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Decentralized state-feedback chaotification method of discrete Takagi–Sugeno fuzzy systems*

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A new chaotification method is proposed for making an arbitrarily given discrete Takagi–Sugeno (TS) fuzzy system chaotic. Based on a given discrete TS fuzzy system, the new chaotification method uses the decentralized state-feedback control and the continuous sawtooth function, instead of the modulo operation, to construct a chaotic nonlinear system, which can generate discrete chaos with the arbitrarily desired amplitude bound. We apply the improved Marotto theorem to mathematically prove that the controlled system is chaotic in the sense of Li and Yorke. In particular, an explicit formula for the computation of chaotification parameters is obtained. A numerical example is used to illustrate the theoretical results.

Keywords: chaos, chaotification, Takagi–Sugeno fuzzy system, chaos control
PACC: 0545

1. Introduction

In addition to the rapid development of chaos suppression and chaos synchronization in the last decade,[1,2] chaotification (also called anticontrol of chaos) by means of making an originally non-chaotic dynamical system chaotic, or enhancing and retaining existing chaos, has attracted certain special attention from the engineering and physics communities lately due to the great potential of chaos in non-traditional application.[2,3] Dynamical systems (maps) with chaotic and/or bifurcate behaviours have been reported to be very useful in some real applications, such as high-performance circuits and devices, liquid mixing, chemical reactions, biological systems, secure information processing, and critical decision-making.[2] These provide a strong motivation for the current research on chaotification of the discrete-time dynamical systems[3−14] and the continuous-time dynamical systems.[15,16]

In the pursuit of chaotifying discrete dynamical systems, the first mathematically rigorous chaotification method was developed by Chen and Lai[4−6] from the engineering feedback control approach. Up to now, the chaotification problem of discrete dynamical systems can be divided into two kinds. The first kind is to consider how to directly design a nonlinear state-feedback controller such that the controlled discrete dynamical systems become chaotic such as in Refs.[3,12–14]. The second kind is to consider how to first design a linear state-feedback controller to ensure that all the Lyapunov exponents of the controlled system become strictly positive so that the system trajectory is expanding in all directions, and then apply a nonlinear operation to fold the trajectories back into a compact region whenever the expansion takes them beyond it. A combination of these two expanding and folding effects is expected to create complex chaotic dynamics within the bounded region of system orbits, thereby leading to chaotic dynamics such as in Refs.[4–11]. Lu *et al*[7] proposed a method of producing spatiotemporal chaos via weak local state-feedback control for coupled logistic map chain (CLMC) model and proved theoretically that all the Lyapunov exponents of the controlled system are strictly positive. References [10] and [11] have given the chaotification algorithms for any given linear time-invariant discrete-time systems.

In this paper, we are interested in the second kind of chaotification problem. Interactions between

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fuzzy logic and chaos theory have been explored since 1990s. The explorations have been carried out mainly on fuzzy modelling of chaotic systems using the Takagi–Sugeno (TS) fuzzy model,[17] suppression control of chaos via an linear matrix inequality (LMI)-based fuzzy control system design.[18,19] Recently, Li *et al*[9] use the centralized control and mod-operation to chaotify a TS fuzzy system and apply the Marotto theorem[20] to prove their results. Moreover, Refs.[21–23] have pointed out that the Marotto theorem[20] has a problem since Marotto had used the following property $A' \Rightarrow B'$ in his theorem:

A': All eigenvalues of the real matrix A are larger than 1 in absolute value;

B': Matrix A satisfies $\|Ax\|_2 \geq \mu \|x\|_2$ for some $\mu > 1$, for all $x \in R^n$.

