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Intensity correlation time of a single-mode laser driven by two coloured noises with coloured cross-correlation with direct signal modulation*

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In this paper, the intensity correlation time T is studied for the gain-noise model of a single-mode laser driven by coloured pump noise and coloured quantum noise with coloured cross-correlation with a direct signal modulation. By using the linear approximation method, it is found that when the pump noise is modulated directly by a signal, the effects of the cross-correlation between the pump noise and the quantum noise will disappear. In addition, there exists a maximum (i.e. resonance) in the curve of the intensity correlation time T versus the pump noise self-correlation time τ_1. Furthermore, when $\tau_1 \leq \tau_2$, the intensity correlation time T increases monotonically with the increase of D and decreases monotonically with the increase of Q, but when $\tau_1 > \tau_2$, the intensity correlation time T increases monotonically with the increase of Q and decreases monotonically with the increase of D.

Keywords: intensity correlation time, direct modulation signal, linear approximation method
PACC: 4250, 0540, 4260K

1. Introduction

Recently, the subject of the influence of noise on a laser system has attracted considerable attention.[1−11] There exist two kinds of noises, one is the quantum noise originating from spontaneous emission, and generally modelled as an additive noise, and the other is the pump noise originating from external sources and generally modelled as a multiplicative noise. Since Fulinski and Telejko [12] proposed that the two different noises could be cross-correlated under certain conditions, Luo and Zhu *et al*[13,14] have found that the cross-correlation between the pump noise and quantum noise can influence the normalized intensity fluctuation in a laser system. Zhou *et al*[15,16] found that there was a minimum in the curve of the relative intensity fluctuation versus noise by studying a single-mode laser system driven by two white noises with coloured cross-correlation. Liang *et al*[17,18] investigated the statistical properties and stochastic resonance of a single-mode laser driven by two coloured noises with coloured cross-correlation.

The intensity correlation time T is a fundamental statistical variable, from which one can obtain the dynamical behaviours of a laser system. Because it can be measured experimentally, the intensity correlation time T is of practical value. In this paper, we focus on the studies of the intensity correlation time T for the gain-noise model of a single-mode laser driven by coloured pump noise and coloured quantum noise with coloured cross-correlation with a direct signal modulation. By using the linear approximation method, some significant results can be obtained. We find that when the pump noise is modulated directly by a signal, the effect of the cross-correlation between the pump noise and the quantum noise will disappear. Besides, there exists a maximum (i.e. resonance) in the curve of the intensity correlation time T versus the pump noise self-correlation time τ_1. Furthermore, when $\tau_1 \leq \tau_2$,

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the intensity correlation time T increases monotonically with the increase of D and decreases monotonically with the increase of Q, but when $\tau_1 > \tau_2$, T increases monotonically with the increase of Q and decreases monotonically with the increase of D.

2. The intensity correlation time T of a single-mode laser with a direct signal modulation

The intensity equation for the gain-noise model of a single-mode laser with a bias signal modulation is given by

$$\frac{\mathrm{d}I}{\mathrm{d}t} = -2KI + \frac{2\Gamma}{1+\beta I} I + D + \frac{2I}{1+\beta I} A\cos(\Omega t)\,\xi(t) + 2\sqrt{I}\,\eta(t), \quad (1)$$

where the statistical property of the pump noise $\xi(t)$ and the quantum noise $\eta(t)$ are assumed as follows:

$$\begin{aligned} \langle \xi(t)\rangle &= \langle \eta(t)\rangle = 0, \\ \langle \xi(t)\xi(t')\rangle &= \frac{Q}{2\tau_1}\exp\left(-\frac{|t-t'|}{\tau_1}\right), \\ \langle \eta(t)\eta(t')\rangle &= \frac{D}{2\tau_2}\exp\left(-\frac{|t-t'|}{\tau_2}\right), \\ \langle \xi(t)\eta(t')\rangle &= \langle \eta(t)\xi(t')\rangle = \frac{\lambda\sqrt{QD}}{2\tau_3}\exp\left(-\frac{|t-t'|}{\tau_3}\right), \\ & (-1 \le \lambda \le 1). \end{aligned} \quad (2)$$

