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Entropy evolution properties in a system of two entangled atoms interacting with light field*

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In this paper, we use the field entropy as a measurement of the degree of entanglement between the light field and the atoms of the system which is composed of two dipole–dipole interacting two-level atoms initially in an entangled state interacting with the single mode coherent field in a Kerr medium. The influence of the coupling constant of dipole–dipole interaction between atoms and the coupling strength of the Kerr medium with the light field and the intensity of the light field on the field entropy are discussed by numerical calculations. It is shown that when the coupling strength of the Kerr medium with the light field is large enough, and the light field is strong, the degree of entanglement between the atoms with the light field becomes weaker. The degree of entanglement only changes slightly with the change of the coupling constant of dipole–dipole interaction between atoms.

Keywords: field entropy, entangled state, dipole–dipole interaction, Kerr medium
PACC: 4250, 0365

1. Introduction

A number of entanglement measures have been discussed in the literature, such as the von Neumann reduced entropy and the relative entropy of entanglement.[1] The quantum entropy has been used to study the dynamic characteristics of a two-level atom interacting with a light field.[2] It is shown that the quantum entropy is a useful physics quantity for measuring the pure degree of the entanglement state. Field entropy has been studied in many papers.[3−7] However, these works are limited to the cases of a single atom being initially in a single state or two atoms being initially in a product state interacting with a single mode field or two mode fields. Recently, the two two-level atom entangled state has aroused great interest. The entangled state describes the quantum nonlocal properties and is widely applied in quantum computation, quantum cryptography, quantum teleportation and so on.[8−11] On the other hand, it is helpful to understand the characteristics of a light field interacting with matter to study the quantum properties of the system which includes electric and vacuum fluctuation inducing dipole interaction between the two atoms and the Kerr effect. The research also has important potential applications in the field of preparation of quantum state and optical communication.[12] In this paper, in order to be close to the fact of experiment, we consider the situation of two two-level atoms being initially in an entangled state interacting with a coherent light field in a Kerr medium. The influence of dipole–dipole interaction between the two atoms and the Kerr medium interaction with the single mode field on field entropy are discussed respectively using full quantum theory.

2. The model

We consider that two two-level atoms A and B are simultaneously injected into a single mode light

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cavity which is filled with a Kerr medium, and suppose that the distance between atoms is less than the wavelength of the field, and dipole interaction between atoms cannot be neglected and there is the same coupling of the two atoms interacting with field. Under these conditions, the Hamiltonian of the system can be written as ($\hbar = 1$)

$$\begin{aligned}\hat{H} =&\omega\hat{a}^{+}\hat{a} + \omega_0 \sum_{j=A}^{B} \hat{S}_z^j + \chi\hat{a}^{+2}\hat{a}^2 \\ &+ g \sum_{j=A}^{B} (\hat{a}^{+}\hat{S}_-^j + \hat{a}\hat{S}_+^j) \\ &+ g_a \sum_{\substack{j,k=A \\ j\neq k}}^{B} \hat{S}_+^j \hat{S}_-^k, \end{aligned} \tag{1}$$

where $\hat{a}^{+}$ and $\hat{a}$ are the creation and annihilation operators of the monochromatic photons with frequency ω, ω_0 denotes the atomic transition frequency, $\hat{S}_z^j$, $\hat{S}_+^j$, $\hat{S}_-^j$ $(j = A, B)$ denote the inversion and transition operators of jth atom, g is the coupling constant of the field with the atom, g_a is the coupling constant of the dipole–dipole interaction between atoms, and χ is the coupling intensity of the single mode field interacting with the Kerr medium.

If we consider that at time $t = 0$ the two atoms and field are decoupled, and assume that the two atoms are initially in entangled state

$$|\varphi_a(0)\rangle = \frac{1}{\sqrt{2}}(|e_A, g_B\rangle + |g_A, e_B\rangle), \tag{2}$$

and the field is initially in coherent state $|\alpha\rangle$, then the initial state vector of the system is

$$|\varphi(0)\rangle_e = \frac{1}{\sqrt{2}} \sum_{0}^{\infty} F_n(|e_A, g_B\rangle + |g_A, e_B\rangle)\,|n\rangle\,, \tag{3}$$

$$F_n = \exp(-\bar{n}/2)(\bar{n}^n/n!)^{1/2}\exp(in\phi), \tag{4}$$

where ϕ and $\bar{n}$ represent the direction angle of the excitation and the initial average photon number, respectively.

