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Frequency conversion of probe wave produced by 2D interaction with relativistic ionization front*

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Frequency conversion of probe electromagnetic wave induced by relativistic ionization front is theoretically analysed based on ray-tracing equations in different regimes. Downshifting as well as upshifting in frequency produced by the front is predicted. The reflected and transmitted angles can be also dramatically changed in certain cases.

Keywords: relativistic ionization front, frequency conversion, ray-tracing equations
PACC: 4215D, 5240D, 1110E, 0330

1. Introduction

With the increasing availability of high power lasers, a large number of interesting phenomena about plasma have been studied both theoretically and experimentally.[1−3] At the present time the intensity of ultrashort pulse laser can be so high as to fully ionize the gas in a focal region into plasma within 1fs. A plasma–neutral-gas interface is synchronously formed in the place where high power laser pulse arrives. Such a relativistic ionization front moves along the laser front in the gas at a speed close to c, the light speed in vacuum. Lampe *et al*[4] have proposed that electromagnetic waves can be reflected by relativistic ionization front in which overdense plasma exists, resulting in double-Doppler-frequency upshifting and pulse-width compression. For the case of underdense density, Mori *et al*[5] have indicated that the transmitted wave is also considerably frequency-upshifted by the oncoming relativistic ionization front. Sprangle *et al*[6] have reported that the frequency upconversion of probe pulse can be produced in co-propagating interaction with the ionization front. And Yu W *et al*[7] have explored the interaction of electromagnetic waves with counter-travelling ionization front under the moving boundary conditions. Subsequently the two different teams have separately carried out theoretical studies of the electromagnetic probe pulse with TE[8] and TM[9] polarizations. So far two experiments devoted to the interactions of electromagnetic waves with relativistic ionization front have been reported: one used microwave as probe pulse,[10] the other used ultrashort laser in 2D configuration (in which the direction of incident wave was variable with that of the front).[11]

The frequency variation of probe pulse induced by relativistic ionization front may have several attractive potential applications. Tunability of ultrashort pulses in a very large band ranging from UV to IR can be realized through changing the interaction parameters. Pulsed laser reflected by the front can produce x-ray or sub-fs laser pulse. The frequency conversion may be used to characterize the plasma in the ionization front, so there appears a new diagnostic method for plasma. Comprehensive study of the mechanism of interaction between the probe wave and relativistic ionization front is useful to develop the above-mentioned potential applications.

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2. Basic theory

The method based on ray-tracing equations of Hamiltonian form is convenient to deduce the frequency variation of electromagnetic wave in a perturbed plasma.[12] It can be used to deal with the slow-varying density plasma in the ionization front; moreover, it is polarization-independent in describing the interactions. A brief introduction about this method is given below.

Propagating in the xy plane, the incident probe electromagnetic wave determined by the wave vector k, the coordinate x and the frequency Ω satisfies the following equations:

$$\Omega = [k^2c^2 + \omega_p^2(x,t)]^{1/2}, \tag{1}$$

$$\frac{\mathrm{d}k}{\mathrm{d}t} = -\frac{\partial\Omega}{\partial x} = -\frac{1}{2\Omega}\frac{\partial\omega_p^2}{\partial x}, \tag{2}$$

$$\frac{\mathrm{d}x}{\mathrm{d}t} = \frac{\partial\Omega}{\partial k} = \frac{kc^2}{\Omega}, \tag{3}$$

$$\frac{\mathrm{d}\Omega}{\mathrm{d}t} = \frac{\partial\Omega}{\partial t} = \frac{1}{2\Omega}\frac{\partial\omega_p^2}{\partial t}, \tag{4}$$

where we assume that the relativistic ionization front moves along the positive x axis, and the plasma frequency determined by the plasma density is described by $\omega_p^2(x-\beta ct) = \omega_p^2[1-u(x-\beta ct)]$. The incident angle of probe pulse is defined as $\cos\theta_i = k\cdot e_x/|k|$, as shown in Fig.1.

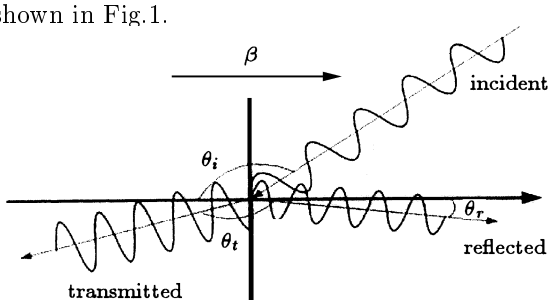


Fig.1. Principle of counter-oblique-interaction configuration.

