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Localized excitations with and without propagating properties in (2+1)-dimensions obtained by a mapping approach*

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By means of an extended mapping approach, a new type of variable-separation excitation is derived with two arbitrary functions in a (2+1)-dimensional modified dispersive water-wave system. Based on the derived variable-separation excitation, abundant nonpropagating and propagating solitons such as dromions, rings, peakons and compactons are revealed by selecting appropriate functions in this paper.

Keywords: extended mapping approach, modified dispersive water-wave system, nonpropagating soliton, propagating soliton
PACC: 0340, 0365, 0200

1. Introduction

Higher-dimensional localized excitations attract considerable interest. For instance, the discovery of the Bäcklund transformation by Boiti *et al*[1] for dromion-type coherent soliton solutions of the Davery–Stewartson system provided renewed interest in three-dimensional soliton systems. In a recent study on higher-dimensional nonlinear physical models, much effort has been focused on propagating solitons.[2−4] However, their nature is colourful, and there may exist many nonpropagating solitons, just as Wu *et al*[5] reported about the nonpropagating hydrodynamical solitons in their experiment. Actually, there are many studies on nonpropagating hydrodynamical solitons both in theory and experiment,[6] However, to the best of our knowledge, very few reports on nonpropagating solitons in higher-dimensional nonlinear physical models have been found, especially, in some well-known nonlinear dynamic systems. In this paper, as a concrete example, we will report and discuss these phenomena in the following well-known (2+1)-dimensional modified dispersive water-wave (MDWW) system:[7]

$$u_{yt} + u_{xxy} - 2v_{xx} - (u^2)_{xy} = 0,$$

$$v_t - v_{xx} - 2(uv)_x = 0. \tag{1}$$

The MDWW system was used to model nonlinear and dispersive long gravity waves travelling in two horizontal directions on shallow waters of uniform depth, and can also be derived from the well-known Kadomtsev–Petviashvili (KP) equation using the symmetry constraint.[8] Abundant propagating localized excitations were derived by Tang *et al*[2] with the help of Painlevé–Bäcklund transformation and a multilinear variable separation approach. However, as far as we know, nonpropagating solitons in the MDWW system have not been reported in the literature. It is worth mentioning that this system has been widely applied in many branches of physics, such as plasma physics, fluid dynamics, nonlinear optics, etc. So, a good understanding of more solutions of the MDWW system (1) is very helpful, especially for coastal and civil engineers in applying the nonlinear water model in harbour and coastal design. Meanwhile, finding

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more types of solutions to system (1) is of fundamental interest in fluid dynamics.

As is well known, to search for solitary wave solutions for a nonlinear physical model, we can exploit different approaches. One of the most efficient methods for finding soliton excitations of a physical model is the so-called mapping transformation method.[9,10] The basic idea of the algorithm is that, for a given nonlinear partial differential equation (NPDE) with independent variables $x \equiv (x_0 = t, x_1, x_2, \cdots, x_m)$, and dependent variable u,

$$P(u, u_{x_i}, u_{x_i x_j}, \cdots) = 0, \tag{2}$$

where P is in general a polynomial function of its argument, and the subscripts denote the partial derivatives. Lu and Zhang [10] assume its solution in the following form:

$$u = \sum_{i=0}^{n} \alpha_i(x)\phi^i(\omega(x)), \tag{3}$$

with

$$\phi' = \sigma + \phi^2, \tag{4}$$

where $\alpha_i(x) \equiv \alpha_i$ and $\omega(x) \equiv \omega$ are arbitrary functions of $x \equiv (x_0 = t, x_1, x_2, \cdots, x_m)$, σ is a constant and the prime denotes ϕ differentiated with respect to ω. To determine u explicitly, one may take the following procedure. First, similar to the usual mapping approach, determine n by balancing the highest nonlinear term and the highest-order partial term in the given NPDE. Second, substituting Eqs.(3) and (4) into the given NPDE and collecting coefficients of polynomials of ϕ, then eliminating each coefficient to derive a set of partial differential equations of $\alpha_i (i = 0, 1, \cdots, n)$ and ω. Third, solving the system of partial differential equations to obtain α_i and ω. Finally, as Eq.(4) possesses the general solution

