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Manifesting mutual relation of spatiotemporally chaotic signals by symbolic analysis*

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Based on the coupled map lattice models, the mutual relation of spatiotemporally chaotic signals is studied using the symbolic analysis method. Numerical computation results show that the conditional entropy of two directly coupled lattices or that of two indirectly coupled but near lattices has a sharp minimum, whereas the conditional entropy of two distant lattices has no sharp minimum. Thus, the mutual relation of spatiotemporally chaotic signals produced by the coupled map lattice systems can be manifested by the symbolic analysis method.

Keywords: spatiotemporal chaos, symbolic analysis, mutual relation, coupling, entropy
PACC: 0545, 0250

1. Introduction

Spatiotemporal chaos is a complex dynamical phenomenon that exists extensively in spatiotemporally extended dynamical systems, such as in fluids, optics, and biology systems, etc. In recent years this phenomenon has attracted much interest. The coupled map lattices models are common and important to the study of spatiotemporal chaos. Based on these models, many investigations have been made on the mechanism of spatiotemporal chaos, on the control and synchronization of spatiotemporal chaos, and on the applications of spatiotemporal chaos in secure communication.[1−9]

In the study of spatiotemporal chaos, how to manifest the mutual relation of two chaotic signals is of significance, whereas the methods of correlation functions and Fourier transform are often inadequate for this purpose. Lehrman and Rechester[10] have first used the symbolic analysis to manifest the coupling of two different signals in the Lorenz equation, and found that the conditional entropy as a function of the shift parameter n_0 has a sharp minimum, from which one can determine whether or not two different signals are produced by the same dynamical system. Later, Liu and Chen[11] used this method to manifest the coupling of two signals of two generalized synchronization Lorenz and Rössler systems, and showed that the conditional entropy has a sharp minimum with the shift of time parameter n_0, from which the generalized synchronization can be identified. Azad *et al*[12] manifested the dynamical coupling between the climatic time-series data of New Delhi and Sundernagar using this method. The purpose of this paper is to verify the capability of this technique in manifesting the mutual relation of different spatiotemporally chaotic signals produced by the extended systems. The computed results show that this method is effective in manifesting the mutual relation of spatiotemporally chaotic signals produced by the coupled map lattices.

2. Theory of symbolic analysis

For two discrete time series $\{X_n\}$ and $\{Y_n\}$, $(n = 0, 1, 2, \cdots)$, generally, one can partition their phase spaces into domains that are each uniquely identified with a symbol via the coarse-graining method,[9−14] and convert $\{X_n\}$ and $\{Y_n\}$ into two symbolic series, so the information concerning the orbits can be suitably encoded with the symbolic series. By analysing the relation between the two symbolic series, one can manifest whether or not there exists a dynamical coupling between time series $\{X_n\}$ and $\{Y_n\}$.

Let a set of $m + 1$ critical points be $\{X_{C_0}, X_{C_1}, \cdots, X_{C_m}\}$, and denote a symbol with an integer,

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the time series $\{X_n\}$ can be converted into a symbolic series $\{S_n\}$, $(n = 0, 1, 2, \cdots)$ by the rule defined as

$$\text{If} \quad X_{C_k} \leq X_j < X_{C_{k+1}}, \quad \text{then} \quad S_j \equiv k, \qquad (1)$$

where $k \in \{0, 1, 2, \cdots, m-1\}$. The resulting symbolic series can be partitioned into subsequences of a given length L, which can be denoted and identified with the following formula[9−14]:

$$l_X(L, i) = \sum_{p=1}^{L} m^{L-p} S_{p+i}. \qquad (2)$$

Here i indicates that the subsequence starts at the position i along the sequence $\{S_1, S_2, S_3, \ldots\}$. From the formula (2), we can see that each subsequence is uniquely denoted by an integer in the set $\{0, 1, 2, \cdots, m^L - 1\}$. Substituting these subsequences with the corresponding integers and denoting the symbolic series formed ordinally by these integers $\{l_X(n)\}$, we convert the time series $\{X_n\}$ into the symbolic series $\{l_X(n)\}$. The information content of the time series $\{X_n\}$ contained in the symbolic series $\{l_X(n)\}$ can be quantified by the information entropy,[9−12]

$$E = -\frac{1}{L} \sum_{l_X} P_{l_X} \ln P_{l_X}, \qquad (3)$$

where P_{l_X} is the probability of finding a particular subsequence l_X, which refers to how many times this subsequence can be found in the long symbolic series $\{l_X(n)\}$ divided by the number of all subsequences.