Introducing the canonical transformation defined by the generating function $F_2 = p_x(x-\beta ct) + p_y y$, in which $k_x = p_x$, $k_y = p_y$, $\eta_x = x-\beta ct$, $\eta_y = y$, we obtain the new Hamiltonian:

$$\overline{\Omega} = [(p_x^2+p_y^2)c^2 + \omega_p^2(\eta_x)]^{1/2} - \beta p_x c, \tag{5}$$

which is independent of t and η_y, so $\overline{\Omega}$ and p_y are both invariant quantities which can be used to deduce the frequency conversion relations in the interactions of probe electromagnetic wave with relativistic ionization front.

3. Counter-oblique incidence

In this case the probe wave is incident from the neutral gas. Here we assume the dielectric relations in vacuum to be obeyed, when the oncoming ionization fronts is transmitted into the plasma or reflected back into the vacuum. The incident angle is restricted in the interval $\pi/2 < \theta_i \leq \pi$.

3.1. Reflected wave

For the frequency upshifting, we have

$$\frac{\omega_r}{\omega_i} = \frac{(1+\beta^2) - 2\beta\cos\theta_i}{1-\beta^2}, \tag{6}$$

and for the reflected angle, we obtain

$$\tan\theta_r = \frac{(1-\beta^2)\sin\theta_i}{2\beta - (1+\beta^2)\cos\theta_i}, \tag{7}$$

where the subscripts i, r denote the quantities relative to the incident wave and reflected wave respectively (in the following, the subscript t denotes quantities of transmitted wave). It is easy to know from the above formulae that $\frac{\omega_r}{\omega_i} \gg 1$, and$\tan\theta_r \ll 1$, which mean that the frequency upshifting of the probe wave is very large, as can be seen from Fig.2, and the reflected angle approaches the propagating direction of the ionization front though the incident angle is much smaller than 180 degrees, as displayed in Fig.3. The frequency upshifting is independent of the plasma density in the ionization front. But in the case of underdense plasma, according to the authors' further results, the portion of reflected energy is very small, far less than unity. In order to achieve considerable reflection the plasma density must be high enough.

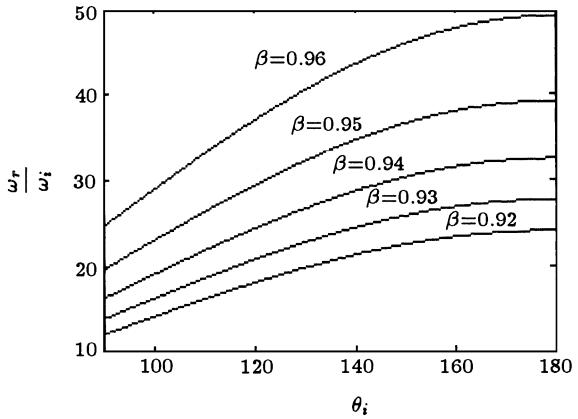


Fig.2. Frequency upshifting of reflected wave in counter-oblique interaction configurations.

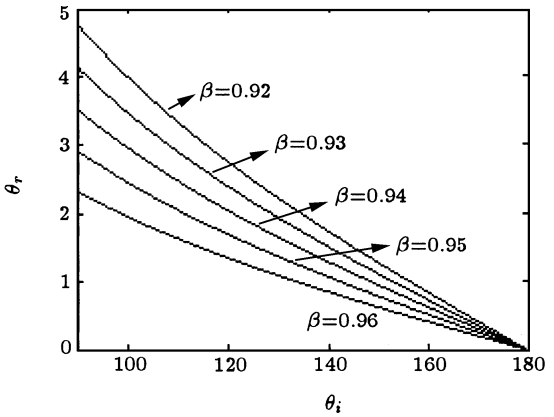


Fig.3. Reflected angles in counter-oblique interaction configurations.

3.2. Transmitted wave

For the frequency upshifting, we have

$$\frac{\omega_t}{\omega_i} = \frac{(1-\beta\cos\theta_i) - \beta[(\beta-\cos\theta_i)^2 - (1-\beta^2)\omega_p^2/\omega_i^2]^{1/2}}{1-\beta^2}, \tag{8}$$

and for the transmitted angle, we acquire

$$\tan\theta_t = \frac{(1-\beta^2)\sin\theta_i}{\beta(1-\beta\cos\theta_i) - [(\beta-\cos\theta_i)^2 - (1-\beta^2)\omega_p^2/\omega_i^2]^{1/2}}. \tag{9}$$

In this case the relation $(\beta-\cos\theta_i)^2 \gg (1-\beta^2)\omega_p^2/\omega_i^2$ always holds, so it leads to the simplification of the frequency upshifting formula:

$$\frac{\omega_t}{\omega_i} \approx 1 + \frac{\beta}{2(\beta-\cos\theta_i)^2}N, \tag{10}$$

where $N = \dfrac{\omega_p^2}{\omega_i^2}$ is defined as the normalized plasma density.