$$\phi = \begin{cases} -\sqrt{-\sigma}\tanh(\sqrt{-\sigma}\omega), & \sigma < 0, \\ -\sqrt{-\sigma}\coth(\sqrt{-\sigma}\omega), & \sigma < 0, \\ \sqrt{\sigma}\tan(\sqrt{\sigma}\omega), & \sigma > 0, \\ -\sqrt{\sigma}\cot(\sqrt{\sigma}\omega), & \sigma > 0, \\ \dfrac{-1}{\omega}, & \sigma = 0, \end{cases} \tag{5}$$

substituting α_i, ω and Eq.(5) into Eq.(3), one can obtain the exact solutions of the given NPDE.

2. New variable-separation excitations for the (2+1)-dimensional MDWW system

To solve the (2+1)-dimensional MDWW system, we first consider the following Painlevé–Bäcklund transformation for u and v in Eqs.(1):

$$u = (\ln f)_x + u_0, \quad v = (\ln f)_{xy} + v_0, \tag{6}$$

which can be derived from the standard Painlevé truncated expansion, where the functions $u_0 = u_0(x, t)$ and $v_0 = 0$ are seed solutions of the MDWW system (1). Based on Eq.(6) and the seed solutions, we can straightforwardly obtain a simple relation for u and v: $v = u_y$.

Inserting $v = u_y$ into Eqs.(1) yields a single nonlinear equation:

$$\partial_y(u_t - u_{xx} - 2uu_x) = 0. \tag{7}$$

Now we apply the extended mapping approach to Eq.(7). By balancing the highest nonlinear term and the highest-order partial term, ansatz (3) becomes

$$u = f(x, y, t) + g(x, y, t)\phi(q(x, y, t)), \tag{8}$$

where $f \equiv f(x, y, t)$, $g \equiv g(x, y, t)$ and $q \equiv q(x, y, t)$ are arbitrary functions of $\{x, y, t\}$ to be determined. Substituting Eq.(8) together with Eq.(4) into Eq.(7), and collecting coefficients of polynomials of ϕ, then setting each coefficient to zero, we have

$$6gq_xq_y(q_x + g) = 0, \tag{9}$$

$$\begin{aligned} &-2gq_yq_t + 4gq_xg_y + 2g^2q_{xy} + 2gq_yq_{xx} + 2g_yq_x^2 \\ &+ 4gq_xq_{xy} + 4g_xq_yq_x + 4fgq_xq_y + 4g_xgq_y = 0, \end{aligned} \tag{10}$$

$$\begin{aligned} &gq_{xxy} + g_yq_{xx} + 2f_xgq_y + 2fgq_{xy} + 2gq_xf_y \\ &+ 2gg_{xy} + 2fg_yq_x - g_yq_t + 8g^2q_xq_y\sigma \\ &+ 2g_xq_{xy} + 8gq_yq_x^2\sigma + 2fg_xq_y + 2g_xg_y \\ &+ 2g_{xy}q_x - g_tq_y + g_{xx}q_y - gq_{yt} = 0, \end{aligned} \tag{11}$$

$$\begin{aligned} &2g_yq_x^2\sigma - 2gq_yq_t\sigma + 2fg_{xy} + 2g_xf_y - g_{yt} \\ &+ 4g_xq_xq_y\sigma + g_{xxy} + 4gq_xg_y\sigma + 2f_xg_y \\ &+ 2g^2q_{xy}\sigma + 4gq_xq_{xy}\sigma + 4g_xgq_y\sigma \\ &+ 4fgq_yq_x\sigma + 2gf_{xy} + 2gq_yq_{xx}\sigma = 0, \end{aligned} \tag{12}$$

$$\begin{aligned} &g_yq_{xx}\sigma + gq_{xxy}\sigma - g_tq_y\sigma \\ &+ 2g^2q_xq_y\sigma^2 + f_{xxy} - f_{yt} - gq_{yt}\sigma \\ &+ 2fg_yq_x\sigma + 2f_xf_y + 2f_xgq_y\sigma \\ &+ 2fg_xq_y\sigma - g_yq_t\sigma + 2gq_xf_y\sigma \\ &+ 2fgq_{xy}\sigma + g_{xx}q_y\sigma + 2gq_x^2q_y\sigma^2 \\ &+ 2g_{yx}q_x\sigma + 2g_xq_{xy}\sigma + 2ff_{xy} = 0. \end{aligned} \tag{13}$$