We only consider the resonance situation: namely, the transition frequency of atom and effective action resonance frequency of the field satisfy $\omega_a = \omega$. Then, along with time evolution, the state vector of the system at any time $t > 0$ is given by

$$\begin{aligned}|\varphi(t)\rangle = \sum_{n=0}^{\infty} F_n[&A(t)\,|e_A, e_B, n-1\rangle + B(t)\,|e_A, g_B, n\rangle \\ &+ C(t)\,|g_A, e_B, n\rangle + D(t)\,|g_A, g_B, n+1\rangle]. \end{aligned} \tag{5}$$

Based on the initial conditions

$$A(0) = D(0) = 0, \quad B(0) = C(0) = \frac{1}{\sqrt{2}}, \tag{6}$$

the solution of Schrödinger equation in the interaction representation is given by

$$\begin{cases} A(t) = a_{11}\mathrm{e}^{p_1 t} + a_{12}\mathrm{e}^{p_2 t} + a_{13}\mathrm{e}^{p_3 t}, \\ B(t) = C(t) = a_{21}\mathrm{e}^{p_1 t} + a_{22}\mathrm{e}^{p_2 t} + a_{23}\mathrm{e}^{p_3 t}, \\ D(t) = a_{31}\mathrm{e}^{p_1 t} + a_{32}\mathrm{e}^{p_2 t} + a_{33}\mathrm{e}^{p_3 t}, \end{cases} \tag{7}$$

where

$$\begin{cases} a_{1j} = -\mathrm{i}g\sqrt{2n}(p_j - z_a) \prod\limits_{\substack{k,j=1 \\ k\neq j}}^{3} \dfrac{1}{(p_j - p_k)}, \\ a_{2j} = \dfrac{1}{\sqrt{2}}(p_j - z_b)(p_j - z_a) \prod\limits_{\substack{k,j=1 \\ k\neq j}}^{3} \dfrac{1}{(p_j - p_k)}, \\ a_{3j} = -\mathrm{i}g\sqrt{2(n+1)}(p_j - z_b) \prod\limits_{\substack{k,j=1 \\ k\neq j}}^{3} \dfrac{1}{(p_j - p_k)}, \\ \qquad (j = 1, 2, 3) \end{cases} \tag{8}$$

with

$$\begin{aligned} Z_a =& -\mathrm{i}\chi n(n+1), \\ Z_b =& -\mathrm{i}\chi n(n-1)(n-2), \\ p_j =& R\cos\left[u + \frac{2}{3}(j-1)\pi\right] - \frac{b}{3}, \quad (j = 1, 2, 3), \\ R =& \left(-\frac{4}{3}P\right)^{1/2}, \\ u =& \frac{1}{3}\arccos\left[-\frac{Q}{2}\left(-\frac{P}{3}\right)^{-3/2}\right], \\ Q =& d - \frac{bc}{3} + \frac{2b^3}{27}, \\ P =& c - \frac{b^2}{3}, \\ b =& \mathrm{i}[g_a + (3n^2 - 3n + 2)\chi], \\ c =& 2g^2(2n+1) - 2g_a\chi(n^2 - n + 1) - 3n^2\chi^2(n-1)^2, \\ d =& \mathrm{i}(n+1)\chi[2g^2(2n^2 - 3n + 2) \\ &- g_a n\chi(n-1)(n-2) \\ &- n^2\chi^2(n^3 - 4n^2 + 5n - 2)]. \end{aligned}$$

The properties of the quantum entanglement between two entangled atoms and a single mode coherent field are discussed in the next section using the above results.

3. Field entropy

We use the von Neumann entropy as a measure of the degree of entanglement between the field and the

two atoms of the system under consideration. The von Neumann entropy S of a system is defined as[13]

$$S(t) = -\mathrm{Tr}(\hat{\rho}(t)\ln\hat{\rho}(t)), \tag{9}$$

where $\hat{\rho}$ is the density operator for a given quantum system. For a system in which both the entangled atoms and field start from pure states, the entropy of the system is always zero, i.e. $S(t)=0$. Therefore the entropies of two interacting subsystems are precisely equal, i.e. $S_a(t) = S_f(t)$. In the following, we only consider the time behaviour of the field entropy $S_f(t)$. In order to derive a calculation formalism of the field entropy, we must obtain the eigenvalues of the reduced field density operator. From Eqs.(5), we see that the reduced field density operator is

