Vol 14 No 2, February 2005
1009-1963/2005/14(02)/0439-09

Chinese Physics

© 2005 Chin. Phys. Soc.
and IOP Publishing Ltd

Magnetic extraction of energy from black hole accretion disc and its application to astrophysics*

Ye Yong-Chun(叶永春), Wang Ding-Xiong(汪定雄)†, and Gong Xiao-Long(龚小龙)

Department of Physics, Huazhong University of Science and Technology, Wuhan 430074, China

(Received 21 April 2004; revised manuscript received 31 October 2004)

A new configuration of magnetic field (NCMF) in black hole (BH) magnetosphere is proposed by considering the effects of the screw instability. Three mechanisms of extracting energy magnetically are involved in the NCMF: (1) the Blandford–Znajek (BZ) process; (2) the magnetic coupling (MC) process; (3) a new scenario (henceforth the DL process) for extracting rotational energy from the disc, which is related to the open field lines connecting the disc with the astrophysical load. The expressions for the powers and torques of the above energy mechanisms are derived by using two kinds of the equivalent circuits. It turns out that the power and efficiency of extracting energy magnetically from the BH accretion disc are all augmented in the NCMF. It is shown that a very steep emissivity can be produced in a NCMF, which is consistent with the recent *XMM-Newton* observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15.

Keywords: black holes, black hole physics, accretion discs, emissivity index
PACC: 9760L, 0470, 9710H

1. Introduction

As is well known, the Blandford–Znajek (BZ) mechanism is the process that the rotating black hole (BH) exerts a torque on the astrophysical loads, and the rotational energy of the BH is extracted and transported to the astrophysical loads via Poynting flux, which has been considered as a plausible way to power the extragalactic jets.[1−5] Recently, the BZ mechanism has been used as a central engine for powering gamma-ray bursts (GRBs), where rotating energy of a stellar BH with a magnetic field of 1×10^{11}T is extracted along the magnetic field lines supported by a magnetized accretion disc.[6,7]

Recently, some authors[8−12] have argued that with the existence of the closed field lines a fast rotating BH will exert a torque on the disc to transfer energy and angular momentum from the BH to the disc. Henceforth this energy mechanism is referred to as the magnetic coupling (MC) process. The MC process can be used to explain a very steep emissivity in the inner region of the disc, which is consistent with the recent *XMM-Newton* observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15.[12−14]

In Ref.[11] we worked out an equivalent circuit to calculate the BZ and MC powers. Very recently, we have discussed the condition for the coexistence of the BZ and MC processes (CEBZMC), and found that the state of the CEBZMC always accompanies a screw instability of the magnetic field connecting a rotating BH with its surrounding disc.[15,16] It turns out that the screw instability will occur at some place far away from the inner edge of the disc, if the BH spin and the power-law index for the variation of the magnetic field are greater than some critical values.

In this paper a new scenario for extracting rotational energy of the disc matter is proposed, in which the configuration of the magnetic field is restricted by the screw instability. To facilitate the description, this mechanism is referred to as the DL process, in which the energy and angular momentum are extracted magnetically from the disc to the load. The effects of the screw instability on the magnetic field configuration and on the power and efficiency of extracting energy from the BH-disc system are discussed in detail.

This paper is organized as follows. In Section 2 we propose a new configuration of magnetic field (NCMF), where three energy mechanisms are involved. In Section 3 the expressions for powers, torques and efficiencies of extracting energy from the

*Project supported by the National Natural Science Foundation of China (Grant Nos 10173004, 10373006 and 10121503).
†E-mail: dxwang@hust.edu.cn

BH-disc system are derived in the BZ, the MC and the DL processes, separately. We compare these powers, torques and efficiencies in the NCMF with those in our previous work in detail. It is shown that the power, torque and the efficiency of extracting energy magnetically in the NCMF are always greater than those in CEBZMC. In Section 4 the emissivity index is calculated in the NCMF, and is compared with that in CEBZMC. In Section 5 we summarize our main results. Throughout this paper the geometric units $G = c = 1$ are used.