When τ_1, τ_2 and τ_3 approach zero, Eqs.(2) are translated into the following forms:

$$\begin{aligned} \langle \xi(t)\xi(t')\rangle &= Q\delta(t-t'), \\ \langle \eta(t)\eta(t')\rangle &= D\delta(t-t'), \\ \langle \xi(t)\eta(t')\rangle &= \langle \eta(t)\xi(t')\rangle = \lambda\sqrt{DQ}\,\delta(t-t'). \end{aligned} \quad (3)$$

In Eqs.(1), (2) and (3), I is the laser intensity; K represents the loss coefficient; $\beta = \tilde{A}/\Gamma$, $\tilde{A}$ and Γ represent the self-saturation and gain coefficients respectively; Q and D are the intensities of the pump noise and quantum noise respectively; λ is the cross-correlation coefficient between both noises; τ_1, τ_2 and τ_3 are the self-correlation time and the cross-correlation times of the pump noise and quantum noise respectively; A is the amplitude of a periodic signal and Ω is the frequency of the signal. The term $A\cos(\Omega t)$ is multiplied by the pump noise $\xi(t)$, i.e. $\xi(t)$ is modulated directly by a signal.

Let

$$I = I_0 + \delta(t), \quad (4)$$

where $I_0 = (\Gamma - K)/\beta K$ is the deterministic steady-state intensity, and $\delta(t)$ is a perturbational term. We linearize Eq.(1) around the deterministic steady-state intensity I_0, thus the linear equation of the laser intensity is found to be

$$\frac{\mathrm{d}\delta(t)}{\mathrm{d}t} = -\gamma\delta(t) + D + \frac{2I_0}{1+\beta I_0} A\cos(\Omega t)\,\xi(t) + 2\sqrt{I_0}\,\eta(t), \quad (5)$$

where the attenuation coefficient $\gamma = 2K(\Gamma - K)/\Gamma$. Integrating Eq.(5), we have

$$\begin{aligned} \delta(t) =& \exp(-\gamma t)\frac{D}{\gamma}\left[\exp(\gamma t) - 1\right] \\ &+ \exp(-\gamma t)\frac{2I_0}{1+\beta I_0} \\ &\times \int_0^t A\cos(\Omega u)\exp(\gamma u)\xi(u)\mathrm{d}u \\ &+ \exp(-\gamma t)2\sqrt{I_0}\int_0^t \exp(\gamma u)\eta(u)\mathrm{d}u. \end{aligned} \quad (6)$$

According to the intensity correlation function defined by

$$C(t) = \lim_{t'\to\infty} \overline{\langle I(t'+t)I(t')\rangle}, \quad (7)$$

where

$$\overline{\langle I(t'+t)I(t')\rangle} = \frac{\Omega}{2\pi}\int_{t'}^{t'+\frac{2\pi}{\Omega}} \langle I(t'+t)I(t')\rangle\,\mathrm{d}t'$$

and combining Eqs.(4), (6) and (7), the intensity correlation function is obtained to be in the following form (only in the case of $\gamma \neq \tau_i^{-1}$, $i = 1, 2, 3$) :

$$\begin{aligned} C(t) =& \frac{I_0^2A^2Q}{2(1+\beta I_0)^2\gamma\tau_1}\left(\frac{\gamma_1}{\Omega^2+\gamma_1^2} - \frac{\gamma_1'}{\Omega^2+\gamma_1'^2}\right)\mathrm{e}^{-\gamma|t|} \\ &+ \frac{I_0^2A^2Q}{(1+\beta I_0)^2\tau_1}\frac{1}{\Omega^2+\gamma_1^2}\frac{1}{\Omega^2+\gamma_1'^2} \\ &\times \mathrm{e}^{-\tau_1^{-1}t}\left[(\Omega^2+\gamma_1\gamma_1')\cos(\Omega t) + \frac{2\Omega}{\tau_1}\sin(\Omega t)\right] \\ &+ 2I_0D\frac{\tau_2\mathrm{e}^{-\tau_2^{-1}|t|} - \gamma^{-1}\mathrm{e}^{-\gamma|t|}}{(\gamma^2\tau_2^2 - 1)}, \end{aligned} \quad (8)$$

where $\gamma_1 = \gamma + \tau_1^{-1}$, $\gamma_1' = \gamma - \tau_1^{-1}$.