The coding ability of the symbolic language defined by the rule (1) depends on the number of the critical points and their values. Generally, we can find out the optimum language through maximizing the entropy. For convenience, we equally partition the phase space of a chaotic time series into 10 domains that are each signified by a symbol, and take the length of each subsequence to be 2 in the later calculation.

Similarly, for the given time series $\{Y_n\}$, $(n = 0, 1, 2, \cdots)$, we can convert it into a symbolic series $\{l_Y(n)\}$ by the coarse-graining method.

For the time series $X = \{X_n\}$ and $Y = \{Y_n\}$, $(n = 0, 1, 2, \cdots)$, their joint entropy is defined as

$$E(XY) = -\frac{1}{L} \sum_{l_X} \sum_{l_Y} P(l_X l_Y) \ln P(l_X l_Y), \qquad (4)$$

where $P(l_X l_Y)$ is the joint probability for the variable X to occupy state l_X and for the variable Y to occupy state l_Y. $P(l_X l_Y)$ can be computed by the number of the sequences combined in which the variable X occupies state l_X while the variable Y occupies state l_Y, divided by the total number of the sequences of the symbolic series $\{l_X(n)\}$ and $\{l_Y(n)\}$ that are combined together.

According to the information theory,[15] the conditional entropy of $\{X_n\}$ and $\{Y_n\}$ can be calculated by the formula

$$E(Y|X) = E(XY) - E(X). \qquad (5)$$

If $\{X_n\}$ is not coupled with $\{Y_n\}$, then $\{X_n\}$ and $\{Y_n\}$ are not correlated with each other, so their symbolic series $\{l_X(n)\}$ and $\{l_Y(n)\}$ are not correlated with each other, either. In contrast, if there exists a coupling between $\{X_n\}$ and $\{Y_n\}$, then they are correlated with each other, so their symbolic series $\{l_X(n)\}$ and $\{l_Y(n)\}$ are also correlated with each other. In this case the conditional entropy $E(Y|X)$ will be remarkably smaller than that when $\{X_n\}$ and $\{Y_n\}$ are not correlated with each other. Hence, the existence of a sharp minimum of the conditional entropy as a function of the shift parameter n_0 can be considered as an indication whether or not there exists a mutual relation between the time series $\{X_n\}$ and $\{Y_n\}$.

3. Manifesting mutual relation of spatiotemporally chaotic signals

Coupled map lattices are common and important models in studying spatiotemporal chaos. Here we investigate the mutual relation of spatiotemporally chaotic signals based on these models.

We begin with the following one-way coupled map lattices:[1−4]

$$\begin{aligned} x_{n+1}(i) &= (1-\varepsilon) f[x_n(i)] + \varepsilon f[x_n(i+1)], \\ & \qquad i = 1, 2, \cdots, 19, \\ x_{n+1}(20) &= (1-\varepsilon) f[x_n(20)] + \varepsilon f[x_n(1)], \end{aligned} \qquad (6)$$

where n, i, and ε are the discrete time step, the lattice site index, and the coupling coefficient, respectively. The local mapping function $f(x)$ is chosen to be the logistic map:

$$f(x) = 4x(1-x). \qquad (7)$$

We take $\varepsilon = 0.99$ and randomly choose the initial condition $x_0(i)$ in the interval $(0, 1)$ for $i = 1, 2, \cdots, 20$. Numerically iterate the system (6). When the transient dies out, the system runs into the spatiotemporally chaotic states. We take the tenth lattice as an example to consider the relation between the time series produced by this lattice and other lattices.