In this paper, for an arbitrarily given discrete TS fuzzy system, we propose a new chaotification method belonging to the second kind mentioned above, which uses the decentralized state-feedback control and the continuous sawtooth function instead of the discontinuous modulus function to construct a chaotic nonlinear system. In each dimension of the subsystem, the decentralized controller uses only one local state variable. We apply the improved Marotto theorem to mathematically prove that the proposed feedback anticontrol method can yield chaos in the sense of Li and Yorke[24] with the arbitrarily desired amplitude bound. In particular, an explicit formula for the computation of chaotification parameters is also obtained. Finally, a numerical example is used to illustrate the theoretical analysis and the controller design.

Throughout this paper, all vector inequalities such as $x \geq 0$ are component-wise inequalities. $\|\cdot\|$, $\|x\|_2$ and $\|x\|_\infty$denote any operator norm, 2-norm and infinity-norm, respectively.

2. Problem description and chaotification algorithm design

Consider a general TS fuzzy system[25] described by

Rule i: IF $x_1(k)$ is M_{i1}, $\cdots$, and $x_n(k)$ is M_{in}, THEN

$$x_{k+1} = A_i x_k + u_k, \tag{1}$$

where $x_k = [x_k(1), x_k(2), \cdots, x_k(n)]^{\mathrm{T}}$, $u_k = [u_k(1), u_k(2), \cdots, u_k(n)]^{\mathrm{T}}$, $i = 1, 2, \cdots, r$, r is the number of IF-THEN rules, M_{ij} are fuzzy sets, and the equation $x_{k+1} = A_i x_k + u_k$ is the output from the ith IF-THEN rule. Now, given a pair of (x_k, u_k), the final output of the fuzzy system is inferred from

$$x_{k+1} = \frac{\sum_{i=1}^{r} w_i(k) \{A_i x_k + u_k\}}{\sum_{i=1}^{r} w_i(k)}, \tag{2}$$

where $w_i(k) = \prod_{j=1}^{n} M_{ij}(x_k(j))$, $M_{ij}(x_k(j))$ is the degree of membership of $x_k(j)$ in M_{ij}, with

$$\begin{cases} \sum_{i=1}^{r} w_i(k) > 0, \\ w_i(k) \geq 0, \quad i = 1, 2, \cdots, r. \end{cases}$$

By using $h_i(k) = w_i(k) / \sum_{i=1}^{r} w_i(k)$ instead of $w_i(k)$, Eq.(2) is rewritten as

$$\begin{aligned} x_{k+1} &= \sum_{i=1}^{r} h_i(k) \{A_i x_k + u_k\} \\ &= \left[\sum_{i=1}^{r} h_i(k) A_i\right] x_k + u_k. \end{aligned} \tag{3}$$

Note that

$$\begin{cases} \sum_{i=1}^{r} h_i(k) = 1, \\ h_i(k) \geq 0, \quad i = 1, 2, \cdots, r, \end{cases} \tag{4}$$

in which $h_i(k)$ can be regarded as the normalized weight of the IF-THEN rules.

The problem investigated is to chaotify arbitrarily given discrete TS fuzzy systems (3) with $\|u_k\|_\infty \leq u_{\max}$, where $u_{\max}$ is an arbitrarily small control magnitude bound. For this purpose, we use the decentralized control and the continuous sawtooth function to obtain the closed-loop controlled system:

$$x_{k+1} = D\Phi\left(\left[\sum_{i=1}^{r} h_i(k) A_i\right] x_k + \sigma x_k\right), \tag{5}$$

where $\Phi(y) = [\varphi(y_1), \cdots, \varphi(y_n)]^{\mathrm{T}}$, for any $y = [y_1, \cdots, y_n]^{\mathrm{T}} \in R^n$, $\sigma > 0$ is a constant to be designed, $D = \mathrm{diag}(d_1, \cdots, d_n) \in R^{n \times n}$, $d_i > 0$, $\varphi : R \to R$ is a continuous sawtooth function defined by (see Fig.1)

$$\varphi(y) = (-1)^m (y - 2m\varepsilon)$$

for

$$(2m-1)\varepsilon \leq y \leq (2m+1)\varepsilon, \quad m = 0, \pm 1, \pm 2 \cdots \tag{6}$$

where $\varepsilon > 0$ can be designed such that $\|u_k\|_\infty = \|\sigma x_k\|_\infty \leq d_{\max}\sigma\varepsilon \leq u_{\max}$, $d_{\max} = \max_{i \in [1, n]} d_i$. Later,

we will apply the improved Marotto theorem to mathematically prove that the controlled discrete TS fuzzy system (5) is chaotic in the sense of Li and Yorke.