According to Eqs.(9)–(13) and through careful calculations, we finally obtain the following exact solution:

$$g = -q_x, \tag{14}$$

$$f = \frac{-q_{xx} + q_t}{2q_x}, \tag{15}$$

$$q = \chi(x,t) + \varphi(y), \tag{16}$$

where $\chi \equiv \chi(x,t)$, $\varphi \equiv \varphi(y)$ are two arbitrary variable-separation functions of $\{x,t\}$ and y respectively. According to the solutions of Eq.(4), one can obtain new types of variable-separation excitations of the (2+1)-dimensional MDWW system (1).

Case 1 For $\sigma < 0$, we can derive the following solitary wave solutions of Eqs.(1):

$$u_1 = \frac{-\chi_{xx} + \chi_t}{2\chi_x} + \sqrt{-\sigma}\chi_x \tanh(\sqrt{-\sigma}(\chi+\varphi)), \tag{17}$$

$$v_1 = -\sigma\chi_x\varphi_y + \sigma\chi_x\varphi_y \tanh^2(\sqrt{-\sigma}(\chi+\varphi), \tag{18}$$

$$u_2 = \frac{-\chi_{xx} + \chi_t}{2\chi_x} + \sqrt{-\sigma}\chi_x \coth(\sqrt{-\sigma}(\chi+\varphi)), \tag{19}$$

$$v_2 = -\sigma\chi_x\varphi_y + \sigma\chi_x\varphi_y \coth^2(\sqrt{-\sigma}(\chi+\varphi), \tag{20}$$

where $\chi \equiv \chi(x,t)$ and $\varphi \equiv \varphi(y)$ are two arbitrary functions of the indicated variables.

Case 2 For $\sigma > 0$, we can obtain the following periodic wave solutions of Eqs.(1)

$$u_3 = \frac{-\chi_{xx} + \chi_t}{2\chi_x} - \sqrt{\sigma}\chi_x \tan(\sqrt{\sigma}(\chi+\varphi)), \tag{21}$$

$$v_3 = -\sigma\chi_x\varphi_y - \sigma\chi_x\varphi_y \tan^2(\sqrt{\sigma}(\chi+\varphi)), \tag{22}$$

$$u_4 = \frac{-\chi_{xx} + \chi_t}{2\chi_x} + \sqrt{\sigma}\chi_x \cot(\sqrt{\sigma}(\chi+\varphi)), \tag{23}$$

$$v_4 = -\sigma\chi_x\varphi_y - \sigma\chi_x\varphi_y \cot^2(\sqrt{-\sigma}(\chi+\varphi)), \tag{24}$$

with two arbitrary functions $\chi(x,t)$ and $\varphi(y)$.

Case 3 For $\sigma = 0$, we obtain the following variable-separation solution of Eqs.(1):

$$u_5 = \frac{-\chi_{xx} + \chi_t}{2\chi_x} + \frac{\chi_x}{\chi+\varphi}, \tag{25}$$

$$v_5 = -\frac{\chi_x\varphi_y}{(\chi+\varphi)^2}, \tag{26}$$

with two arbitrary functions $\chi(x,t)$ and $\varphi(y)$.