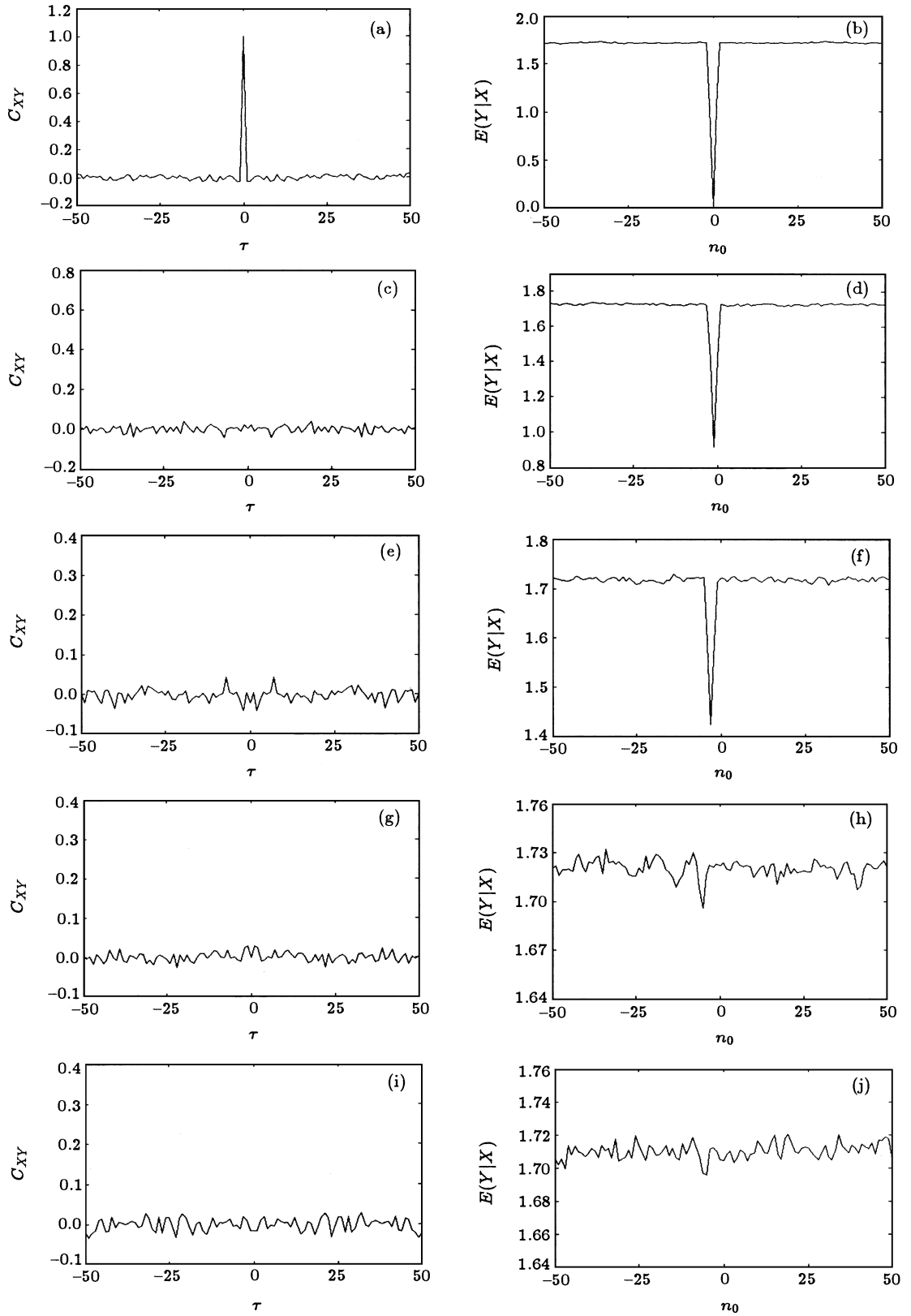


Fig.1. The correlation function vs time τ and conditional entropy vs shift time n_0 for two chaotic time series X and Y produced by the one-way coupled map lattices. (a) $X = \{x_n(10)\}$, $Y = \{x_{n+\tau}(10)\}$; (b) $X = \{x_n(10)\}$, $Y = \{x_{n+n_0}(10)\}$; (c) $X = \{x_n(10)\}$, $Y = \{x_{n+\tau}(11)\}$; (d) $X = \{x_n(10)\}$, $Y = \{x_{n+n_0}(11)\}$; (e) $X = \{x_n(10)\}$, $Y = \{x_{n+\tau}(13)\}$; (f) $X = \{x_n(10)\}$, $Y = \{x_{n+n_0}(13)\}$; (g) $X = \{x_n(10)\}$, $Y = \{x_{n+\tau}(15)\}$; (h) $X = \{x_n(10)\}$, $Y = \{x_{n+n_0}(15)\}$; (i) $X = \{x_n(10)\}$, $Y = \{x_{n+\tau}(16)\}$; (j) $X = \{x_n(10)\}$, $Y = \{x_{n+n_0}(16)\}$.