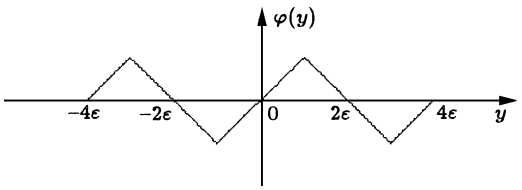


Fig.1. The sawtooth function.

3. Verification of chaos in the sense of Li–Yorke

In this section, the improved Marotto theorem is first reviewed, and then applied to prove that the controlled system (5) is chaotic in the sense of Li and Yorke.

3.1. Chaos in the sense of Li and Yorke: the improved Marotto theorem

Consider a general n-dimensional discrete-time autonomous system of the form

$$x_{k+1} = g(x_k), \quad k = 0,\ 1,\ 2,\ \cdots \tag{7}$$

where $x_k \in R^n$ and $g : R^n \to R^n$ is continuous mapping. Let g^t denote the t times of compositions of g itself. A point, x^*, is said to be a p-periodic point of g if $g^p(x^*) = x^*$ but $g^t(x^*) \neq x^*$ for $1 \leq t < p$. If $p = 1$, that is, $g(x^*) = x^*$, x^* is called a fixed point. Let $g'(x)$ and $\det(g'(x))$ be the Jacobian of g evaluated at the point x and its determinant, respectively, and let $B_r(x)$ denote a closed ball in R^n of radius r centred at point x. From Ref.[20], the improved Marotto theorem can be given as follows.

Theorem 1[20] If the system (7) has a fixed point x^*, which satisfies:

(i) there exists a real number $r > 0$ such that g is differentiable in $B_r(x^*)$ with

$$\left\|(g'(\xi))^{-1}\right\| < 1$$

where $g'(\xi) \in R^{n\times n}$ is an $n \times n$ matrix whose ith row vector is $\nabla g_i(\xi^i)$, $i = 1,\ 2,\ \cdots,\ n$, $\forall \xi^1,\ \xi^2, \cdots, \xi^n \in B_r(x^*)$;

(ii) there exists a point $x^0 \in B_r(x^*)$ with $x^0 \neq x^*$ such that $g^M(x^0) = x^*$ and $\det((g^M)'(x^0)) \neq 0$ for some positive integer M, then the system (7) is chaotic in the sense of Li–Yorke, namely,

(a) there is a positive integer n such that for every integer $p \geq n$, g has a point of period p;

(b) there is a scrambled set of g (an uncountable set S containing no periodic points of g) such that

(b1) $g(S) \subset S$,

(b2) for every $x, y \in S$ with $x \neq y$,

$$\lim_{k\to\infty} \sup \left\|g^k(x) - g^k(y)\right\| > 0,$$

(b3) for every $x \in S$ and any periodic point $y_{\rm per}$ of g

$$\lim_{k\to\infty} \sup \left\|g^k(x) - g^k(y_{\rm per})\right\| > 0;$$

(c) there is an uncountable subset S_0 of S such that for any $x, y \in S_0$,

$$\lim_{k\to\infty} \inf \left\|g^k(x) - g^k(y)\right\| = 0.$$

3.2. Verification of chaos in the designed controlled systems

Lemma 1[26] Let (X, d) be a complete metric space and V be a nonempty closed subset of X. If $f : V \to X$ is continuous, satisfies $f(V) \supset V$, and f is expanding in V, then f has a unique fixed point in V.