3. Nonpropagating solitons and propagating solitons in the (2+1)-dimensional MDWW system

For simplification in discussion, we analyse mainly some special nonpropagating localized excitations of solutions expressed by v_1, while the corresponding propagating localized excitations will be briefly discussed in the end of this section. Now we rewrite solutions v_1 in a simple form V, namely

$$V \equiv v_1 = -\chi_x\varphi_y\sigma\mathrm{sech}^2(\sqrt{-\sigma}(\chi+\varphi)), \ (\sigma < 0). \tag{27}$$

Due to the arbitrariness of the functions $\chi(x,t)$ and $\varphi(y)$ included in above case, the physical quantity v_1 may possess rich localized structures of coherent solitons. For example, when $\chi(x,t) = \alpha(kx+ct)$ and $\varphi(y) = \beta(y)$, where α and β are arbitrary functions of the indicated arguments, then the solution v_1 may contain abundant localized coherent excitations with propagating property. Obviously, one of the simplest travelling solitary wave excitations can be easily obtained when $\chi(x,t) = kx + ct$ and $\varphi(y) = ly$, where k, l and c are arbitrary constants. Meanwhile, still based on the above solution, we may also derive rich non-travelling excitations, which are not propagating solitary waves,[6] just as Wu *et al*[5] reported about the nonpropagating hydrodynamical solitons in 1984. For instance, when the arbitrary functions are selected to be $\chi(x,t) = \varsigma(x) + \tau(t)$ and $\varphi(y) = \beta(y)$, where ς, τ and β are also arbitrary functions of the indicated arguments, then we can obtain many kinds of nonpropagating solitary wave solutions like dromions, rings, peakons, compactons and so on. Moreover, if $\tau(t)$ is considered to be a periodic function or a solution of a chaos system like Lorenz chaos system,[11] then the nonpropagating solitons may possess periodic or chaotic behaviours.

3.1. Dromion and dromion lattice excitations

According to the solution V (27) and setting $\sigma = -1$, we first discuss its dromion excitation which is one of the significant localized excitations localized exponentially in all directions. When the simple selections of the functions χ and φ are given to be

$$\begin{aligned} \chi =& 0.1\tanh(x+6) + 0.15\tanh(x) + \sin(t), \\ \varphi =& 0.1\tanh(y), \end{aligned} \tag{28}$$

we can obtain a two-dromion excitation for the physical quantity V, as shown in Fig.1(a). The corresponding evolutional plot is depicted in Fig.1(b).

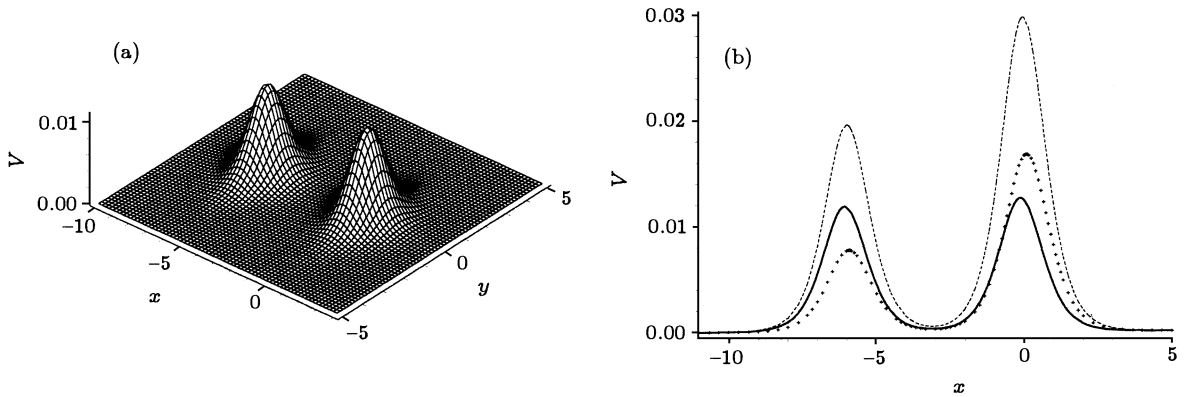


Fig.1. (a) A two-dromion plot of the solution V expressed by Eq.(27) under the condition (28) at time $t = 0$. (b) The corresponding evolutional plot related to (a) at $y = 0$ and at times $t = -3$, $t = 0$, $t = 3$.

