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Gravitational effect of centre mass with electric charge and a large number of magnetic monopoles*

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In this paper, using an elegant mathematical method advanced by us, we calculate the orbital effect in the gravitational field of the centre mass with electric charge and a large number of magnetic monopoles. Generalizing the effect in the Schwarzschild field, we obtain interesting results by discussing the parameters of the celestial body that provide a feasible experimental verification of the general relativity.

Keywords: precession of perihelion, centre mass, electric charge, magnetic monopoles
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1. Introduction

The general theory of relativity is now accepted as the most satisfactory theory of gravitation. This acceptance rest partly on its conceptual and structural elegance, and partly on its agreement with experimental observation. Four "crucial tests" are usually cited as experimental verifications of the theory:[1−3] the red shift of strong gravitational potential;[4,5] the deflection of light rays that pass close to the Sun;[1,2] the precession of the perihelion of the orbit of the planet Mercury[3,6,7] and the delay time of radar waves reflected from planets.[8,9]

Mercury is the closest planet to the Sun. Following Newtonian mechanics, the orbit of Mercury is a proper ellipse, neglecting the perturbation of other planets. However, in 1859, the French astronomer discovered that the angular displacement of the perihelion of the Mercury is $42.6'' \pm 1''$ per century, which cannot be explained by Newtonian gravitational theory, and became one of the difficult problems in classical mechanics.

This problem was not resolved until Einstein established the general theory of relativity. In terms of general relativity, we consider the spacetime in the gravitational field induced by the Sun to be described by the Schwarzschild metric which has the form (in this paper, we take $G = c = 1$)

$$\mathrm{d}s^2 = \left(1 - \frac{2M}{r}\right)\mathrm{d}t^2 - \left(1 - \frac{2M}{r}\right)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2). \quad (1)$$

It is distinctly different from the Minkowski metric which has the form

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{d}r^2 - r^2(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\varphi^2). \quad (2)$$

That is, the gravitational field of the Sun curves the spacetime, the smaller the quantity r, the more distinct the difference between the two metrics. In the case r becomes infinite, these two metrics are identical—flat spacetime metric. That is to say, the nearer to the Sun, the stronger the gravitational field, and the spacetime curves more. For Mercury, being the nearest planet to the Sun, its orbit deflects notably from the calculated result in the flat spacetime, which is why the planet Mercury's orbit shows the precession of the perihelion.

In 1931 Dirac[10] predicted the existence of magnetic monopoles. From the grand unified theory (GUT),[11] t'Hooft and Polyakov affirmed the existence of magnetic monopoles with large mass ($M = 1 \times 10^{16}$GeV).[12,13] In recent years Rubakov separately presented the process of the decay of a nucleon, $pM \to \pi^0 e^+ M$[4] whose reaction cross section

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reaches the same order of magnitude as usual strong action, $\sigma \sim 10^{-26}\mathrm{cm}^3$. S. Dimopoles *et al* and K. Kolb *et al* separately researched a problem that a neutron star captures magnetic monopoles so that the decay of nucleons is impelled. They stated that if magnetic monopoles in the interstellar space are once captured by a neutron star, they will remain in the neutron star for a long time to catalyze the decay of nucleons whose products will transform into x-rays or ultraviolet photons to be emitted by several means. They assumed that background x-rays and ultraviolet photons are all produced in this way from neutron stars in the universe. In the very primitive period of the universe the magnetic field might be very strong, the separation of the positive and negative magnetic monopoles was possible, hence, we can assume that magnetic monopoles of one sign (say, positive magnetic charge) are dominant in the Milky Way system.[15] The effect of the field induced by the magnetic charge cannot be neglected.

In this paper, we propose an elegant mathematical method to solve the general problem of the orbital effect in the gravitational field induced by the centre mass with electric charge and many magnetic monopoles. The geodesic motion of the test particle is analysed in the next section. We obtain the precession effect of the perihelion in a class of interesting fields (Schwarzschild spacetime, Reissner–Nordström–Kasuya (R–N–K) and Reissner–Nordström) in Section 3. Finally, we discuss the results applied to all kinds of conditions by substituting the parameters into the expressions.