The frequency upshifting is small, which is proportional to the normalized plasma density. More upshifting requires greater plasma density or shorter wavelength probe wave. Correspondingly, the transmitted angle changes little.

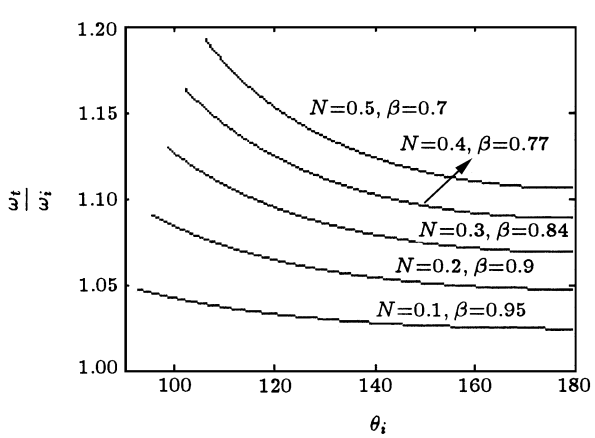


Fig.4. Frequency upshifting of transmitted wave in counter-oblique interaction configurations.

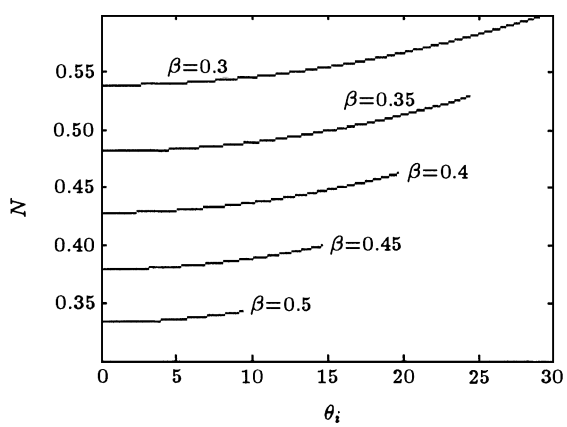


Fig.5. Reflecting condition for N in co-oblique-interaction configurations.

4. Co-oblique incidence

The incident angle of probe pulse satisfies the inequality $0 \le \theta_0 < \pi/2$. Compared with the counter-interaction configuration, this case is very complicated. There exist three types of interactions: (I) the ionization front is overtaken by the probe wave which is reflected back into plasma and propagates backward, resulting in frequency downshifting; (II) the probe pulse transmits the front and propagates in vacuum, leading to frequency upshifting; (III) the front

overtakes the probe pulse and the transmitted wave still propagates in the plasma, in which the transmitted wave falls behind the front.

4.1. Reflected wave

For the reflected frequency, we have

$$\frac{\omega_r}{\omega_i} = \frac{(1+\beta^2) - 2\beta a_0}{1-\beta^2}, \tag{11}$$

and for the reflected angle, we acquire

$$\tan\theta_r = \frac{(1-\beta^2)\sin\theta_i}{2\beta - (1+\beta^2)a_0}, \tag{12}$$

where $a_0 \equiv \dfrac{k_{0x}c}{\omega_i} = \left(1 - \dfrac{\omega_p^2}{\omega_i^2}\right)^{1/2} \cos\theta_i$, which is the normalized x-component of the probe wave vector in the plasma. The condition for the existence of reflected wave is $a_0 > \dfrac{2\beta}{(1+\beta^2)} > \beta$, which requires β to be less than 0.55. For a certain value of β, only for a special range of plasma density there exists the reflected wave, as shown in Fig.5. In this case the reflected wave is downshifting, as shown in Fig.6, and the reflected angle approaches 90 degrees.

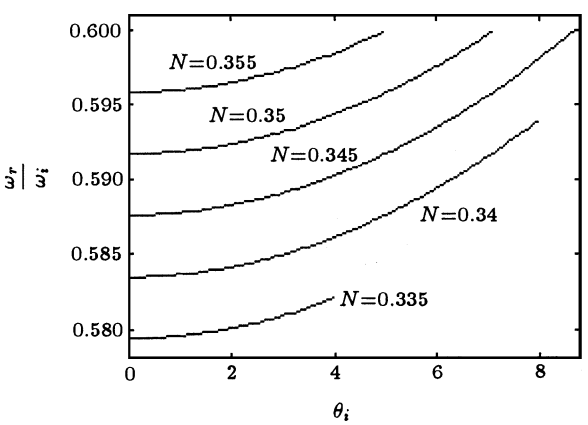


Fig.6. Frequency downshifting of reflected wave for β=0.5 in co-oblique interaction configurations.