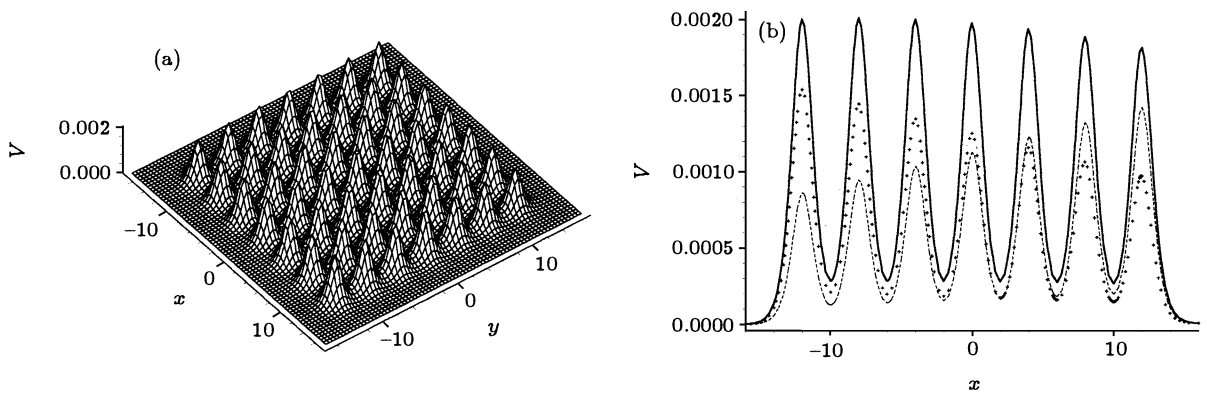


Fig.2. (a) A dromion-lattice plot of the solution V expressed by Eq.(27) the conditions (29) and (30) at time $t = 0$. (b) The corresponding evolutional plot related to (a) at $y = 0$ and at times $t = -3$, $t = 0$ and $t = 3$.

Remark In the following plots, we utilize different curves to express different cases: the dotted curve indicates an initial case and the solid curve represents the ultimate case while the dashed one corresponds an intermediate one of the given times.

Along with the above lines, if taking

$$\chi = \sum_{m=-M}^{M} 0.1 \tanh(x + 4m) + \sin(t), \qquad (29)$$

$$\varphi = \sum_{n=-N}^{N} 0.1 \tanh(y + 4n), \ M = N = 3 \qquad (30)$$

then we can obtain a 7×7 "dromion lattice" excitation for the physical field V. The corresponding dromion lattice plot is presented in Fig.2.

3.2. Ring soliton and standing breather excitations

In high dimensions, in addition to the point-like localized coherent excitations, there may be some other types of physically significant localized excitations. For instance, in (2+1)-dimensional case, there are some types of ring soliton solutions which are not equal to zero identically at some closed curves and decay exponentially away from these curves. Based on the solution V (27) ($\sigma = -1$), for example, when the functions χ and φ are considered to be

$$\chi = -x^2 + t^2, \quad \varphi = -y^2, \qquad (31)$$

we can obtain a ring soliton excitation for the physical quantity V.

Meanwhile, the ring soliton solution is also a standing breather excitation of the (2+1)-dimensional MDWW system. The related evolutional plots are presented in Fig.3.