2. The geodesic motion of the test particle

First, we discuss the orbital motion confined to the type of conic sections, by the first integral of the equation of geodesic motion we obtain the proper energy ε and the proper vector area h; by the second integral of it we get the orbital equation. In the spherical coordinate system, we assume that the test particle moves in the plane $\theta = \dfrac{\pi}{2}$, the orbit has the form of a conical curve

$$\begin{aligned} u &= \frac{1}{r} = \frac{1}{p}(1 + e\cos\psi), \\ \psi &= \varphi + \alpha(\varphi) = \varphi + \triangle\varphi, \end{aligned} \tag{3}$$

where p and e are, respectively, the focus parameter and eccentricity of the ellipse, ψ is real abnormal angular displacement, including (compared with the Newtonian theory) the additional angular displacement $\alpha(\varphi)$ (additional abnormal). When $e < 1$, $\alpha(\varphi)$ is the additional abnormal angular displacement of the perihelion which represents the precession. In the case of circular motion, the periodic concept is degenerated, but the general conception of additional abnormal is effective: $\lim\limits_{e\to 0}\alpha = 0$.

We integrate the equation of the motion, that is

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}\lambda}\left(g_{\sigma\mu}\frac{\mathrm{d}x^\sigma}{\mathrm{d}\lambda}\right) = \frac{1}{2}\frac{\partial g_{\tau\sigma}}{\partial x^\mu}\frac{\mathrm{d}x^\tau}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\sigma}{\mathrm{d}\lambda}, \\ &g_{\mu\nu}\frac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}\frac{\mathrm{d}x^\nu}{\mathrm{d}\lambda} = 1. \end{aligned} \tag{4}$$

In the spherical coordinate system $(x^0, r, \theta, \varphi)$, for stationary symmetric radiation field, $g_{\tau\sigma}$ does not include x^0 and φ, the components of $(0, 3)$ are respectively,

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}\lambda}(g_{0\sigma}\dot{x}^\sigma) = 0, \\ &\frac{\mathrm{d}}{\mathrm{d}\lambda}(g_{3\sigma}\dot{x}^\sigma) = 0. \end{aligned} \tag{5}$$

Their integration yields

$$g_{0\sigma}\dot{x}^\sigma = -\varepsilon, \quad g_{3\sigma}\dot{x}^\sigma = h, \tag{6}$$

where ε and h are integration constants. Substituting Eq.(6) into $g_{\mu\nu}\dfrac{\mathrm{d}x^\mu}{\mathrm{d}\lambda}\dfrac{\mathrm{d}x^\nu}{\mathrm{d}\lambda} = 1$, we obtain the orbital equation

$$\left(\frac{\mathrm{d}u}{\mathrm{d}\varphi}\right)^2 = F(u), \tag{7}$$

where $F(u)$ is an expression about $u = r^{-1}$. In different ways, we can obtain the solution of Eq.(7) in the form of Eq.(3).

Substituting Eq.(3) into Eq.(7), it becomes

$$\begin{aligned} &\frac{e^2}{p^2}\sin^2\theta\left(\frac{\mathrm{d}\psi}{\mathrm{d}\varphi}\right)^2 \\ =&A(\varepsilon, h, p, e) + B(\varepsilon, h, p, e)\frac{e}{p}\cos\psi \\ &+ C(\varepsilon, h, p, e)\frac{e^2}{p^2}\sin^2\psi. \end{aligned} \tag{8}$$

Comparing coefficients of each term of the equation, we obtain

$$\begin{aligned} &A(\varepsilon, h, p, e) = 0, \\ &B(\varepsilon, h, p, e) = 0, \\ &\psi'^2 = C(\varepsilon, h, p, e). \end{aligned} \tag{9}$$

Or we can obtain another expression of Eq.(3),

$$\alpha'^2 + 2\alpha' - C(\varepsilon, h, p, e) + 1 = 0, \tag{10}$$

where $\alpha' = \dfrac{\mathrm{d}\alpha}{\mathrm{d}\varphi}$, so that we obtain the expression of the periodic additional abnormal

$$\frac{\Delta\alpha}{2\pi} = \frac{1}{2\pi}\int_0^{2\pi} C^{-1/2}\mathrm{d}\psi - 1, \quad \int \mathrm{d}\alpha = -\Delta\alpha. \quad (11)$$

Substituting the metric into Eq.(9), (10) and (11), we can obtain the expression of additional displacement of the perihelion. Substituting the values of the parameters of some planet into the expression we will obtain the precession effect of the planet.

