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An exponential distribution network*

Liu Jing-Zhou(刘景舟)[a)b)†] and Tang Yi-Fa(唐贻发)[b)‡]

[a)] *Department of Physics and Institute of Theoretical Physics, Beijing Normal University, Beijing* 100875, *China*
[b)] *State Key Laboratory of Scientific and Engineering Computing, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing* 100080, *China*

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A complex network with an exponential distribution $p(k) \propto \mathrm{e}^{-\frac{k}{k_c}}$ with $k_c = 3.50 \pm 0.02$ is introduced and found to have assortative correlation $\left\langle k_i^{nn} \right\rangle = B + qk_i$ $(q > 0)$ from numerical simulation.

Keywords: network, degree correlation
PACC: 0130, 1240, 0500, 0520

1. Introduction

Quite a number of classical regular network models have been developed. However there still exist many network systems beyond these models in both the natural world and human society. These systems, described as complex networks,[1−5] exhibit more complex topological features and strange dynamical characteristics, such as high cluster coefficient, short average path length, strong degree correlation,[3−7] and no epidemic and percolation thresholds.[8−11]

In recent years, a major scientific effort has been devoted to the study of the characterization and the modelling of some complex networks, in which the sites (nodes) represent the real individuals and the edges(links) stand for the possible relation or interaction between the two individuals in the system, such as social networks, the Internet, food webs, the distribution networks, metabolic and protein networks, the networks of air-line routes, the power grid and neural systems. Some complex network models have been examined. Most of them are based on two kinds of mechanisms: incremental growth and preferential attachment. Among these models, Barabási and Albert's (BA) scale-free network[12] is the most adopted. The algorithm of the BA scale-free network is as follows: (1) growth—starting with a small number of nodes, a new node linked to m different present nodes is added at each time step; (2) preferential attachment—the probability that a new node will be connected to any node i depends on its degree k_i, which refers to the number of its nearest neighbours, and there exists the relation $\Pi(k_i) \propto k_i$. For a large k the network shows power-law degree distribution $p(k) \propto k^{-\nu}(\nu = 3)$, which has been found in many real systems.[1−5]

Apart from the power-law degree distribution network, also widely studied is the exponential network, whose node's connectivity distribution is exponential.[1,13,14] The degree distributions of some real networks, such as the power grid and C. elegants, are found to be consistent with those of an exponential network. Here we propose a simple model that generates an exponential distribution network with strong correlation.

2. The model

As we have mentioned in the above section, most network models are generated by two kinds of mechanisms: incremental growth and preferential attachment. The incremental growth describes that a network is assembled through the addition of new nodes to the system, while the preferential attachment means that the nodes to be linked are not chosen in equal probability. Here we propose a simple network model. The algorithm is below.

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†E-mail: liujz606@eyou.com
‡E-mail: tyf@lsec.cc.ac.cn.

(1) **Growth** starting with a few fully connected nodes (m_0), at each time step we add a new node linked to $m(\leq m_0)$ different nodes already present in the system.

(2) **Preferential attachment** when choosing the node with which the new node will be connected, we assume that the probability Π that a new node will be connected to node i is determined by the average nearest neighbours degree (ANND) k^{nn} of node i, as

$$\Pi(k_i) \propto k_i^{nn}, \tag{1}$$

where $k_i^{nn} = \dfrac{1}{k_i}\sum_{j\in\Omega} k_j$, and $j \in \Omega$ denotes that the sum goes over all nodes in the nearest neighbours of node i.

3. Degree distribution

According to the above generating procedure, there are $N = (t+m_0)$ nodes and $\left[2mt+\dfrac{1}{2}m_0(m_0-1)\right]$ edges in the system after t steps. For a large t and small m, the average degree of the network $\langle k\rangle$ is $2m$. We simulate this on computer. In Fig.1, we plot the degree distribution $p(k)$ vs degree k. In Fig.1(a), we set $m_0 = 3$, $m = 3$, and the number of node N to be 10000, 20000, 30000 represented by different symbols respectively. In Fig.1(b), we set $m_0 = 4$, $m = 2$, $N = 15000$. From the Figs.1(a) and 1(b), we can obtain the $\ln p(k)$ scaled as follows

$$\ln p\,(k) = A - k/k_c, \tag{2}$$

where A is a constant independent of m and m_0, and $k_c = 3.50 \pm 0.02$. Therefore, we have