For the transmitted frequency, we have

$$\frac{\omega_t}{\omega_i} = \frac{(1-\beta\cos\theta_i) - \beta[(\beta-\cos\theta_i)^2 - (1-\beta^2)\omega_p^2/\omega_i^2]^{1/2}}{1-\beta^2}, \tag{15}$$

and for the transmitted angle, we have

$$\tan\theta_t = \frac{(1-\beta^2)\sin\theta_i}{\beta(1-\beta\cos\theta_i) + [(\beta-\cos\theta_i)^2 - (1-\beta^2)\omega_p^2/\omega_i^2]^{1/2}}. \tag{16}$$

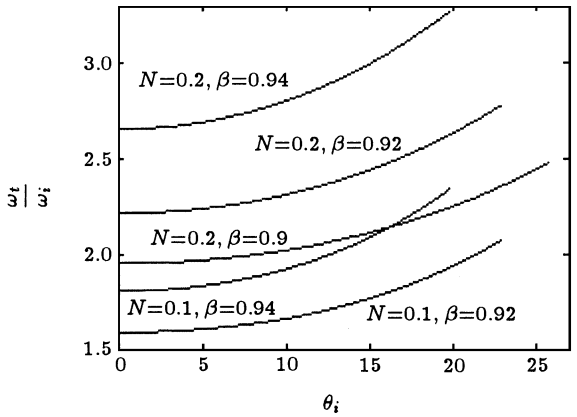


Fig.7. Frequency upshifting of transmitted wave in co-oblique interaction configurations for the type II.

4.2. Transmitted wave

For the type II of interaction, the transmitted wave overtakes the front and propagates in vacuum, requiring that $\cos\theta_i > \beta$.

For the transmitted frequency, we have

$$\frac{\omega_t}{\omega_i} = \frac{(1-\beta a_0) + \beta[(\beta-a_0)^2 + (1-\beta^2)\omega_p^2/\omega_i^2]^{1/2}}{1-\beta^2}, \tag{13}$$

and for the transmitted angle, we obtain

$$\tan\theta_t = \frac{(1-\beta^2)\sin\theta_i}{\beta(1-\beta a_0) + [(\beta-a_0)^2 + (1-\beta^2)\omega_p^2/\omega_i^2]^{1/2}}. \tag{14}$$

The curves plotted in Fig.7 show that, under similar conditions, frequency upshifting in this case is greater than that in counter-interaction configuration, and the reflected wave propagates more closely along the axis.

In the type III interaction, there exists $\cos\theta_i < \beta$, which implies that the x-component of the probe wave velocity is less than the velocity of ionization front. In this case the interaction happens when the probe wave propagating in vacuum is overtaken by the front. The transmitted wave travelling in the plasma requires $\left(1 - \dfrac{\omega_p^2}{\omega_i^2}\right)^{1/2} \cos\theta_t < \beta$.

In this case the probe pulse also undergoes considerable frequency upshifting, as shown in Fig.8.

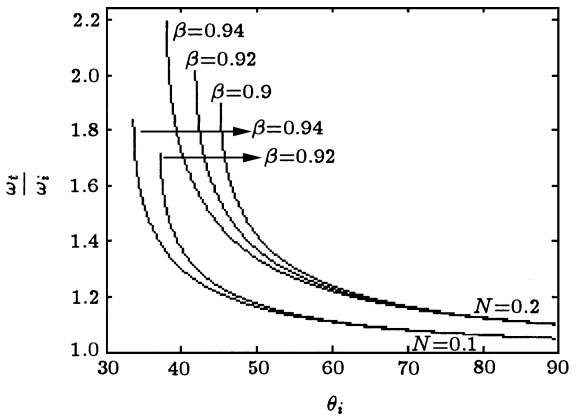


Fig.8. Frequency upshifting of transmitted wave in co-oblique interaction configurations for type III.

5. Conclusions

Based on ray-tracing equations of Hamiltonian form, the article has presented the theoretical investigation about the 2D interactions of probe electromagnetic wave with relativistic ionization front, indicating that besides the frequency upshifting being introduced, frequency downshifting can also take place in some special cases. When reflected by the oncoming ionization front the probe wave undergoes the maximum frequency upshifting which can reach more than several ten times the frequency of the incident wave. In co-interaction (parallel) configurations, when the probe pulse transmits the front the frequency will be upshifted several times at most, greater than that in counter-transmission interaction. Frequency downshifting is surely produced for some appropriate parameters of plasma density and front velocity. Though this method is convenient to discuss the frequency conversion of the probe wave induced by the relativistic ionization front, it has some limits, e.g. it is difficult to deduce the so-called free streaming mode produced in the interactions, and may not be used to develop the coefficient of the induced waves.

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