3. The precession effect in R-N-K field

3.1. For Schwarzschild spacetime

The metric is

$$\begin{aligned}\mathrm{d}s^2 =&\left(1-\frac{2M}{r}\right)\mathrm{d}t^2 - \left(1-\frac{2M}{r}\right)^{-1}\mathrm{d}r^2\\ &- r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2),\end{aligned} \quad (12)$$

and the nonzero components of $g_{\mu\nu}$ and $g^{\mu\nu}$ of the Schwarzschild metric are found as follows:

$$\begin{aligned} g_{00} &= \frac{1}{g^{00}} = 1-\frac{2M}{r},\\ g_{11} &= \frac{1}{g^{11}} = -\left(1-\frac{2M}{r}\right)^{-1},\\ g_{22} &= \frac{1}{g^{22}} = -r^2,\\ g_{33} &= \frac{1}{g^{33}} = -r^2\sin^2\theta = -r^2. \end{aligned} \quad (13)$$

Substituting Eq.(13) into Eq.(7), we obtain

$$\begin{aligned}\left(\frac{\mathrm{d}u}{\mathrm{d}\varphi}\right)^2 =&F(u),\\ =&\frac{\varepsilon^2-1}{h^2}+\frac{2m}{h^2}u-u^2+2mu^3.\end{aligned} \quad (14)$$

Substituting Eq.(3) into Eq.(14), the latter becomes

$$\begin{aligned}&\frac{e^2}{p^2}\sin^2\psi\left(\frac{\mathrm{d}\psi}{\mathrm{d}\varphi}\right)^2\\ =&\frac{\varepsilon^2-1}{h^2}+\frac{2M}{ph^2}-\frac{1+e^2}{p^2}+\frac{2M(1+3e^2)}{p^3}\\ &+\left(\frac{2M}{h^2}-\frac{2}{p}+\frac{6M}{p^2}+\frac{2Me^2}{p^2}\right)\frac{e}{p}\cos\psi\\ &+\left[1-\frac{2M}{p}(3+e\cos\psi)\right]\frac{e^2}{p^2}\sin^2\psi.\end{aligned} \quad (15)$$

Comparing Eq.(15) with Eq.(8) we obtain

$$\begin{aligned}&\frac{\varepsilon^2-1}{h^2}+\frac{2M}{ph^2}-\frac{1+e^2}{p^2}+\frac{2M(1+3e^2)}{p^3}=0,\\ &\frac{M}{h^2}-\frac{1}{p}+\frac{3M}{p^2}+\frac{Me^2}{p^2}=0,\\ &1-\frac{2M}{p}(3+e\cos\psi)=C.\end{aligned} \quad (16)$$

Solving Eq.(16), we obtain

$$\begin{aligned}\varepsilon^2 =&\left[\left(1-\frac{2M}{p}\right)^2-\frac{4M^2}{p^2}e^2\right]\left[1-\frac{M(3+e^2)}{p}\right]^{-1},\\ h^2 =&Mp\left[1-\frac{M(3+e^2)}{p}\right]^{-1},\\ C =&1-\frac{2M}{p}(3+e\cos\psi).\end{aligned} \quad (17)$$

Substitution of Eq.(17) into Eq.(11) yields

$$\left(\frac{\triangle\alpha}{2\pi}\right)_m = \frac{3M}{p}+\frac{27M^2}{2p^2}+\frac{3M^2}{4p^2}e^2, \quad (18)$$

the first term on the right-hand side of the expression is the approximate solution which Einstein obtained from the field equation in 1915, the other two terms are higher-order compensations. Later, researchers found that the results agree with what were observed. That provides a test of the general theory of relativity.

3.2. For Reissner–Nordström–Kasuya (R–N–K) spacetime

The metric takes the form

$$\mathrm{d}s^2 = V(r)\mathrm{d}t^2 - V(r)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \quad (19)$$

where

$$V(r) = 1-\frac{2M}{r}+\frac{k}{r^2}, \quad k = q_e^2+q_m^2,$$

with q_e the electric charge, q_m the magnetic charge of the field source and m is the total mass. The nonzero components of $g_{\mu\nu}$ and $g^{\mu\nu}$ of the R-N-K metric are found as follows:

$$\begin{aligned} g_{00} &= \frac{1}{g^{00}} = 1-\frac{2M}{r}+\frac{k}{r^2},\\ g_{11} &= \frac{1}{g^{11}} = -\left(1-\frac{2M}{r}+\frac{k}{r^2}\right)^{-1},\\ g_{22} &= \frac{1}{g^{22}} = -r^2,\\ g_{33} &= \frac{1}{g^{33}} = -r^2\sin^2\theta = -r^2. \end{aligned} \quad (20)$$