The intensity correlation time is defined by

$$T = \int_0^\infty \Phi(t)\mathrm{d}t, \quad (9)$$

where $\Phi(t) = C(t)/C(0)$, and $C(0)$ is the relative intensity fluctuation, which is given by

$$C(0)=\frac{I_0^2A^2Q}{(1+\beta I_0)^2\gamma\,\tau_1^2}\frac{(\Omega^2-\gamma_1\gamma_1')}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}+\frac{I_0^2A^2Q}{(1+\beta I_0)^2\tau_1}\frac{(\Omega^2+\gamma_1\gamma_1')}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}+\frac{2I_0D}{\gamma(\gamma\tau_2+1)}. \quad (10)$$

Combining Eqs.(8), (9) and (10), the intensity correlation time T is obtained as follows:

$$T=\frac{1}{C(0)}\left[\frac{I_0^2A^2Q}{(1+\beta I_0)^2\gamma^2\,\tau_1^2}\frac{(\Omega^2-\gamma_1\gamma_1')}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)}+\frac{I_0^2A^2Q}{(1+\beta I_0)^2}\frac{3\Omega^2+\gamma_1\gamma_1'}{(\Omega^2+\gamma_1^2)(\Omega^2+\gamma_1'^2)(\Omega^2\tau_1^2+1)}+\frac{2I_0D}{\gamma^2}\right]. \quad (11)$$

3. Discussion and conclusions

From Eq.(11), we see clearly that there exists neither λ term nor τ_3 term in the expression of T, which implies that the effects of the cross-correlation between the pump noise and the quantum noise disappear when the pump noise is modulated directly by a signal. The reason for the cross-correlation disappearing is as follows. When we calculate the stationary intensity correlation function $C(t)$, as shown in Eq.(7), we need to take periodic mean over t' and then set $t'\to\infty$ (stationary state). And from Eq.(1) we see that, as the periodic signal $A\cos(\Omega t)$ modulates directly the pump noise $\xi(t)$, in the cross-correlation (the $\frac{\lambda\sqrt{QD}}{\tau_3}$terms) would appear $\mathrm{e}^{-\alpha t'}$ terms or $\cos(\Omega\,t')$ terms. However, the $\mathrm{e}^{-\alpha t'}$ terms or $\cos(\Omega\,t')$ terms become zero when we take periodic mean over t' and set $t'\to\infty$ in calculating $C(t)$, so the cross-correlation disappears.

Then we will discuss the effects of noises on the intensity correlation time T according to the expression of T in Eq.(11).

3.1. The analyses of T–τ_1 curves with the values of different parameters

According to Eq.(11), by respectively taking Q, D, Ω, τ_2 as parameters, we depict T–τ_1 curves in Fig.1. From Fig.1(a) we can see obviously that there exists a maximum in each T–τ_1 curve, indicating that the resonance appears in the T–τ_1 curve. Figure 1(a) also shows that with the increasing of the value of Q, the resonant peaks increase and the peak positions shifts toward the right. Figures 1(b) and 1(c) show that with the increasing of the values of D and Ω, the resonant peaks decrease and the peak positions shift toward the left in the T–τ_1 curves. Figure 1(d) indicates that with the increasing of the value of τ_2, the resonant peaks increase and the peak positions shift toward the right in the T–τ_1 curves. Moreover, From Fig.1 (a) and Fig.1(b) we can also see that with the different values of Q and D, the different T–τ_1 curves tend to the same point as τ_1 increases respectively. And as shown in Fig.1(c), with the different values of Ω, the starting-points of T–τ_1 curves tend to the same one, and at the same time, the different T–τ_1 curves converge to the same point with τ_1 increasing. Figure 1(d) shows that with the different values of τ_2, the starting-points of T–τ_1 curves approach the same one.

