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Robust scheme for the generation of entangled states for N atoms in a cavity*

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A scheme is proposed for the generation of W entangled states for several atoms trapped in a cavity by detecting photon decay. The scheme works in the regime, where the cavity decay rate is larger than the atom-cavity coupling strength. Thus, the requirement for the quality factor of the cavity is greatly loosened, which is of importance in view of experiment. Another advantage of this scheme is that the atoms are always populated in two ground states coupled by Raman transitions, thus the spontaneous emission can also be suppressed.

Keywords: entangled states, photon, spontaneous emission
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1. Introduction

Entanglement is one of the most striking features of quantum mechanics. If two subsystems are entangled, the whole state vector can not be separated into a product of the states of the two subsystems. In this case the two subsystems are no longer independent even if they are far spatially separated. A measurement on one subsystem not only gives information of the other subsystem, but also provides possibilities of manipulating it. The generation of entangled states for two or more particles is fundamental to demonstrate quantum nonlocality.[1] Apart from the test of fundamental aspects of quantum mechanics, entangled state is useful for the implementation of quantum information, such as quantum cryptography[2] and teleportation.[3] Recently, the schemes have been proposed for the generation of entanglement between two distant atoms by single-photon interference at photodetectors.[4−8]

In this paper, we propose a scheme for the generation of the entangled state for many atoms in an optical cavity via detecting one photon leaking from the cavity. Our scheme works in the weak-coupling regime, where the atom-cavity coupling strength is smaller than the cavity decay rate. This is of importance in view of experiment. Another advantage of this scheme is that the atoms are always populated in two ground states coupled by Raman transitions, thus the spontaneous emission can also be suppressed.

Assume that N Λ-type three-level atoms are trapped in a single-mode optical cavity. Photons are allowed to leak from the cavity and then are detected by a photodetector. The atoms have one excited state $|r\rangle$ and two ground states $|e\rangle$ and $|g\rangle$. The transition $|g\rangle \rightarrow |r\rangle$ is coupled to the cavity mode with the coupling constant g, while the transition $|e\rangle \rightarrow |r\rangle$ is driven by a classical laser field with a Rabi frequency Ω. Assume the classical field and cavity mode are detuned from the respective transitions by Δ. In the case $\Delta \gg \Omega$ and $\Delta \gg g$, the upper level $|r\rangle$ can be adiabatically eliminated. This leads to the Hamiltonian[4]

$$H_{\rm e} = \Big[\frac{\Omega^2}{\Delta}\sum_{j=1}^{N}|e_j\rangle\langle e_j| + \frac{g^2}{\Delta}a^+a\sum_{j=1}^{N}|g_j\rangle\langle g_j| + \lambda(aS^+ + a^+S^-)\Big], \quad (1)$$

where

$$S^+ = \sum_{j=1}^{N}|e_j\rangle\langle g_j|, \quad (2)$$

$$S^- = \sum_{j=1}^{N}|g_j\rangle\langle e_j|, \quad (3)$$

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a and a^+ are the annihilation and creation operators for the cavity mode, respectively, and $\lambda = \frac{\Omega g}{\Delta}$. The first and second terms on the right-hand side in Eq.(1) describe the Stark shifts for the states $|e\rangle$ and $|g\rangle$, induced by the classical field and cavity field, respectively. The third term describes the Raman coupling of the two ground states.

The cavity is initially in the vacuum state and each atom is initially in the state $|e\rangle$. Then the initial state of the cavity and the atoms is $|0_c\rangle|0_N\rangle$. We here have defined the symmetrical state $|k_N\rangle$ with k atoms being in the state $|g\rangle$, i.e. the well known Dicke state.[9] For example, $|0_N\rangle$ and $|1_N\rangle$ are given by

$$|0_N\rangle = |e_1\rangle|e_2\rangle...|e_N\rangle, \tag{4}$$

$$\begin{aligned}|1_N\rangle =&\frac{1}{\sqrt{N}}(|g_1\rangle|e_2\rangle...|e_N\rangle + |e_1\rangle|g_2\rangle|e_3\rangle...|e_N\rangle \\ &+ ... + |e_1\rangle|e_2\rangle...|e_{N-1}\rangle|g_N\rangle).\end{aligned} \tag{5}$$

The Dicke state $|1_N\rangle$ is also referred to as the W state.[10] The W state has some interesting properties. For example, it retains bipartite entanglement when any one of the three qubits is traced out. The W states can also be used to test quantum nonlocality without inequality.[11] Such states might also be useful in quantum information processing.[12] Wang has shown that such states can be generated in the quantum Heisenberg XY model.[13] Guo *et al*[14] have proposed a scheme to generate such states for three or four atoms interacting dispersively with a vacuum cavity. Such states can be also generated via the resonant interaction of atoms with a cavity field.[15]