$$\begin{aligned}\hat{\rho}_f(t) =& \mathrm{Tr}_{\mathrm{atom}}[\hat{\rho}(t)]\\ =& |\mu(t)\rangle\langle\mu(t)| + |\nu_1(t)\rangle\langle\nu_1(t)|\\ &+ |\nu_2(t)\rangle\langle\nu_2(t)| + |\tau(t)\rangle\langle\tau(t)|\\ =& |\mu(t)\rangle\langle\mu(t)| + |\nu(t)\rangle\langle\nu(t)| + |\tau(t)\rangle\langle\tau(t)|,\end{aligned} \tag{10}$$

where

$$\begin{cases} |\mu(t)\rangle = \sum_{n=1}^{\infty} F_n A_n(t)\,|n-1\rangle,\\ |\nu(t)\rangle = |\nu_1(t)\rangle + \mathrm{i}\,|\nu_2(t)\rangle\\ \qquad\quad = (1+\mathrm{i})\sum_{n=0}^{\infty} F_n B_n(t)\,|n\rangle,\\ |\tau(t)\rangle = \sum_{n=0}^{\infty} F_n D_n(t)\,|n+1\rangle, \end{cases} \tag{11}$$

with $|\nu_2(t)\rangle = |\nu_1(t)\rangle = \sum_{n=0}^{\infty} F_n C_n(t)\,|n\rangle$.

We assume that the eigenstate of the reduced field density operator can be written in the following form:

$$|\pi_f(t)\rangle = \lambda_1\,|\mu(t)\rangle + \lambda_2\,|\nu(t)\rangle + \lambda_3\,|\tau(t)\rangle. \tag{12}$$

If we apply the density matrix given by equation (10) to the state equation (12), we find that

$$\begin{aligned}\hat{\rho}_f(t)|\pi_f(t)\rangle =& \Big(\langle\mu(t)|\mu(t)\rangle + \frac{\lambda_2}{\lambda_1}\langle\mu(t)|\nu(t)\rangle\\ &+ \frac{\lambda_3}{\lambda_1}\langle\mu(t)|\tau(t)\rangle\Big)\lambda_1|\mu(t)\rangle\\ &+ \Big(\langle\nu(t)|\nu(t)\rangle + \frac{\lambda_1}{\lambda_2}\langle\nu(t)|\mu(t)\rangle\\ &+ \frac{\lambda_3}{\lambda_2}\langle\nu(t)|\tau(t)\rangle\Big)\lambda_2|\nu(t)\rangle\\ &+ \Big(\langle\tau(t)|\tau(t)\rangle + \frac{\lambda_1}{\lambda_3}\langle\tau(t)|\mu(t)\rangle\\ &+ \frac{\lambda_2}{\lambda_3}\langle\tau(t)|\nu(t)\rangle\Big)\lambda_3|\tau(t)\rangle.\end{aligned} \tag{13}$$

Consequently, for $|\pi_f(t)\rangle$ to be an eigenstate of $\hat{\rho}_f(t)$ we must have the relation

$$\begin{aligned}\pi_f(t) =& \Big(\langle\mu(t)|\mu(t)\rangle + \frac{\lambda_2}{\lambda_1}\langle\mu(t)|\nu(t)\rangle\\ &+ \frac{\lambda_3}{\lambda_1}\langle\mu(t)|\tau(t)\rangle\Big)\\ =& \Big(\langle\nu(t)|\nu(t)\rangle + \frac{\lambda_1}{\lambda_2}\langle\nu(t)|\mu(t)\rangle\\ &+ \frac{\lambda_3}{\lambda_2}\langle\nu(t)|\tau(t)\rangle\Big)\\ =& \Big(\langle\tau(t)|\tau(t)\rangle + \frac{\lambda_1}{\lambda_3}\langle\tau(t)|\mu(t)\rangle\\ &+ \frac{\lambda_2}{\lambda_3}\langle\tau(t)|\nu(t)\rangle\Big),\end{aligned} \tag{14}$$

where $\pi_f(t)$ is an eigenvalue of the density operator $\hat{\rho}_f(t)$. Then the eigenvalues of the density matrix of the field are given by

$$\begin{cases} \pi_f^1(t) = \frac{\theta_1}{3} - \frac{2}{3}(\theta_1^2 - 3\theta_2)^{\frac{1}{2}}\cos\beta,\\ \pi_f^2(t) = \frac{\theta_1}{3} + \frac{1}{3}(\theta_1^2 - 3\theta_2)^{\frac{1}{2}}(\cos\beta + \sqrt{3}\sin\beta),\\ \pi_f^3(t) = \frac{\theta_1}{3} + \frac{1}{3}(\theta_1^2 - 3\theta_2)^{\frac{1}{2}}(\cos\beta - \sqrt{3}\sin\beta), \end{cases} \tag{15}$$

