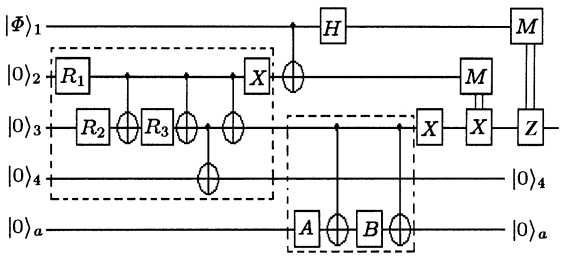


Fig.2. The same as Fig.1, but Von Neumann measurement M and classically controlled operation are made at the end instead.

Here, the double-line coming out of the measurement M denotes the classical information.[23,31] In Fig.2, Bob, depending on the measurement results performed by Alice and Charlie on their own qubits, can realize the controlled operation locally. If and only if the outcome of Alice's measurement is $|1\rangle$, can Bob operate the controlled-X or controlled-Z ($Z = HXH$)[23] on condition that Charlie obtains the result of $|0\rangle_4$. If the measurement result of the auxiliary particle is $|1\rangle_a$, the teleportation fails. If the result is $|0\rangle_a$, the final state of particle 3 at Bob's side will be collapsed into $|\Phi\rangle_3 = x|0\rangle_3 + y|1\rangle_3$, which is the desired state. Clearly, Bob can obtain the signal state from Alice by himself with the co-operation of Charlie.

Similarly, if $a = b = c = 1/\sqrt{3}$, the auxiliary particle is not needed and our quantum channel turns into $|W'\rangle_{234} = \left[\frac{1}{\sqrt{3}}(|001\rangle + |010\rangle + |100\rangle)\right]_{234}$, then $\cos\theta_1 = 1/\sqrt{3}$ and $\cos\theta_2 = 1/\sqrt{2}$. Based on the above argumentation, the networks of probabilistic and controlled teleportation of an unknown state consisting of a single particle via W state are shown in Figs.3 and 4, respectively. Here, the two top lines represent Alice's system, the third line is Bob's system, while the bottom line is Charlie's system.

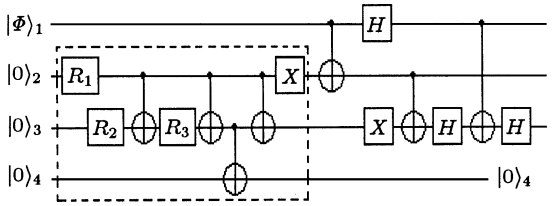


Fig.3. Quantum circuit for controlled teleportation of an unknown qubit via W state.

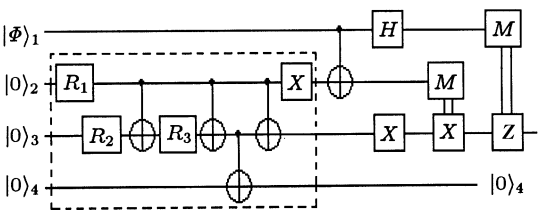


Fig.4. The same as Fig.3, but Von Neumann measurement M and classically controlled operation are made at the end instead.

3. Conclusions

In this paper, we have shown that the W state can be used to realize the probabilistic and controlled teleportation of a single qubit. The feature of this scheme is that teleportation between two sides depends on the agreement of the third side (Charlie), who may participate the process of quantum teleportation as a supervisor. Without the co-operation of Charlie, Bob cannot receive the signal state from Alice by himself. Recently, schemes showing how to prepare the W state have been reported in Refs.[32–37], and Eibl *et al*[38] has experimentally realized a three-qubit entangled W state. The quantum logic networks for implementing this scheme are also provided and thus our scheme may be realized by experiment.

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