Theorem 2 Suppose that $h_i(k)$, $i = 1, 2, \cdots, r$, are continuously differentiable in the neighbourhood of the fixed point, $x^* = 0$ of the controlled system (5). If

$$\begin{aligned}\sigma > \sigma^* =& \max\left\{\frac{(2 + d_{\min}\alpha)}{d_{\min}}, \alpha + \frac{1}{d_{\min}}\right.\\ &\left. + \sqrt{\frac{4\alpha}{d_{\min}} + \frac{1}{d_{\min}}}, \frac{1}{d_{\min}} + \alpha\right\}\end{aligned} \tag{8}$$

where $\alpha = \max\limits_{1\leq i\leq r}\{\|A_i\|_\infty\}$, $d_{\min} = \min\limits_{i\in[1,\,n]} d_i$, then the controlled TS fuzzy system (5) is chaotic in the sense of Li and Yorke.

Remark It is surprising to note that σ is independent of ε.

Proof The controlled system (5) is

$$\begin{aligned}x_{k+1} =& D\,\Phi\left(\sum_{i=1}^{r} h_i(k)\left\{A_i x_k + u_k\right\}\right)\\ =& D\,\Phi\left(\left[\sum_{i=1}^{r} h_i(k) A_i\right] x_k + u_k\right) \equiv g(x_k).\end{aligned} \tag{9}$$

Obviously, $x^* = 0$ is a fixed point of Eq.(9). Let $b = \left[\begin{array}{cccc} 1 & 1 & \cdots & 1 \end{array}\right]^{\rm T} \in R^n$. In what follows, we prove the following.

(a) There exists a point $-(\sigma + \alpha)^{-1} b\varepsilon < x_0 < (\sigma + \alpha)^{-1} b\varepsilon$, such that $g^2(x_0) = 0 = x^*$.

(b) There exists a constant r, $0 < r < (\sigma + \alpha)^{-1}\varepsilon$ such that $x^0 \in B_r(0) \equiv \{x \in R^n \mid \|x\|_\infty \leq r\}$, $g(x)$ is

differentiable for all $x \in B_r(0)$ and $\left\|(g'(\xi))^{-1}\right\|_\infty < 1$ for $\xi = [\xi^1, \cdots, \xi^n] \in R^{n\times n}$ with $\xi^i \in B_r(0)$.

(c) $\det\left((g^2)'(x^0)\right) \neq 0$.

Proof of a) The proof is completed in two steps.

Step (i) Firstly, we prove that there exists a point x_1, $-\varepsilon\, d_{\min} b < x_1 \leq \varepsilon\, d_{\min} b$, such that

$$\bar{g}(x_1) \equiv \left[\sum_{i=1}^{r} h_i(k) A_i\right] x_1 + \sigma x_1 - 2\varepsilon b = 0, \qquad (10)$$

which implies that

$$\begin{aligned} g(x_1) =& D\,\Phi\left(\sum_{i=1}^{r} h_i(k)\left\{A_i x_1 + \sigma x_1\right\}\right) \\ =& D\,\Phi(2\varepsilon b) = 0. \end{aligned} \qquad (11)$$

To do so, we define $V = [-\varepsilon\, d_{\min} b, \varepsilon\, d_{\min} b]$, $X = R^n$, $d(\cdot) = \|\cdot\|_\infty$ and consider the following map:

$$\Phi(x) = \left[\sum_{i=1}^{r} h_i(k) A_i\right] x + (\sigma+1)x - 2\varepsilon b.$$