where

$$\begin{cases} \beta = \frac{1}{3}\arccos\left(\frac{\xi}{\varsigma}\right),\\ \xi = 9\theta_1\theta_2 - 2\theta_1^2 - 27\theta_3\\ \varsigma = 2(\theta_1^2 - 3\theta_2)^{\frac{3}{2}}, \end{cases} \tag{16}$$

with

$$\begin{aligned}\theta_1 =& -(\langle\mu(t)|\mu(t)\rangle + \langle\nu(t)|\nu(t)\rangle + \langle\tau(t)|\tau(t)\rangle),\\ \theta_2 =& \langle\mu(t)|\mu(t)\rangle\langle\tau(t)|\tau(t)\rangle\\ &+ \langle\mu(t)|\mu(t)\rangle\langle\nu(t)|\nu(t)\rangle\\ &+ \langle\nu(t)|\nu(t)\rangle\langle\tau(t)|\tau(t)\rangle\\ &- |\langle\mu(t)|\nu(t)\rangle|^2 - |\langle\mu(t)|\tau(t)\rangle|^2\\ &- |\langle\nu(t)|\tau(t)\rangle|^2,\\ \theta_3 =& \langle\mu(t)|\mu(t)\rangle\langle\nu(t)|\nu(t)\rangle\langle\tau(t)|\tau(t)\rangle\\ &+ \langle\mu(t)|\nu(t)\rangle\langle\tau(t)|\mu(t)\rangle\langle\nu(t)|\tau(t)\rangle\\ &- \langle\mu(t)|\mu(t)\rangle|\langle\nu(t)|\tau(t)\rangle|^2\\ &- \langle\nu(t)|\nu(t)\rangle|\langle\tau(t)|\mu(t)\rangle|^2\\ &- \langle\tau(t)|\tau(t)\rangle|\langle\mu(t)|\nu(t)\rangle|^2.\end{aligned}$$

The field entropy $S_f(t)$ may be expressed in terms of the eigenvalues $\pi_f^i(t)$ $(i=1,2,3)$ for the reduced field density operator as

$$\begin{aligned}S_f(t) =& -\{\pi_f^1(t)\ln[\pi_f^1(t)] + \pi_f^2(t)\ln[\pi_f^2(t)]\\ &+ \pi_f^3(t)\ln[\pi_f^3(t)]\}.\end{aligned} \tag{17}$$

If $S_f(t)$ takes its minimum value 0, the coherent field and the two atoms are in a disentangled state; If $S_f(t)$ takes its maximum value 1, the coherent field and the two atoms are in the maximum entangled state.

Figures 1, 2 and 3 present the numerical results of the time evolution of the field entropy for different values of the three parameters g_a, χ and $\bar{n}$, respectively. It is shown that the field entropy is strongly dependent on the interaction parameters g_a, χ and the initial average photon number $\bar{n}$. In Figs.1–3, the ordinates represent (a) S_f+4, (b) S_f+3, (c) S_f+2, (d) S_f+1, (e) S_f.

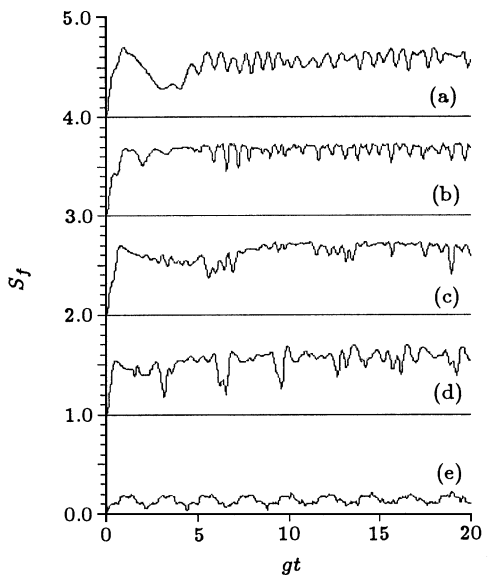


Fig.1. Time evolution of the field entropy for g_a/g=1, $\bar{n}$ = 5. (a) χ/g = 0.01, (b) χ/g = 0.1, (c) χ/g = 0.5, (d) χ/g = 1, (e) χ/g = 5.