We now consider the cavity decay. Suppose that no photon has been detected until the moment t. Before the detection of photons the system is governed by the non-Hermitian Hamiltonian[4]

$$H_n = H_e - i\kappa a a^+, \tag{6}$$

where κ is the cavity decay rate. Assume that $\Omega = g$. Then the non-Hermitian Hamiltonian is mathematically equivalent to the Hamiltonian that describes the nonresonant interaction of N two-level atoms with a single-mode field. The "energy" of the state $|0_c\rangle|0_N\rangle$ is $-N\lambda$ and the "energy" of the state $|1_c\rangle|1_N\rangle$ is $-N\lambda - i\kappa$. Thus the "detuning" between the two states is $i\kappa$. It can be easily shown that

$$\langle 0_c|\langle 0_N|H_e|1_c\rangle|1_N\rangle = \sqrt{N}\lambda. \tag{7}$$

On condition that $\sqrt{N}\lambda \ll \kappa$, i.e. the Raman coupling strength is much smaller than the detuning, the Raman coupling can be considered as a perturbation. The first two eigenstates of the system are obtained using the perturbation theory,

$$|\phi_1\rangle \simeq |0_c\rangle|0_N\rangle + \frac{\sqrt{N}\lambda}{i\kappa}|1_c\rangle|1_N\rangle, \tag{8}$$

$$\begin{aligned}|\phi_2\rangle \simeq&|1_c\rangle|1_N\rangle - \frac{\sqrt{N}\lambda}{i\kappa}|0_c\rangle|0_N\rangle \\ &+ \frac{\sqrt{2(N-1)}\lambda}{i\kappa}|2_c\rangle|2_N\rangle.\end{aligned} \tag{9}$$

The corresponding "eigenenergies" are

$$E_1 = N\lambda + \frac{N\lambda^2}{i\kappa}, \tag{10}$$

$$E_2 = N\lambda - i\kappa - \frac{N\lambda^2}{i\kappa}. \tag{11}$$

In this case the initial state can be expressed in terms of $|\phi_1\rangle$ and $|\phi_2\rangle$

$$|0_c\rangle|0_N\rangle \simeq |\phi_1\rangle - \frac{\sqrt{N}\lambda}{i\kappa}|\phi_2\rangle. \tag{12}$$

The evolution of the system is

$$\begin{aligned}|0_c\rangle|0_N\rangle \rightarrow&e^{-iN\lambda t - N\lambda^2 t/\kappa}|\phi_1\rangle \\ &- \frac{\sqrt{N}\lambda}{i\kappa}e^{-iN\lambda t-(\kappa - N\lambda^2/\kappa)t}|\phi_2\rangle \\ \simeq&e^{-iN\lambda t}\{e^{-N\lambda^2 t/\kappa}|0_c\rangle|0_N\rangle \\ &+ \frac{\sqrt{N}\lambda}{i\kappa}[e^{-N\lambda^2 t/\kappa} \\ &- e^{-(\kappa - N\lambda^2/\kappa)t}]|1_c\rangle|1_N\rangle\}.\end{aligned} \tag{13}$$

Here the probability that the cavity contains two photons is so small that it has been neglected (on the order of $(\sqrt{N}\lambda/\kappa)^4$). The detection of one photon leaking out from the cavity at the moment t implies that the atomic system collapses down to the W state $|1_N\rangle$.

After a time t the probability of the atomic system remaining in the state $|0_c\rangle|0_N\rangle$ is given by $e^{-2N\lambda^2 t/\kappa}$. If one wishes to wait for a sufficiently long time t_D the probability of detecting a photon reaches 1. Thus, one can achieve a W state for the N atoms with a probability approximate to 1. The mean time before the detecting event is

$$T_D \simeq \frac{\kappa}{2N\lambda^2}. \tag{14}$$

Therefore, the time T_D is shortened with the increase of the atomic number N.

In summary, we have proposed a scheme for the generation of a W entangled state for many atoms

in a cavity via the detection of one photon leaking out from the cavity. In comparison with the previous schemes,[14,15] our scheme works in the weak-coupling regime, greatly loosening the requirement for the quality factor of the cavity, which is of importance in view of experiment. Furthermore, the atoms are always populated in two ground states coupled by Raman transitions, thus the spontaneous emission can also be suppressed.

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