The autocorrelation function and mutual correlation function are given as[3]

$$C_{i\cdot i}(\tau) = \hat{C}_{i\cdot i}(\tau)/\hat{C}_{i\cdot i}(0),$$
$$\hat{C}_{i\cdot i}(\tau) = \frac{1}{T}\sum_{n=1}^{T}[x_n(i) - x_A(i)][x_{n+|\tau|}(i) - x_A(i)], \tag{8}$$
$$C_{i\cdot j}(\tau) = \hat{C}_{i\cdot j}(\tau)/\sqrt{\hat{C}_{i\cdot i}(0)\hat{C}_{j\cdot j}(0)},$$
$$\hat{C}_{i\cdot j}(\tau) = \frac{1}{T}\sum_{n=1}^{T}[x_n(i) - x_A(i)][x_{n+|\tau|}(j) - x_A(j)], \tag{9}$$

where $x_A(i)$ is the average value of $x_n(i)$, i and j are the lattice indices. In Figs.1(a), 1(c), 1(e), 1(g), and 1(i), we have plotted the correlation functions $C_{10\cdot 10}(\tau)$, $C_{10\cdot 11}(\tau)$, $C_{10\cdot 13}(\tau)$, $C_{10\cdot 15}(\tau)$ and $C_{10\cdot 16}(\tau)$, respectively. We can see that the correlation function method is inadequate to the manifestation of the mutual relation of the time series produced by the lattices. Again we have plotted the conditional entropy of the above series vs shift time n_0. In the computation we divided the range of every series into ten domains of equal length, with each domain uniquely marked with an integer in the set $\{0, 1, 2, \cdots, 9\}$, and the length of the subsequences taken to be 2. The conditional entropy $E(Y|X)$ vs shift time n_0 for $X = \{x_n(10)\}$ and $Y = \{x_{n+n_0}(10)\}$ is plotted in Fig.1(b), where we can see a sharp minimum that is equal to zero. This means that $X = \{x_n(10)\}$ is completely coupled with itself. In Fig.1(d) we plotted $E(Y|X)$ vs n_0 for $X = \{x_n(10)\}$ and $Y = \{x_{n+n_0}(11)\}$, where the sharp minimum indicates that $\{x_n(10)\}$ is strongly coupled with $\{x_n(11)\}$. This coincides with Eq.(6). In Fig.1(f) we plotted $E(Y|X)$ vs n_0 for $X = \{x_n(10)\}$ and $Y = \{x_{n+n_0}(13)\}$, where we also can see a sharp minimum. This indicates that there exists a dynamical coupling between $\{x_n(10)\}$ and $\{x_n(13)\}$ even if they are indirectly coupled. In Figs.1(h) and 1(j), we plotted $E(Y|X)$ vs n_0 for $X = \{x_n(10)\}$ and $Y = \{x_{n+n_0}(15)\}$, and for $X = \{x_n(10)\}$ and $Y = \{x_{n+n_0}(16)\}$, respectively. We can see that the minimum has become indistinguishable, so the dynamical coupling between $\{x_n(10)\}$ and $\{x_n(15)\}$ (as well as between $\{x_n(10)\}$ and $\{x_n(16)\}$) is weak. In addition, from Figs.1(b), 1(d), 1(f), 1(h), and 1(j), we can see that the minimum of the conditional entropy increases with the increasing of the distance between two lattices. That indicates that the coupling strength decreases as the distance increases. From the above analysis we can derive that the symbolic analysis method is applicable for manifesting the mutual relation of spatiotemporally chaotic signals produced by the one-way coupled map lattices, whereas the correlation function method is inadequate for this purpose.

Now we consider the following case of multi-way coupled map lattices, which represents the laser oscillation output in a ring cavity:[7]

$$\begin{aligned} I_{n+1}(i,j) =& (1-D)f(I_n(i,j)) + \frac{D}{4}[f(I_n(i-1,j)) \\ &+ f(I_n(i+1,j)) + f(I_n(i,j+1)) \\ &+ f(I_n(i,j-1))], \end{aligned} \tag{10}$$

where i and j are the lattice indexes in the $X - Y$ cross section $(i, j = 1, 2, \cdots, L)$, L is the size of the pattern, n is the discrete time step $(n = 1, 2, \cdots, N)$, and D is the coupling coefficient $(0 < D < 1)$. The local mapping function $f(I_n)$ is chosen to be