Apparently, $(R^n, \|\cdot\|_\infty)$ is a complete metric space, V is a nonempty closed subset of R^n and $\Phi : V \to R^n$ is continuous. Let $\Phi(x) = [\Phi_1(x), \cdots, \Phi_n(x)]^{\mathrm{T}}$, $\left[\sum_{i=1}^{r} h_i(k) A_i\right] x = [f_1(x), \cdots, f_n(x)]^{\mathrm{T}}$. It follows from formula (8) and $\left\|\sum_{i=1}^{r} h_i(k) A_i\right\|_\infty \leq \alpha$ that for $1 \leq j \leq n$,

$$\begin{aligned} &\Phi_j(x) \geq -\alpha\varepsilon\, d_{\min} + (\sigma+1)\varepsilon\, d_{\min} - 2\varepsilon > \varepsilon\, d_{\min}, \\ &\qquad \forall x \in V \ \text{ with } \ x_j = \varepsilon\, d_{\min}; \end{aligned} \qquad (12)$$

$$\begin{aligned} &\Phi_j(x) \leq \alpha\varepsilon\, d_{\min} - (\sigma+1)\varepsilon\, d_{\min} - 2\varepsilon < -\varepsilon\, d_{\min}, \\ &\qquad \forall x \in V \ \text{ with } \ x_j = -\varepsilon\, d_{\min}. \end{aligned} \qquad (13)$$

The above relations (12), (13) and the continuity of f on V imply that for each $\sigma > \sigma^*$,

$$\Phi(V) \supset V = [-\varepsilon\, d_{\min} b, \varepsilon\, d_{\min} b]. \qquad (14)$$

On the other hand, for any $x, y \in V$, by applying the intermediate value theorem to the scalar-valued functions $\Phi(x)_i, i = 1, 2\cdots, n$, we have

$$\Phi(x) - \Phi(y) = \left(Df(\xi) + (\sigma+1)\right)\,(x - y)$$

and $Df(\xi) = \left[Df_1(\xi^1), \cdots, Df_n(\xi^n)\right]^{\mathrm{T}}$, where $\xi = [\xi^1, \cdots, \xi^n] \in R^{n\times n}$, $\xi^j = \alpha_j x + (1-\alpha_j) y \in V$ for some $\alpha_j \in [0,1]$, $i, j = 1, \cdots, n$. Further,

$$\begin{aligned} &\|\,\Phi(x) - \Phi(y)\|_\infty \\ =& \|\,f(x) + (\sigma+1)x - (f(y) + (\sigma+1)y)\|_\infty \\ \geq& \left\|(Df(\xi) + \sigma + 1)^{-1}\right\|_\infty^{-1} \|x - y\|_\infty, \end{aligned}$$

and from (18) we can obtain

$$\begin{aligned} &\left\|(Df(\xi) + \sigma + 1)^{-1}\right\|_\infty \\ =&(\sigma+1)^{-1}\left\|\left((\sigma+1)^{-1} Df(\xi) + I\right)^{-1}\right\|_\infty \\ \leq&\frac{1}{(\sigma+1)\left(1 - \|(\sigma+1)^{-1} Df(\xi)\|_\infty\right)} \\ \leq&\frac{1}{\sigma + 1 - \alpha}. \end{aligned}$$

Therefore, for any $x, y \in V$,

$$\|\,\Phi(x) - \Phi(y)\|_\infty \geq (\sigma + 1 - \alpha)\,\|x - y\|_\infty,$$

and then it follows from condition (8) that there is a positive number λ_0 satisfying $\sigma + 1 - \alpha \geq \lambda_0 > 1$ such that $\|\,\Phi(x) - \Phi(y)\|_\infty > \lambda_0 \|x - y\|_\infty$, $\forall x, y \in V$, i.e., Φ is expanding in V. It follows from Lemma 1 that there exists a unique fixed point x_1, satisfying $-\varepsilon\, d_{\min} b < x_1 \leq \varepsilon\, d_{\min} b$, such that