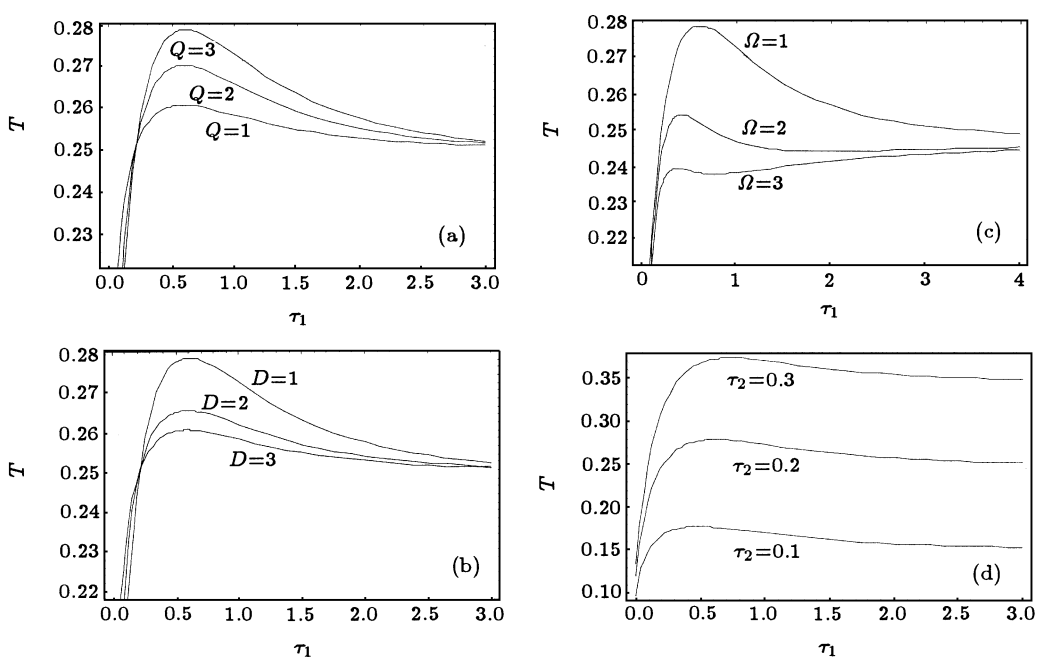


Fig.1. T–τ_1 curves respectively with Q, D, Ω and τ_2 being parameters ($I_0=1$, $A=1$, $\beta=1$, $\gamma=20$, (a) τ_2=0.2, $\Omega=1$, $D=1$, (b) $\tau_2=0.2$, $\Omega=1$, $Q=3$, (c) $\tau_2=0.2$, $Q=3$, $D=1$, (d) $\Omega=1$, $Q=3$, $D=1$).

3.2. The analyses of T–τ_2 curves with the values of different parameters

According to Eq.(11), Fig.2 depicts the T–τ_2 curves with Q, D, Ω and τ_1 being parameters, as shown in Figs 2(a), 2(b), 2(c) and 2(d), respectively. From Fig.2(a) we see that, the intensity correlation time T increases monotonically with τ_2 increasing, and in the mean time, with the increasing of the value of Q, T increases slowly with τ_2 increasing. Figure 2(b) shows that, with the increasing of the value of D, T increases rapidly with τ_2 increasing. Figure 2(c) indicates that with the increasing of the value of Ω, the T–τ_2 curves almost do not change. As shown in the Fig.2(d), with the increasing of the value of τ_1, T increases rapidly with τ_2 increasing. Furthermore, from Fig.2(b), Fig.2(c) and Fig.2(d) we can also see that, with the different values of D, Ω and τ_1, the starting-points of T–τ_1 curves tend to the same one.