$$f(I_n) = I_n \exp\left[\frac{\alpha}{1 + I_n} - \Gamma\right]. \tag{11}$$

We take $D = 0.2$, $\Gamma = 11.0$, $\alpha = 15.0$, $i \times j = 64 \times 64$. The initial condition is chosen so that $I_0(i, j) = 0.5$, and the boundary condition is assumed so that $I_n(0, \cdot) = 0$, $I_n(65, \cdot) = 0$, $I_n(\cdot, 0) = 0$ and $I_n(\cdot, 65) = 0$. Run the system (10). When the transient process dies out, the system goes into the spatiotemporally chaotic states. We take the $(32, 32)$ lattice as an example to calculate the correlation function and conditional entropy of the chaotic series produced by this lattice and other neighbouring lattices. For the result appearing to be more pronounced, we sampled the chaotic series with a frequency of 10Hz to produce the time series for the computations of the correlation function and conditional entropy. We computed the correlation function with Eqs.(8) and (9), and divided the range of the time series into ten equal domains and took the length of the subsequences to be 2. In Figs.2(a) and 2(b), we plotted the autocorrelation function of $\{I_n(32, 32)\}$ vs time τ and the conditional entropy between $\{I_n(32, 32)\}$ and its shift series, respectively. Figures 2(c) and 2(d) show the mutual correlation function and conditional entropy of $\{I_n(32, 32)\}$ and $\{I_n(33, 33)\}$ vs shift time, respectively. Figures 2(e) and 2(f) show the mutual correlation function and conditional entropy of $\{I_n(32, 32)\}$ and $\{I_n(34, 34)\}$