$$\Phi(x_1) = f(x_1) + (\sigma+1)x_1 - 2\varepsilon b = x_1. \qquad (15)$$

Then

$$\bar{g}(x_1) \equiv f(x_1) + \sigma x_1 - 2\varepsilon b = 0, \qquad (16)$$

which implies that

$$g(x_1) = D\,\Phi(2\varepsilon b) = 0. \qquad (17)$$

Step (ii) It is to prove that there exists a point x_0, satisfying $-(\sigma+\alpha)^{-1} b\varepsilon < x_0 < (\sigma+\alpha)^{-1} b\varepsilon$ such that $g(x_0) = x_1$. To do so, we define $V_1 = [-(\sigma+\alpha)^{-1}\varepsilon\, b, (\sigma+\alpha)^{-1}\varepsilon\, b]$, $X = R^n$, $d(\cdot) = \|\cdot\|_\infty$ and consider the following map:

$$\hat{\Phi}(x) = \left[\sum_{i=1}^{r} h_i(k) A_i\right] x + (\sigma+1)x - D^{-1} x_1.$$

Apparently, $(R^n, \|\cdot\|_\infty)$ is a complete metric space, V_1 being a nonempty closed subset of R^n and $\hat{\Phi} : V \to R^n$ is continuous. Similar to Step (i), we can prove that $\hat{\Phi}$ has a unique fixed point x_0 in V_1, i.e.,

$$\hat{\Phi}(x_0) = \left[\sum_{i=1}^{r} h_i(k) A_i\right] x_0 + (\sigma+1)x_0 - D^{-1} x_1 = x_0.$$

Then

$$\left[\sum_{i=1}^{r} h_i(k) A_i\right] x_0 + \sigma x_0 - D^{-1} x_1 = 0, \qquad (18)$$

which implies that

$$g(x_0) = D\Phi\left(\left[\sum_{i=1}^{r} h_i(k)A_i\right]x_0 + \sigma x_0\right) = x_1. \quad (19)$$

From Eqs.(17) and (19), we can obtain that after two-step iterations ($m = 2$),

$$g^2(x_0) = g(x_1) = 0.$$

Proof of b) Now, let r be chosen such that

$$\|x_0\|_\infty \le r \le (\alpha + \sigma)^{-1}\varepsilon.$$

Then $x_0 \in B_r(0)$ and it follows from the definition of g that, for any $x \in B_r(0)$

$$g(x) = D\Phi\left(\left[\sum_{i=1}^{r} h_i(k)A_i\right]x + \sigma x\right)$$

is differentiable and

$$g'(x) = D\cdot\left(\left[\sum_{i=1}^{r} h_i(k)A_i\right] + \sigma\right).$$

Furthermore,

$$g'(\xi) = \left[\sum_{i=1}^{r} h_i(k)A_i\right] + \beta\sigma I,$$

where

$$\xi \in B_r^n(0) = \left\{\xi = [\xi^1, \cdots, \xi^n] \in R^{n\times n} \,\middle|\, \xi^i \in B_r(0)\right\}.$$

Condition (8) implies that $\left\|g'(\xi)^{-1}\right\|_\infty < 1$ for $\xi = [\xi^1, \xi^2, \cdots, \xi^n] \in R^{n\times n}$ with $\xi^i \in B_r(0)$.

Proof of c) g^2 is differentiable at x^0 with

$$|\det g'(x_0)| = |\det D\,(f'(x_0) + \sigma I)| > 1$$

and

$$|\det\{g'(x_1)\}| = |\det\{D(f'(x_1) + \sigma I)\}| > 1.$$

Therefore,

$$\det((g^2)'(x^0)) = \det(g'(x^0))\,\det(g'(x^1)) \neq 0.$$

Thus, based on the improved Marotto theorem, a), b) and c) together imply that the controlled system (5) is chaotic in the sense of Li and Yorke.