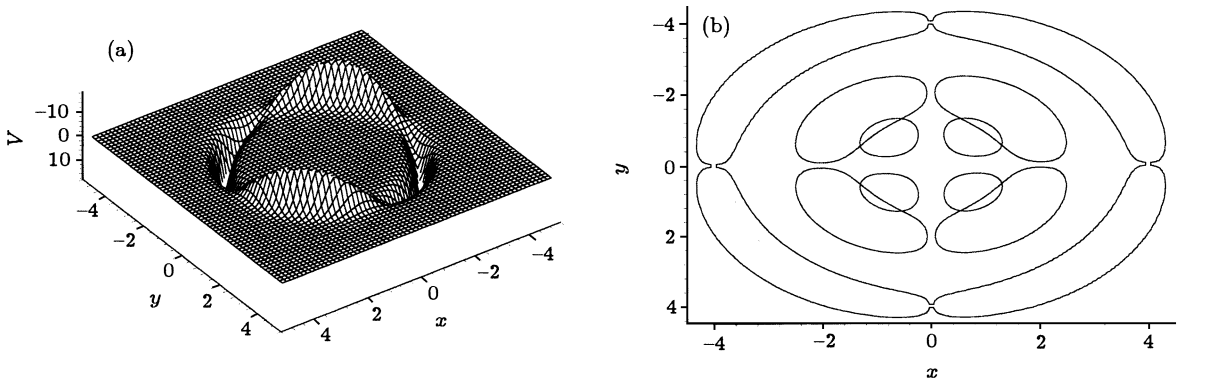


Fig.3. The ring breather evolutional plots of the solution V expressed by Eq.(27) under the condition (31) at time $(a)t = -4$. (b) The evolutional contour plot related to (a); and the values of the contours are set to be $|V| = 0.5$ at times $t = 0$, $t = 2$ or -2 and $t = 4$ or -4 respectively from inside to outside.

3.3. Peakon and compacton excitations

Similarly, according to the solution V (27), we can obtain some weak-localized excitations such as peakon and compaction excitations.[12] For example, when selecting χ and φ to be some piecewise smooth functions, we can derive some multipeakon excitations

$$\chi = \sin(t) + \sum_{i=1}^{M} \begin{cases} X_i(x), & x \le 0, \\ -X_i(-x) + 2X_i(0), & x > 0, \end{cases} \tag{32}$$

$$\varphi = 1 + \sum_{i=1}^{N} \begin{cases} Y_i(y), & y \le 0, \\ -Y_i(-y) + 2Y_i(0), & y > 0, \end{cases} \tag{33}$$

where the functions $X_i(x)$ and $Y_i(y)$ are differentiable functions of the indicated arguments and possess the boundary conditions $X_i(\pm\infty) = C_{\pm i}$, $(i = 1, 2, \ldots, M)$, $Y_i(\pm\infty) = D_{\pm i}$, $(i = 1, 2, \ldots, N)$, where $C_{\pm i}$ and $D_{\pm i}$ are constants and/or even infinity. For instance, when choosing

$$\begin{aligned} X_1 &= 0.1\exp(x+5), \\ X_1 &= 0.3\exp(x), \\ X_3 &= 0.1\exp(x-5), \end{aligned} \tag{34}$$

$$\begin{aligned} Y_1 &= 0.1\exp(y+1), \\ M &= 3, \quad N = 1, \end{aligned} \tag{35}$$

we can derive a three-peakon excitation for the solution V expressed by Eq.(27). The corresponding three-peakon excitation plot is depicted in Fig.4.

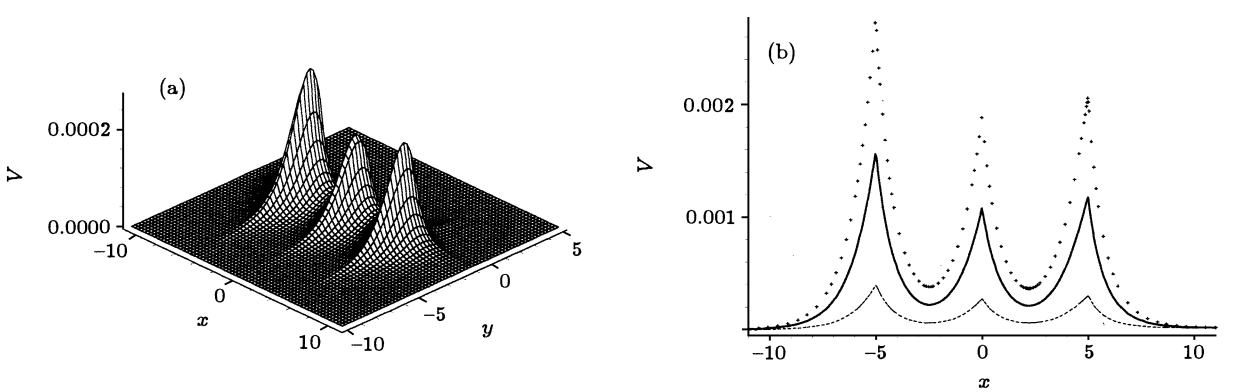


Fig.4. (a) A three-peakon soliton structure of the solution V expressed by (27) under the conditions (34) and (35) at time $t = 0$. (b) The corresponding evolutional plot related to (a) at $y = -2$ and at times $t = -3$, $t = 0$ and $t = 3$.