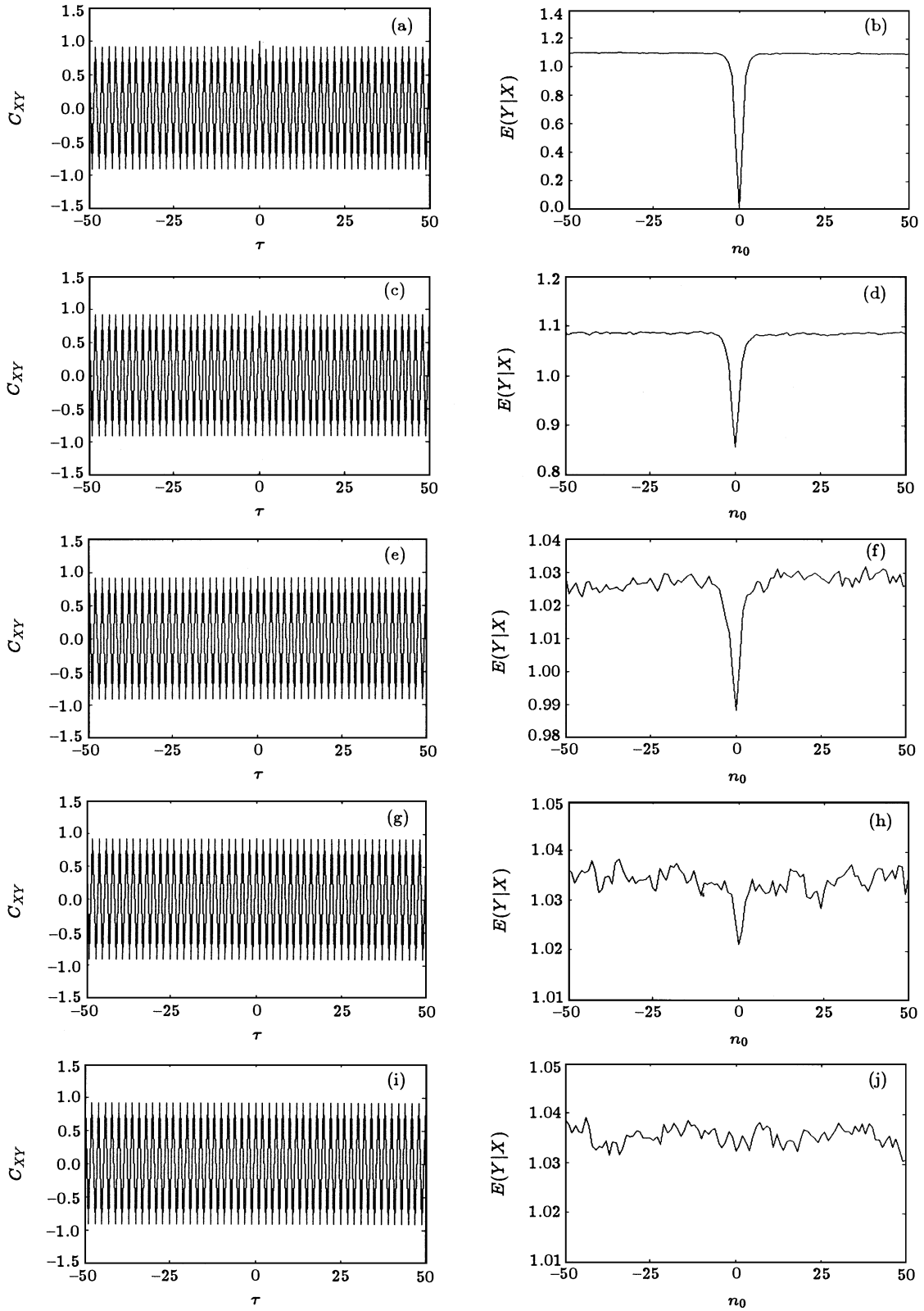


Fig.2. The correlation function vs time τ and conditional entropy vs shift time n_0 for two chaotic time series X and Y produced by the multi-way coupled map lattices. (a) $X = \{I_n(32,32)\}$, $Y = \{I_{n+\tau}(32,32)\}$; (b) $X = \{I_n(32,32)\}$, $Y = \{I_{n+n_0}(32,32)\}$; (c) $X = \{I_n(32,32)\}$, $Y = \{I_{n+\tau}(33,33)\}$; (d) $X = \{I_n(32,32)\}$, $Y = \{I_{n+n_0}(33,33)\}$; (e) $X = \{I_n(32,32)\}$, $Y = \{I_{n+\tau}(34,34)\}$; (f) $X = \{I_n(32,32)\}$, $Y = \{I_{n+n_0}(34,34)\}$; (g) $X = \{I_n(32,32)\}$, $Y = \{I_{n+\tau}(35,34)\}$; (h) $X = \{I_n(32,32)\}$, $Y = \{I_{n+n_0}(35,34)\}$; (i) $X = \{I_n(32,32)\}$, $Y = \{I_{n+\tau}(35,35)\}$; (j) $X = \{I_n(32,32)\}$, $Y = \{I_{n+n_0}(35,35)\}$.

vs shift time, respectively. Figures 2(g) and 2(h) show the mutual correlation function and conditional entropy of $\{I_n(32,32)\}$ and $\{I_n(35,34)\}$ vs shift time, respectively. Figures 2(i) and 2(j) show the mutual correlation function and conditional entropy of $\{I_n(32,32)\}$ and $\{I_n(35,35)\}$ vs shift time, respectively. From Figs.2(a), 2(c), 2(e), 2(g), and 2(i), we can see that the correlation method is inadequate to the manifestation of the dynamical coupling between two chaotic time series. There exists a sharp minimum in Figs.2(b), 2(d), 2(f), and 2(h), respectively, and the value of the minimum increases with the increasing of the distance between two lattices. Hence, we can conclude that the series $\{I_n(32,32)\}$ has the dynamical coupling with the series $\{I_n(32,32)\}$, $\{I_n(33,33)\}$, $\{I_n(34,34)\}$, and $\{I_n(35,34)\}$, respectively, and the coupling strength decreases with the increasing of the distance between two lattices. Here we notice that the series $\{I_n(32,32)\}$ has the dynamical coupling with $\{I_n(34,34)\}$, and $\{I_n(35,34)\}$, respectively, even they are indirectly coupled with each other. In Fig.2(j) a sharp minimum disappears, so we can conclude that the dynamical coupling between $\{I_n(32,32)\}$ and $\{I_n(35,35)\}$ is very weak, this is because the distance between these two lattices is large. From the above analysis we can conclude that the symbolic analysis method is effective in manifesting the mutual relation of two chaotic series produced by the multi-way coupled map lattices, whereas the correlation method is inadequate for this purpose.

4. Conclusion

In this paper we have applied the symbolic analysis method to manifest the mutual relation of spatiotemporally chaotic signals produced by extended systems. For two kinds of coupled map lattices, we computed the conditional entropy and correlation function of two chaotic time signals produced by these spatiotemporally extended systems. The numerical results show that, for two directly coupled lattices or two indirectly coupled but near lattices, the conditional entropy of two chaotic time series produced by these two lattices has a sharp minimum, whereas for two distant lattices the sharp minimum comes to inexistence, so the mutual relation of two spatiotemporally chaotic signals can be manifested from the existence of the sharp minimum. Hence we conclude that the symbolic analysis method is applicable for manifesting mutual relation of spatiotemporally chaotic signals. The results also show that the correlation function is inadequate for this purpose.

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