2. A new configuration of magnetic field and the three energy mechanisms

The configuration of the magnetic field containing the three mechanisms arises from the following assumptions given in Refs.[11] and [15].

(i) The disc is both stable and perfectly conducting, and the closed magnetic field lines are frozen in the disc.[17]

(ii) The disc is thin and Keplerian, and lies in the equatorial plane of the BH with the inner boundary being at the marginally stable orbit.

(iii) The magnetic field is assumed to be constant on the horizon, and to vary in a power law with the radial coordinate of the disc; the magnetosphere is stationary, axisymmetric and force-free outside the BH and the disc.

(iv) The magnetic flux connecting a BH with its surrounding disc takes precedence over that connecting the BH with the remote load.

Assumption (iv) is proposed based on two reasons: (1) The magnetic field on the horizon is brought and held by the surrounding magnetized disc; (2) the disc is much nearer to the BH than the remote load.

It is found that the condition for CEBZMC is satisfied, provided that the BH spin and the power-law index are great enough.

As is well known, the screw instability plays a very important role in determining the configurations of magnetic fields. According to the Kruskal–Shafranov criterion,[18−20] if the toroidal magnetic field is so strong that from one end to the other end the magnetic field lines wind around the symmetry axis once or more, the screw instability will occur. Recently, some authors have discussed the screw instability under different conditions in the BH magnetosphere.[21−23]

Very recently, we have discussed the influence of the screw instability on the magnetic field connecting a rotating BH with its surrounding disc. By using the Kruskal–Shafranov criterion, we have derived the condition for the screw instability in a BH accretion disc as follows:[16]

$$(2\pi\varpi_{_D}/L)\,F\,(a_*;\xi,n) < 1, \tag{1}$$

where

$$F\,(a_*;\xi,n) = \frac{\xi^{1-n}\,(1+q)\left[2\csc^2\theta-(1-q)\right]}{2a_*\,(1-\beta)} \times\sqrt{\frac{1+a_*^2\chi_{ms}^{-4}\xi^{-2}+2a_*^2\chi_{ms}^{-6}\xi^{-3}}{1+a_*^2\chi_{ms}^{-4}+2a_*^2\chi_{ms}^{-6}}}, \tag{2}$$

and L is the poloidal length of the closed field line connecting the BH with the disc. The ratio of the angular velocity of the closed field line to the angular velocity of the horizon is defined as

$$\beta \equiv \Omega_D/\Omega_H = \frac{2\,(1+q)}{a_*}\left[\left(\sqrt{\xi}\chi_{ms}\right)^3+a_*\right]^{-1}, \tag{3}$$

and ϖ_D is the cylindrical radius of the disc and reads

$$\varpi_{_D} = \xi M\chi_{ms}^2\sqrt{1+a_*^2\xi^{-2}\chi_{ms}^{-4}+2a_*^2\xi^{-3}\chi_{ms}^{-6}}. \tag{4}$$

In Eq.(3) M is the BH mass, and $a_* = J/M^2$ is BH spin; in Ref.[24] we have already shown that a_* is a more reasonable parameter than angular momentum J when we describe the BH-disc. The parameter $q \equiv \sqrt{1-a_*^2}$ is a function of the BH spin, and $\xi \equiv r/r_{ms}$ is the radial coordinate on the disc, which is defined in terms of the radius of the marginally stable orbit, and r_{ms} is related to M and a_* by the following relation:[25]

$$\begin{aligned} &r_{ms} \equiv M\chi_{ms}^2,\\ &\chi_{ms} = \{3+A_2 \pm [(3-A_1)(3+A_1+2A_2)]^{1/2}\}^{1/2},\\ &A_1 = 1+\left(1-a_*^2\right)^{1/3}[(1+a_*)^{1/3}+(1-a_*)^{1/3}],\\ &A_2 = \left(3a_*^2+A_1^2\right)^{1/2}. \end{aligned} \tag{5}$$

From criterion (1), it is found that the screw instability will occur, provided that the BH spin a_* and the power-law index n are great enough. According to this criterion the closed field lines connecting the BH horizon cannot reach the region beyond a critical radius r_S, and we think that the configuration of the magnetic field in this region might consist of the open field lines connecting the disc with the astrophysical load. Based on this conclusion, we propose a NCMF as shown in Fig.1. Thus the three mechanisms of extracting energy magnetically from the BH-disc system

are included: (1) the BZ process involves the open field lines connecting the BH with the astrophysical load, (2) the MC process involves the closed field lines connecting the BH with the surrounding disc, and (3) the DL process involves the open field lines connecting the disc with the astrophysical load.