4. A simulation example

We consider a non-chaotic discrete-time TS fuzzy model given by

Rule 1: IF x_k is M_1, THEN

$$X_{k+1} = \begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = A_1 \begin{bmatrix} x_k \\ y_k \end{bmatrix} + u_k,$$

Rule 2: IF x_k is M_2, THEN

$$X_{k+1} = \begin{bmatrix} x_{k+1} \\ y_{k+1} \end{bmatrix} = A_2 \begin{bmatrix} x_k \\ y_k \end{bmatrix} + u_k,$$

where

$$A_1 = \begin{bmatrix} d & 0.1 \\ 0.5 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} -d & 0.1 \\ 0.5 & 0 \end{bmatrix},$$

$x_k \in [-d, d]$, $d > 0$ and membership functions

$$M_1 = \frac{1}{2}\left(1 - \frac{x_k}{d}\right), \quad M_2 = \frac{1}{2}\left(1 + \frac{x_k}{d}\right).$$

The controlled TS fuzzy system is constructed as follows:

$$\begin{aligned} X_{k+1} =& D\Phi\left(\left[\sum_{i=1}^{2} h_i(k)A_i\right]X_k + u_k\right) \\ =& D\Phi\left(\left[\sum_{i=1}^{2} h_i(k)A_i\right]X_k + \sigma X_k\right). \end{aligned} \quad (20)$$

The magnitude of the control input is arbitrarily chosen to be $u_{\max} = 2$. We choose $D = \mathrm{diag}(2.0, 0.5)$, $d = 0.7$ and know that the controlled TS fuzzy system (18) is chaotic if $\sigma > 5.7$ from Theorem 2. In the simulation, we choose $\sigma = 10$ and $\varepsilon = 0.1$. The phase portrait and time wave diagrams are shown in Fig.2. From Fig.2, we can see that $\|u_k\|_\infty \le 2$ and the controlled TS fuzzy system (20) displays complex chaotic behaviors. These numerical results verify the theoretical analysis.