Similarly, if χ and φ are fixed in another types of piecewise smooth functions, then we can derive compacton excitations as

$$\chi = \sin(t) + \sum_{i=1}^{M} \begin{cases} 0, & x \le x_{1i}, \\ H_i(x) - H_i(x_{1i}), & x_{1i} < x \le x_{2i}, \\ H_i(x_{2i}) - H_i(x_{1i}), & x > x_{2i}, \end{cases} \tag{36}$$

and

$$\varphi = 1 + \sum_{j=1}^{N} \begin{cases} 0, & y \le y_{1j}, \\ G_j(y) - G_j(y_{1j}), & y_{1j} < y \le y_{2j}, \\ G_j(y_{2j}) - G_j(y_{1j}), & y > y_{2j}, \end{cases} \tag{37}$$

where the functions H_i and G_j are all arbitrary differentiable functions under the conditions $H_{ix}|_{x=x_{1i}} =$

$H_{ix}|_{x=x_{2i}} = 0$, $G_{jy}|_{y=y_{1j}} = G_{jy}|_{y=y_{2j}} = 0$. So the physical quantity V (27) will become some kinds of higher dimensional multiple compacton solutions. A simple example of compacted solitary waves is depicted in Fig.5 for

$$\chi = \sin(t) + \sum_{i=1}^{M} \begin{cases} 0, & x \le x_{0i} - \dfrac{\pi}{2k_i}, \\ h_i \sin(k_i(x - x_{0i})) + h_i, & x_{0i} - \dfrac{\pi}{2k_i} < x \le x_{0i} + \dfrac{\pi}{2k_i}, \\ 2h_i, & x > x_{0i} + \dfrac{\pi}{2k_i}, \end{cases} \tag{38}$$

$$\varphi = 1 + \sum_{j=1}^{N} \begin{cases} 0, & y \le y_{0i} - \dfrac{\pi}{2l_j}, \\ d_j \sin(l_j(y - y_{0j})) + d_j, & y_{0j} - \dfrac{\pi}{2l_j} < y \le y_{0j} + \dfrac{\pi}{2l_j}, \\ 2d_j, & y > y_{0j} + \dfrac{\pi}{2l_j}, \end{cases} \tag{39}$$

under conditions $M = 2N = 2$, $h_1 = d_1 = 0.3, h_2 = 0.6$, $k_2 = 2k_1 = 2l_1 = 2$, $x_{01} = y_{01} = 0$ and $x_{02} = 4$.