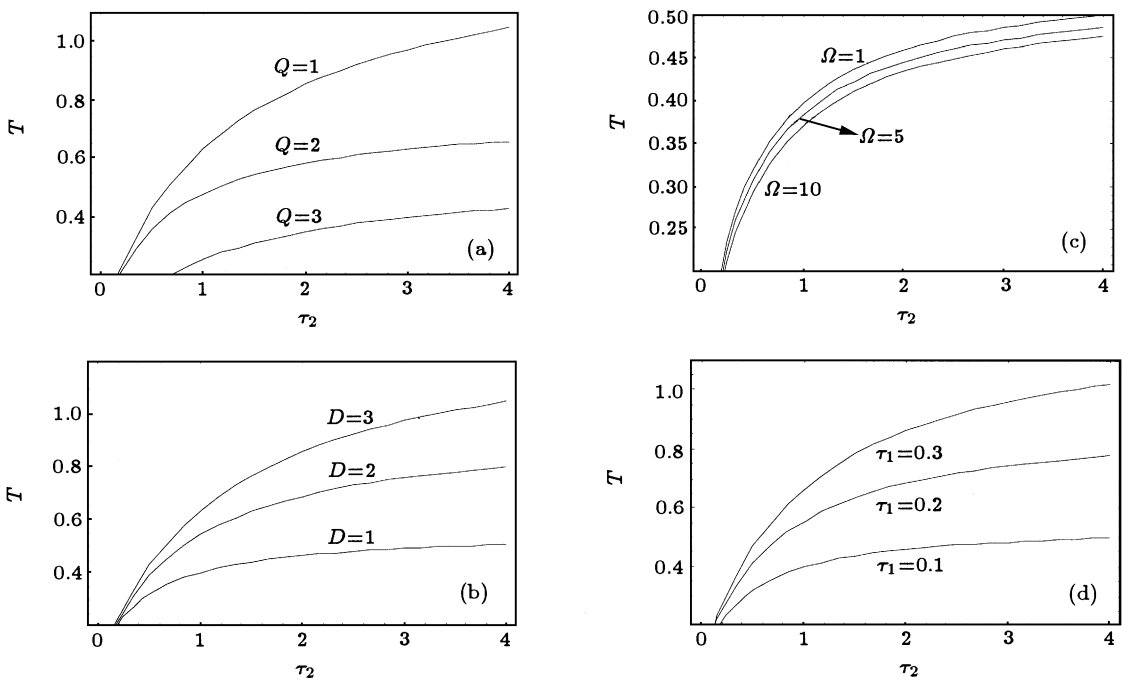


Fig.2. T–τ_2 curves respectively with D, Q, Ω and τ_2 being parameters ($I_0 = 1$, $A = 1$, $\beta = 1$, $\gamma = 20$, (a) $\tau_1 = 0.1, \Omega = 1$, $D = 1$, (b)$\tau_1 = 0.1$, $\Omega = 1$, $Q = 3$, (c)$\tau_1 = 0.1$, $Q = 3$, $D = 1$, (d) $\Omega = 1$, $Q = 3$, $D = 1$).

3.3. The analyses of T–Q curves with different values of the parameter τ_1

Following Eq.(11), taking τ_1 as a parameter, we depict T–Q curves in Fig.3, which show that when $\tau_1 \leq \tau_2$, the intensity correlation time T decreases monotonically with Q increasing, and with increasing value of τ_1, T decreases slowly with Q increasing. On the other hand, when $\tau_1 > \tau_2$, the intensity correlation time T increases monotonically with Q increasing, and in the mean time, with the increasing of the value of τ_1, T increases rapidly with Q increasing. In addition, with the different values of τ_1, the starting-points of the T–Q curves tend to the same point.

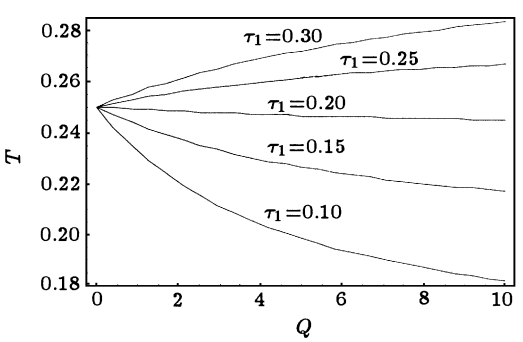


Fig.3. T–Q curves with τ_1 being a parameter ($I_0 = 1$, $A = 1$, $\beta = 1$, $\gamma = 20$, $\tau_2 = 0.2$, $\Omega = 1$, $D = 1$).

Figure 4 depicts the T–Q curves by choosing D and Ω as parameters and taking $\tau_1 \leq \tau_2$, as shown in Fig.4(a) and Fig.4(b), respectively. From Fig.4(a) we see that when $\tau_1 \leq \tau_2$, with the increasing of the value of D, T decreases slowly with Q increasing. As

shown in Fig.4(b), with increasing value of Ω, T decreases rapidly with Q increasing. In addition, Fig.4 also shows that with the different values of D and Ω, the starting-points of the T–Q curves converge to the same one as $\tau_1 \leq \tau_2$.