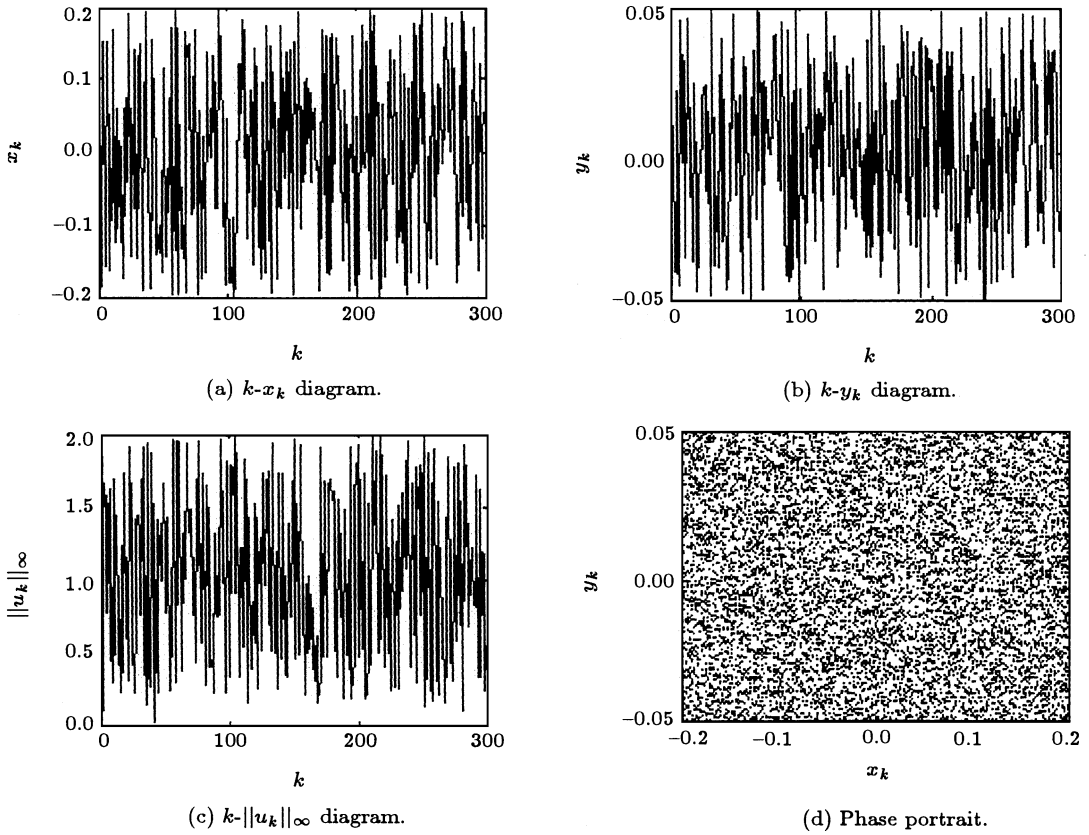


Fig.2. Simulation Results.

5. Conclusions

In this paper, we have proposed another simple and systematic chaotification method for arbitrarily given discrete TS fuzzy systems, which employs the decentralized control and the continuous sawtooth function. By applying the improved Marotto theorem, we have mathematically proven that the proposed algorithm for chaotification leads to chaos in the mathematical sense of Li and Yorke with the arbitrarily desired amplitude bound. In particular, an explicit formula for the computation of control parameters has been given. The simulation results illustrate the effectiveness of the proposed method.

References

[1] Chen G and Fradkov A L Chaos Control and Synchronization Bibliographies (1987-2001), available via the WWW: http://www.ee.cityu.edu.hk/~gchen/chaos-papers.html
[2] Chen G and Dong X 1998 *From Chaos to Order—Perspectives, Methodologies, and Applications* (Singapore: World Scientific)
[3] Wang X F and Chen G 2000 *Int. J. Bifurc. Chaos* **10** 549
[4] Chen G and Lai D 1996 *Int. J. Bifurc. Chaos* **6** 1341
[5] Chen G and Lai D 1997 *IEEE Trans. Circuits Syst.* I **44** 250
[6] Chen G and Lai D 1998 *Int. J. Bifurc. Chaos* **8** 1585
[7] Lu J G, Wang X F, Wei R and Wang Z Q 2000 *Proc. 19th Chinese Control Conference* p171
[8] Wang X F and Chen G 1999 *Int. J. Bifurc. Chaos* **9** 1435
[9] Li Z *et al* 2002 *IEEE Trans. Circuits Syst.* I **49** 249
[10] Wei R and Wang X Y 2004 *Chin. Phys.* **13** 1210
[11] Lu J G 2004 *Int. J. Nonlinear Sci. Numer. Simul.* **5** 313
[12] Li Z *et al* 2002 *Int. J. Bifurc. Chaos* **12** 2283
[13] Wang X F and Chen G 2000 *IEEE Trans. Circuits Syst.* I **47** 410
[14] Lu J G 2005 *Chin. Phys.* **14** 67
[15] Guan X P *et al* 2002 *Acta Phys. Sin.* **51** 2216 (in Chinese)
[16] Wang X F, Chen G and Yu X 2000 *Chaos* **10** 771
[17] Tanaka K *et al* 1998 *IEEE Trans. Circuits Syst.* I **45** 1021
[18] Yue D 2003 *Acta Phys. Sin.* **52** 292 (in Chinese)
[19] Wu Z Q 2002 *Acta Phys. Sin.* **51** 1193 (in Chinese)
[20] Marotto F R 1978 *J. Math. Anal. Appl.* **63** 199
[21] Chen G, Hsu S B and Zhou J 1998 *J. Math. Phys.* **39** 6459
[22] Lin W, Ruan J and Zhao W R 2002 *Int. J. Bifurc. Chaos* **12** 1129
[23] Li C P and Chen G 2003 *Chaos, Solitons Fractals* **18** 69
[24] Li T Y and Yorke J A 1975 *Am. Math. Monthly* **82** 481
[25] Takagi T and Sugeno M 1985 *IEEE Trans. Syst. Man. Cybern.* **15** 116
[26] Shi Y and Chen G 2004 *Chaos, Solitons Fractals* **22** 555