$$p(k) \propto \mathrm{e}^{-k/k_c} \quad (k_c = 3.50 \pm 0.02)\,. \tag{3}$$

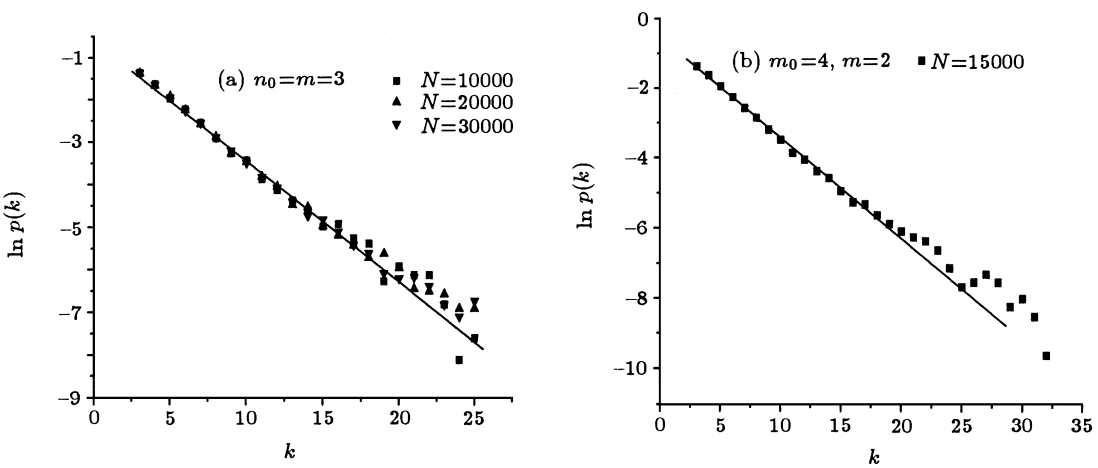


Fig.1. Degree distributions for different N's from the result of simulation. (a) $m_0 = 3$, $m = 3$; N=10000, 20000, and 30000 respectively. (b) $m_0 = 4$, $m = 2$, $N = 15000$. The solid line is obtained by linear fitting of the original data, $p(k) \propto \mathrm{e}^{-k/k_c}$ ($k_c = 3.50 \pm 0.02$).

4. Correlation

Within the research of many social networks, it is found that high degree "nodes" connect more preferably with highly connected ones. These networks show nontrivial degree correlation[15] or "assortative mixing".[16] On the other hand, many other network systems, such as technological and biological networks, show "disassortative mixing".[17] Due to preferential attachment, the current network shows strong "assortative mixing".

Figure 2(a) shows the node degree correlation of the current exponential degree distribution network with $m_0 = 3$, $m = 3$, and $N = 10^5$. For the sake of comparison, we also calculate $\langle k_i^{nn}\rangle$ for a random scale-free network. Figure 2(b) shows the node degree correlation of a scale-free network also with $m_0 = 3$, $m = 3$, and $N = 10^5$. From Figs.2(a) and 2(b), we can easily see that in the random scale-free network the average nearest neighbour degree (ANND) of a node is independent of the node degree;[17] while in the current exponential network, except for some weakly connected nodes, the ANND increases with the node degree. That is, the current exponential degree distribution network shows "assortative mixing":

$$\langle k_i^{nn}\rangle = B + qk_i \quad (q > 0), \tag{4}$$

where B and q are independent of m and m_0. In the end, we should mention that for other parameters, the pictures are found to be the same qualitatively.

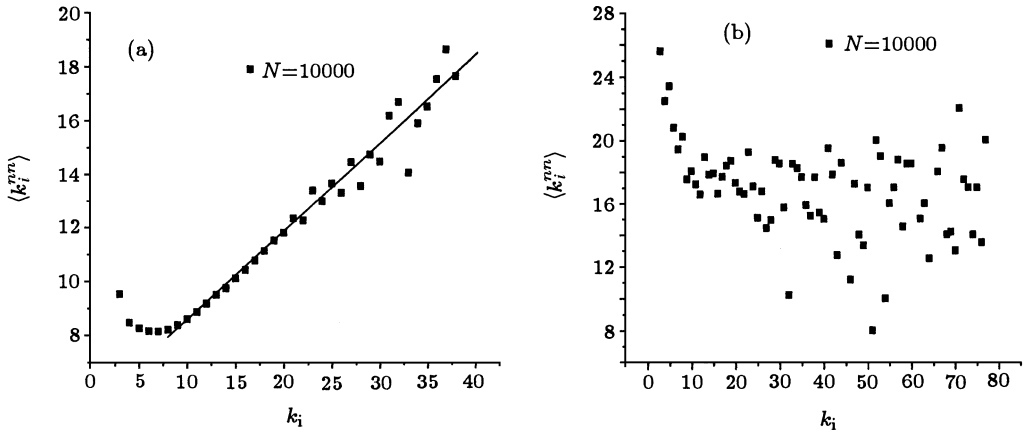


Fig.2. Relation between the node degree and the average nearest neighbour degree: (a) for our current network; (b) for a random scale-free network, where $m_0 = 3$, $m = 3$, N=10000. From Fig.2(a), we obtain $q = 3.0$.

5. Conclusion

In this paper, we have introduced a complex network based on two kinds of mechanisms: incremental growth and preferential attachment. Numerical simulations indicate that this network shows exponential distribution and "assortative mixing", which have been found in some real systems. Although the growing model with random attachment also leads to an exponential degree distribution, the node degrees are uncorrelated. So "assortative mixing" of our model results from the preferential attachment. We should point out that preferential attachment does not always exist in real systems, so there is still much work to be done for exploring the growth of real networks.

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