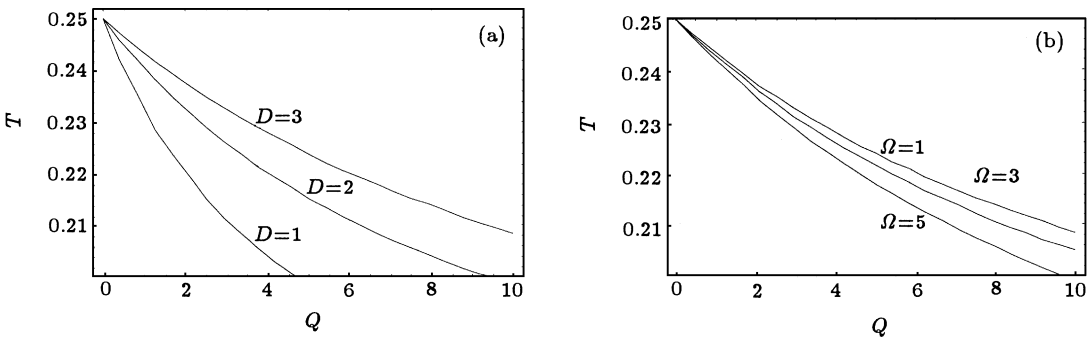


Fig.4. T–Q curves respectively with D and Ω being parameters when $\tau_1 \leq \tau_2$ ($I_0 = 1$, $A = 1$, $\beta = 1$, $\gamma = 20$, $\tau_1 = 0.1$, $\tau_2 = 0.2$, (a) $\Omega = 1$ (b) $D = 3$).

Figure 5 depicts the T–Q curves by choosing D and Ω as parameters and taking $\tau_1 > \tau_2$ as shown in Fig.5(a) and Fig.5(b), respectively. From these figures we can see that when $\tau_1 > \tau_2$, with increasing values of D and Ω, T increases slowly with Q increasing. Meanwhile, Fig.5 also shows that with the different values of D and Ω, the starting-points of T–Q curves approach the same one as $\tau_1 > \tau_2$.

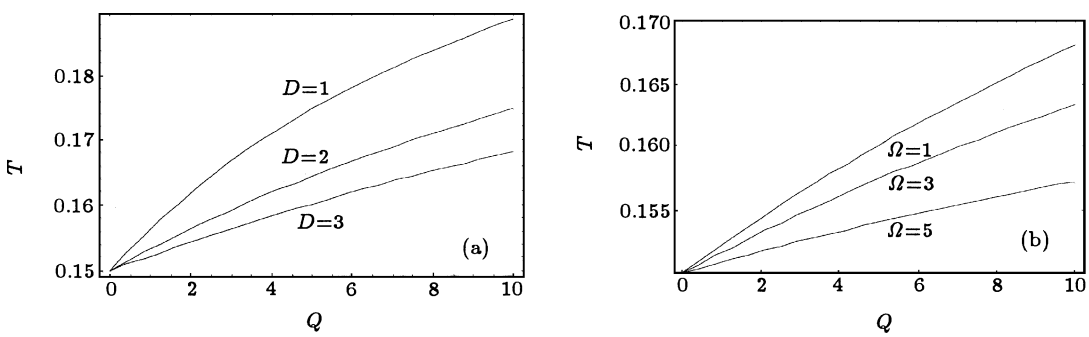


Fig.5. T–Q curves with D and Ω being parameters when $\tau_1 > \tau_2$ ($I_0 = 1$, $A = 1$, $\beta = 1$, $\gamma = 20$, $\tau_1 = 0.2$, $\tau_2 = 0.1$, (a) $\Omega = 1$, (b) $D = 3$).

3.4. The analyses of T–D curves with different values of the parameter τ_1

Following Eq.(11), taking τ_1 as a parameter, we depict T–D curves in Fig.6, which show that when $\tau_1 \leq \tau_2$, the intensity correlation time T increases monotonically with D increasing, and in the meantime, with increasing value of τ_1, T increases slowly with D increasing. On the other hand, when $\tau_1 > \tau_2$, the intensity correlation time T decreases monotonically with D increasing, and in the meantime, with increasing value of τ_1, T decreases faster with D increasing. Also, with the different values of τ_1, the different T–D curves tend to the same point as D increases.