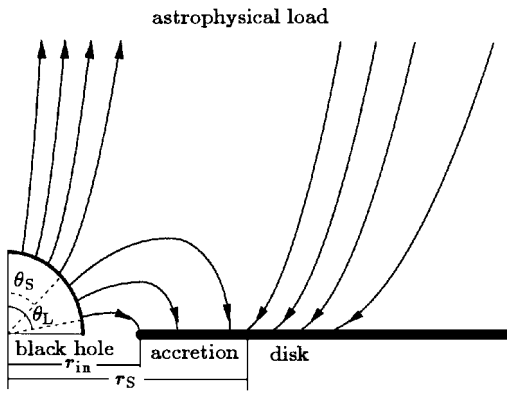


Fig.1. Configuration of the magnetic field restricted by the screw instability.

Following Blandford,[26] we assume that the poloidal magnetic field varies with the parameter ξ on the disc as a power law,[12,15,27]

$$B_D^{\rm p} = B_H^{\rm p}\left[r_{_H}/\varpi_{_D}\left(r_{ms}\right)\right]\xi^{-n}, \quad 1 < \xi < \xi_S, \tag{6}$$

$$B_D^{\rm p} = B_H^{\rm p}\left(r_H/r_S\right)\left(\xi/\xi_S\right)^{-n}, \quad \xi_S < \xi < \infty, \tag{7}$$

where $B_D^{\rm p}$ and $B_H^{\rm p}$ are the poloidal components of the magnetic fields on the disc and on the horizon respectively, and $\xi_S \equiv r_S/r_{ms}$ is the minimum radial coordinate for the screw instability.

3. Powers and efficiencies of extracting energy in the NCMF

3.1. Comparison of the boundary angle in the NCMF with that in CEBZMC

The angular boundary between the open and closed field lines on the horizon in the NCMF is θ_S, which is related to ξ_S by

$$\cos\theta_S - \cos\theta_L = \int_1^{\xi_S} G\left(a_*;\xi,n\right)\mathrm{d}\xi, \tag{8}$$

where

$$G\left(a_*;\xi,n\right) = \frac{\xi^{1-n}\chi_{ms}^2\sqrt{1+a_*^2\chi_{ms}^{-4}\xi^{-2}+2a_*^2\chi_{ms}^{-6}\xi^{-3}}}{2\sqrt{\left(1+a_*^2\chi_{ms}^{-4}+2a_*^2\chi_{ms}^{-6}\right)\left(1-2\chi_{ms}^{-2}\xi^{-1}+a_*^2\chi_{ms}^{-4}\xi^{-2}\right)}}. \tag{9}$$

However, in Ref.[15] the angular boundary between the open and closed field lines on the horizon is θ_M, which is related to infinity via Eq.(6). We find that θ_M is always less than θ_S due to the effects of the screw instability on the magnetic field configurations. The curves of the relative variation of the boundary angle, $(\theta_S - \theta_M)/\theta_M$, with the power-law index n for the given BH spin a_*, and with a_* for the given n are shown in Fig.2.

From Fig.2 we find that $(\theta_S - \theta_M)/\theta_M$ is always positive, and a smaller power-law index n results in a greater relative variation of the boundary angle for a given a_*.