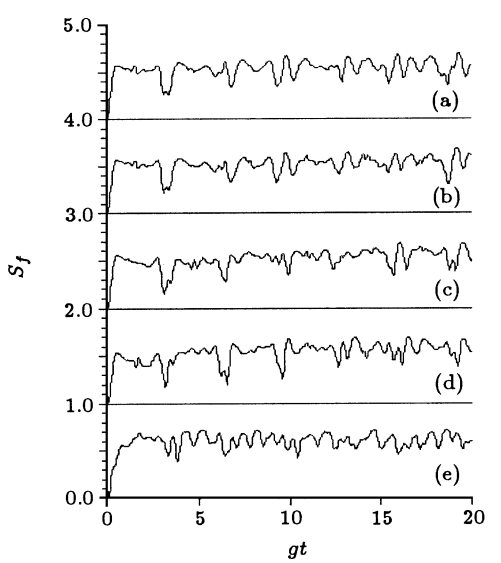


Fig.2. Time evolution of the field entropy for χ/g = 1, $\bar{n}$ = 5. (a) g_a/g = 0, (b) g_a/g = 0.1, (c) g_a/g = 0.5, (d) g_a/g = 1, (e) g_a/g = 5.

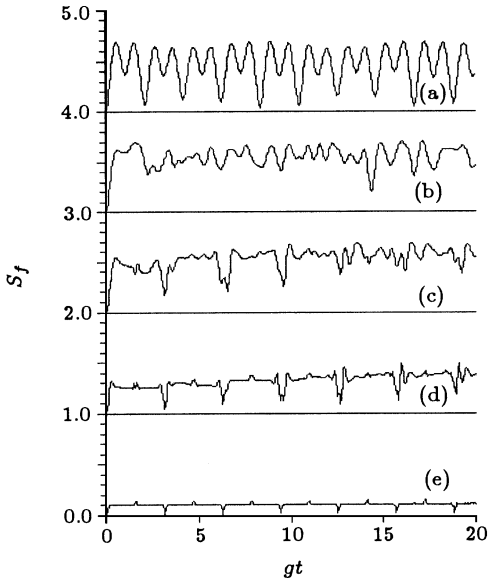


Fig.3. Time evolution of the field entropy with χ/g = 1, g_a/g = 1. (a) $\bar{n}$ = 0.1, (b) $\bar{n}$ = 1, (c) $\bar{n}$ = 5, (d) $\bar{n}$ = 10 (e) $\bar{n}$ = 30.

Firstly, we study the influence of the Kerr medium on the time evolution properties of the field entropy $S_f(t)$. In Fig.1, we find that for g_a/g = 1, $\bar{n}$ = 5 and different ratio χ/g, the time evolution of the field entropy $S_f(t)$ is quite complicated. When χ/g is large enough, even if the other parameters remain unchanged, the field entropy $S_f(t)$ is small enough. Obviously, this is a direct product state.

Secondly, we examine the influence of the coupling constant of the dipole–dipole interaction between atoms. The evolutions of the field entropy $S_f(t)$ are shown in Fig.2. We can see that the evolution of the field entropy $S_f(t)$ is not orderly, the atoms and field are always in the entangled state with the degree of entanglement being larger, when taking different ratio of g_a/g with χ/g = 1 and $\bar{n}$ = 5.

Thirdly, we discuss the effects of the intensity of the field. In Fig.3, we take g_a/g = 1 and χ/g = 1. When $\bar{n}$ is small, the amplitude of vibration of the field entropy $S_f(t)$ shows periodicity. It is obvious that the atoms and field are always in the entangled state. With the increasing of $\bar{n}$, the field entropy $S_f(t)$ decreases gradually. When $\bar{n}$ is large enough, the atoms and field may be in disentangled state.

4. Conclusion

We have investigated the evolution properties of the field entropy in a system of two entangled atoms interacting with a light field where the two two-level atoms are dipole–dipole interacting and initially in an

entangled state interacting with the single mode coherent field in a Kerr medium. We have discussed the influence of the coupling constant of dipole–dipole interaction between atoms, the coupling strength of the Kerr medium with the light field and the intensity of the light field on the field entropy by numerical calculations. The result shows that the field entropy is near zero and the entanglement between the atoms and the light field becomes weaker when the light field is strong, the coupling strength of the Kerr medium with the light field is large enough and the coupling constants of the light field and atoms, and the dipole–dipole interacting between atoms are unchanged. When the coupling constants of the light field interacting with atoms and the Kerr medium with the light field and the intensity of the light field are unchanged, the coupling constant of dipole–dipole interaction between the atoms changes, the evolutions of the field entropy is not orderly, and the atoms and field are always in the entangled state with the degree of entanglement being larger. Our results are of importance for experimental realization of the preparation of entangled states.

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