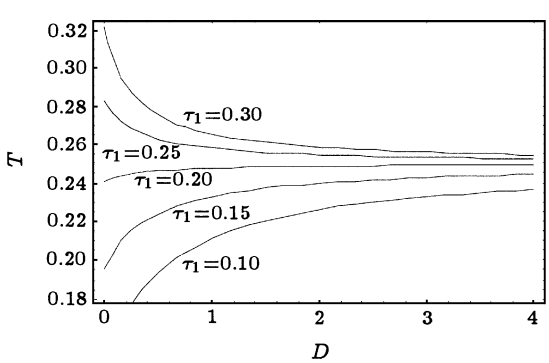


Fig.6. T–D curves with τ_1 being a parameter ($I_0 = 1$, $A = 1$, $\beta = 1$, $\gamma = 20$, $\tau_2 = 0.2$, $\Omega = 1$, $Q = 3$).

Figure 7 depicts the T–D curves by choosing Q and Ω as parameters and taking $\tau_1 \leq \tau_2$, as shown respectively in Fig.7(a) and Fig.7(b). From Fig.7(a) we see that when $\tau_1 \leq \tau_2$, with increasing value of

Q, T increases slowly with D increasing. As shown in Fig.7(b), with increasing value of Ω, T increases rapidly with D increasing. In addition, Fig.7(b) also shows that with the different values of Ω, the different T–D curves tend to the same point as D increases when $\tau_1 \leq \tau_2$.

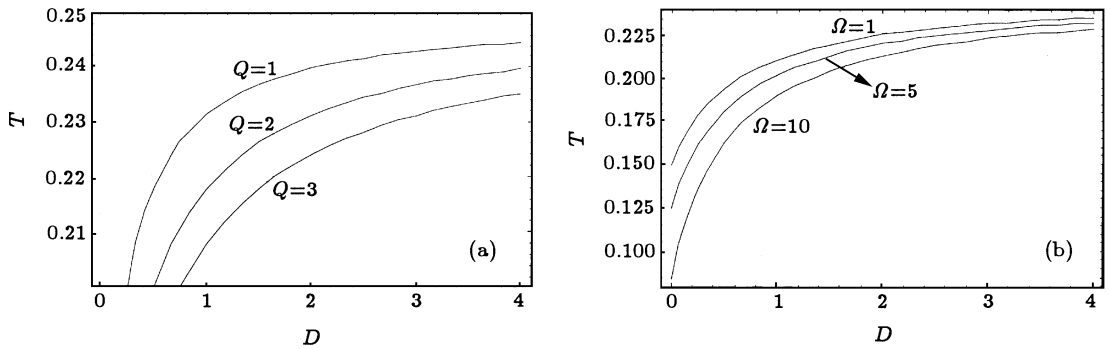


Fig.7. T–D curves with Q and Ω being parameters when $\tau_1 \leq \tau_2$ ($I_0 = 1$, $A = 1$, $\beta = 1$, $\gamma = 20$, $\tau_1 = 0.1$, $\tau_2 = 0.2$, (a) $\Omega = 1$, (b)$Q = 3$).

Figure 8 depicts the T–D curves by choosing Q and Ω as parameters and taking $\tau_1 > \tau_2$, as shown respectively in Fig.8(a) and Fig.8(b). From these figures we see that when $\tau_1 > \tau_2$, with increasing values of Q and Ω, T decreases slowly with D increasing. Figure 8(b) also shows that with the different values of Ω, the different T–D curves tend to the same point as D increases when $\tau_1 > \tau_2$.