Substituting Eq.(20) into Eq.(7), we obtain

$$\begin{aligned}\left(\frac{\mathrm{d}u}{\mathrm{d}\varphi}\right)^2 =&F_{m,k}(u),\\ =&\frac{\varepsilon^2-1}{h^2}+\frac{2m}{h^2}u-\left(1+\frac{k}{h^2}\right)u^2\\ &+2mu^3-ku^4.\end{aligned} \quad (21)$$

Substituting Eq.(3) into Eq.(21), we then obtain

$$\frac{e^2}{p^2}\sin^2\psi\left(\frac{\mathrm{d}\psi}{\mathrm{d}\varphi}\right)^2 = \frac{\varepsilon^2-1}{h^2} + \frac{2M}{ph^2} - \left(1+\frac{k}{h^2}\right)\frac{1+e^2}{p^2} + \frac{2M(1+3e^2)}{p^3} - \frac{k}{p^4}(1+6e^2+e^4) + \left[\frac{2M}{h^2} - \left(1+\frac{k}{h^2}\right)\frac{2}{p} + \frac{2M}{p^2}(3+e^2) - \frac{4ke}{p^3}(1+e^2)\right]\frac{e}{p}\cos\psi + \left[1+\frac{k}{h^2} - \frac{2M}{p}(3+e\cos\psi) + \frac{k}{p^2}(6+4e\cos\psi+e^2+e^2\cos^2\psi)\right]\frac{e^2}{p^2}\sin^2\psi. \quad (22)$$

Comparison of Eq.(22) with Eq.(8) yields

$$\frac{\varepsilon^2-1}{h^2} + \frac{2M}{ph^2} - \left(1+\frac{k}{h^2}\right)\frac{1+e^2}{p^2} + \frac{2M(1+3e^2)}{p^3} - \frac{k}{p^4}(1+6e^2+e^4) = 0,$$
$$\frac{M}{h^2} - \left(1+\frac{k}{h^2}\right)\frac{1}{p} + \frac{M}{p^2}(3+e^2) - \frac{2k}{p^3}(1+e^2) = 0,$$
$$1+\frac{k}{h^2} - \frac{2M}{p}(3+e\cos\psi) + \frac{k}{p^2}(6+4e\cos\psi+e^2+e^2\cos^2\psi) = C. \quad (23)$$

Solving Eq.(23), we obtain

$$\varepsilon^2 = \left\{\left[1-\frac{2M}{p}+\frac{k(1+e^2)}{p^2}\right]^2 - \frac{4e^2}{p^2}\left(M-\frac{k}{p}\right)^2\right\}\left[1-\frac{M(3+e^2)}{p}+\frac{2k}{p^2}(1+e^2)\right]^{-1},$$
$$h^2 = (Mp-k)\left[1-\frac{M(3+e^2)}{p}+\frac{2k}{p^2}(1+e^2)\right]^{-1},$$
$$C = 1-\frac{2M}{p}(3+e\cos\psi)+\frac{k}{h^2}+\frac{2k}{p^2}(3+2e\cos\psi)+\frac{ke^2}{p^2}(1+\cos^2\psi). \quad (24)$$

Substitution of these expressions into the integrand of Eq.(11) yields

$$\left(\frac{\triangle\alpha}{2\pi}\right)_k = -\frac{k}{2Mp} - \frac{6k}{p^2} - \frac{ke^2}{4p^2}, \quad (25)$$

that is the precession effect of the particle in the R–N–K spacetime.

3.3. For Reissner–Nordström (RN) spacetime

The metric is given by

$$\mathrm{d}s^2 = V(r)\mathrm{d}t^2 - V(r)^{-1}\mathrm{d}r^2 - r^2(\mathrm{d}\theta^2+\sin^2\theta\mathrm{d}\varphi^2), \quad (26)$$

where $V(r) = 1-\frac{2M}{r}+\frac{k}{r^2}$, q is the electric (magnetic) charge of the field source. Similarly, we can obtain the precession effect of the particle in the RN spacetime

$$\left(\frac{\triangle\alpha}{2\pi}\right)_q = -\frac{q}{2Mp} - \frac{6q}{p^2} - \frac{qe^2}{4p^2}. \quad (27)$$

We can obtain the precession effects of all kinds of planets by substituting the parameters into the expressions, which is an interesting problem that we will discuss in our next paper.

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