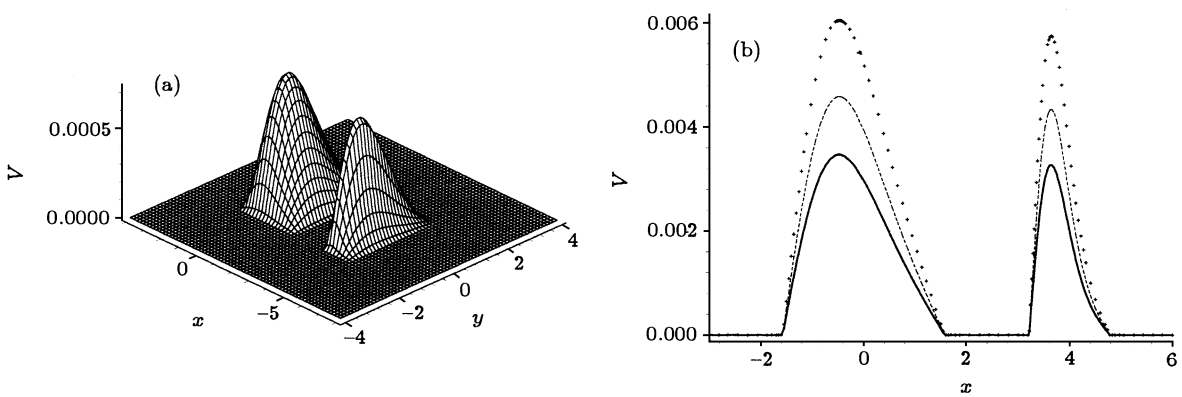


Fig.5. (a) Two-compacton solitary wave excitation determined by solution (27) with function selections (38) and (39) at time $t = \pi/2$. (b) The corresponding evolution plot of the two-compacton excitation related to (a) at times $t = -3, t = 0$ and $t = 3$.

Certainly, in the above cases, if the independent variable x is replaced by $x + ct$ (since the function $\chi(x,t)$ is an arbitrary function $\{x,t\}$), then all the above nonpropagating solitons will become propagating solitons, A simple example for the travelling ring soliton is presented in Fig.6. The corresponding parameters selected are directly presented in the figure captions. Other similar cases are neglected in the present paper.

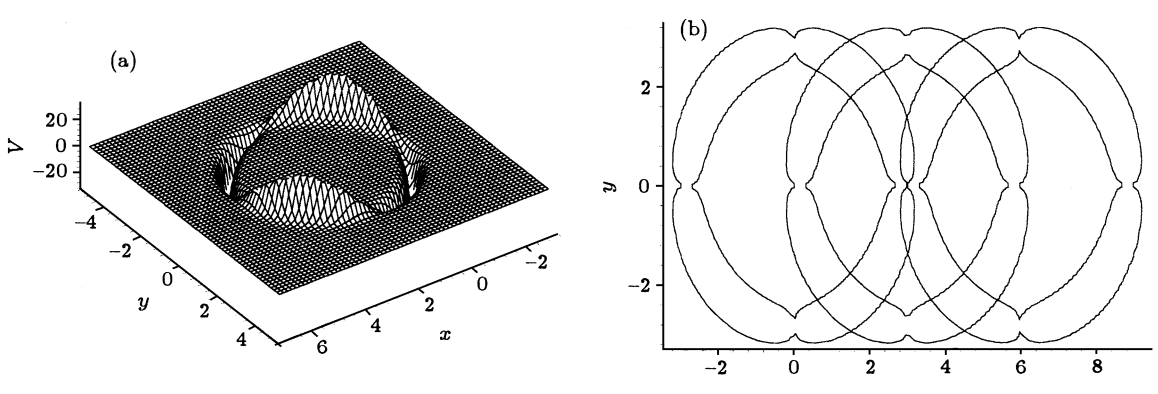


Fig.6. (a) A propagating ring structure of the solution v_1 expressed by Eq.(27) under the condition $\chi = -(x-t)^2 + 4, \varphi = -y^2 + 4$ at time $t = 2$. (b) The corresponding evolutional contour plot related to (a); and the value of the contour figure is set to be $|V| = 0.5$ at times $t = 0$, $t = 3$, $t = 6$ travelling from left to right.

4. Summary and discussion

In summary, by means of an extended mapping approach, the (2+1)-dimensional MDWW system has been successfully solved. Based on the derived new types of variable-separation solutions with two arbitrary functions, we have found some localized excitations, which possess propagating or nonpropagating property, by selecting the arbitrary functions appropriately. Actually, even according to the solution v_5 (26), we can also obtain these propagating and nonpropagating solitons. The main reason is that, comparing the common formula (1.1) in Ref.[2] with the solution v_5 (26), one can easily find that they are essentially equivalent when choosing parameters appropriately. Therefore, all the localized excitations based on the common formula (1.1) in Ref.[2] can be rederived from the solution v_5 (26). This paper is only a beginning work. We hope the mapping approach used in the present work may be further extended to other nonlinear physical systems.

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