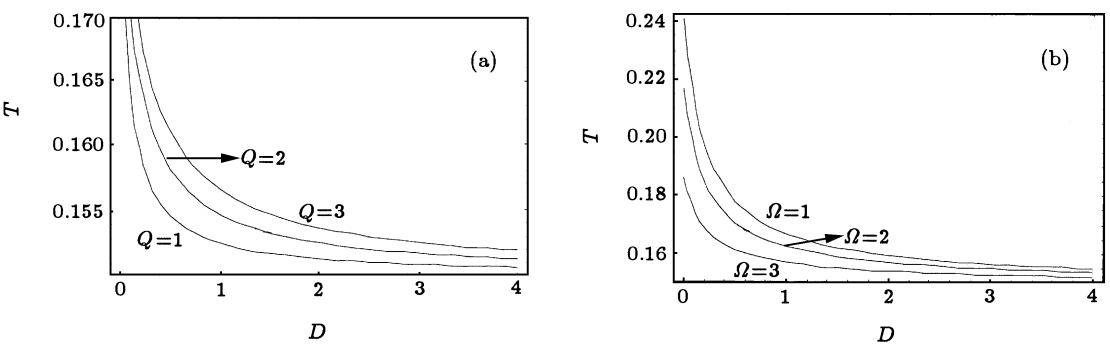


Fig.8. T–D curves with Q and Ω being parameters when $\tau_1 > \tau_2$ ($I_0 = 1$, $A = 1$, $\beta = 1$, $\gamma = 20$, $\tau_1 = 0.2$, $\tau_2 = 0.1$, (a) $\Omega = 1$, (b) $Q = 3$).

4. Summary

From the above analyses, we reach the following conclusions:

1. The effects of the cross-correlation between the pump noise and the quantum noise disappear when the pump noise is modulated directly by a signal.

2. There exists a maximum in the T–τ_1 curve, which implies that the resonance appears in the T–τ_1 curve. With the increasing of the value of Q, the resonant peak increases and the peak position shifts toward the right; with increasing value of D, the resonant peak decreases and the peak position shifts toward the left.

3. The intensity correlation time T increases monotonically with τ_2 increasing. With increasing value of Q, T increases slowly with τ_2 increasing; with increasing value of D, T increases rapidly with τ_2 increasing.

4. When $\tau_1 \leq \tau_2$, the intensity correlation time T decreases monotonically with Q increasing, but when $\tau_1 > \tau_2$, the intensity correlation time T increases monotonically with Q increasing.

5. When $\tau_1 \leq \tau_2$, the intensity correlation time T increases monotonically with D increasing, but when $\tau_1 > \tau_2$, T decreases monotonically with D increasing.

References

[1] Hernandez-Gorcia E, Toral R and San Minguel M 1990 *Phys. Rev.* A **42** 6823

[2] Zhu S Q and Yin J P 1992 *Phys. Rev.* A **45** 4969

[3] Zhu S Q 1993 *Phys. Rev.* A **47** 2405

[4] Hu G 1994 *Stochastic Forces and Nonlinear Systems* (Shanghai Scientific and Technological Education Publishing House, Shanghai) p184 (in Chinese)

[5] Mei D C, Chen L E, Xie G Z, Cao L and Wu D J 1999 *Acta Phys. Sin.* (Overseas Edition) **8** 808

[6] Mei D C, Cao L, Wu D J and Xie G Z 1999 *Acta Phys. Sin.* (Overseas Edition) **8** 174

[7] Cao L and Wu D J 2001 *Phys. Lett.* A **283** 313

[8] Cao L and Wu D J 2003 *Eur. Phys. Lett.* **61** 593

[9] Zhang L Y, Cao L and Wu D J 2003 *Acta Phys. Sin.* **52** 1174 (in Chinese)

[10] Zhang L, Cao L and Wu D J 2003 *Chin. Phys.* **12** 33

[11] Zhang L, Cao L and Wu D J 2004 *Chin. Phys.* **13** 353

[12] Fulinski A and Telejko T 1991 *Phys. Lett.* A **152** 11

[13] Luo X Q, Zhu S Q and Gao W J 2001 *Chin. Phys.* **10** 1011

[14] Luo X Q and Zhu S Q 2002 *Acta Phys. Sin.* **51** 977 (in Chinese)

[15] Zhou X J, Cao L and Wu D J 1998 *Chin. Phys. Lett.* **15** 183

[16] Zhou X J, Cao L and Wu D J 1999 *Acta Opt. Sin.* **19** 7 (in Chinese)

[17] Liang G Y, Cao L and Wu D J 2003 *Chin. Phys.* **12** 1105

[18] Liang G Y, Cao L, Zhang L and Wu D J 2003 *Chin. Phys.* **12** 1109