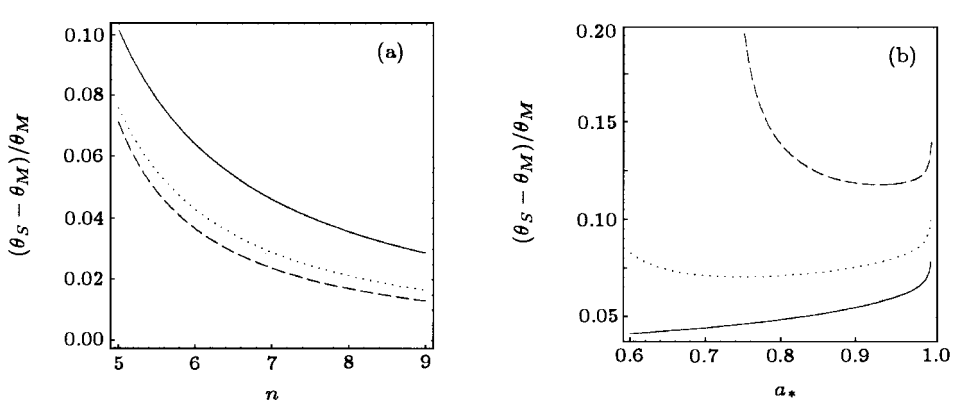


Fig.2. The relative variation of the boundary angle, $(\theta_S - \theta_M)/\theta_M$, due to screw instability, (a) with the power-law index n for a_*= 0.8, 0.9 and 0.998 in dashed, dotted and solid lines, respectively; (b) with the BH spin a_* for n=4.5, 5 and 5.5 in dashed, dotted and solid lines, respectively.

3.2. Comparison of extracting powers in the NCMF with those in CEBZMC

The expressions for the powers and torques in the BZ and MC processes can be derived by using an equivalent circuit as shown in Fig.3 (henceforth circuit I).

In Refs [11] and [27] the powers and torques for the BZ and MC processes in the NCMF are expressed by

$$P^S_{\rm BZ}/P_0 = 2a_*^2 \int_0^{\theta_S} \frac{k\,(1-k)\sin^3\theta {\rm d}\theta}{2-(1-q)\sin^2\theta}, \quad 0<\theta<\theta_S, \tag{10}$$

$$T^S_{\rm BZ}/T_0 = 4a_*\,(1+q) \int_0^{\theta_S} \frac{(1-k)\sin^3\theta {\rm d}\theta}{2-(1-q)\sin^2\theta}, \quad 0<\theta<\theta_S, \tag{11}$$

$$P^S_{\rm MC}/P_0 = 2a_*^2 \int_{\theta_S}^{\theta_L} \frac{\beta\,(1-\beta)\sin^3\theta {\rm d}\theta}{2-(1-q)\sin^2\theta}, \quad \theta_S<\theta<\theta_L, \tag{12}$$

$$T^S_{\rm MC}/T_0 = 4a_*\,(1+q) \int_{\theta_S}^{\theta_L} \frac{(1-\beta)\sin^3\theta {\rm d}\theta}{2-(1-q)\sin^2\theta}, \quad \theta_S<\theta<\theta_L. \tag{13}$$

In Eqs.(10)–(13), $k \equiv \Omega_F/\Omega_H$ is the ratio of the angular velocity of the open field lines to that of the horizon, and $\beta \equiv \Omega_D/\Omega_H$ is the ratio of the angular velocity of the closed field lines to that of the horizon. Usually, $k = 0.5$ is taken for the optimal BZ power,[17] while β depends on the BH spin and the place where the field line penetrates the disc. The parameters P_0 and T_0 are defined as

$$\begin{aligned} P_0 \equiv & (B_H^{\rm p})^2 M^2 \approx B_4^2\,(M/M_\odot)^2 \\ & \times 6.59\times 10^{28}{\rm erg\cdot s^{-1}}, \\ T_0 \equiv & (B_H^{\rm p})^2 M^3 \approx B_4^2\,(M/M_\odot)^3 \\ & \times 3.26\times 10^{23}{\rm g\cdot cm^2\cdot s^{-2}}, \end{aligned} \tag{14}$$

where B_4 is the strength of the poloidal magnetic field on the horizon in units of T.

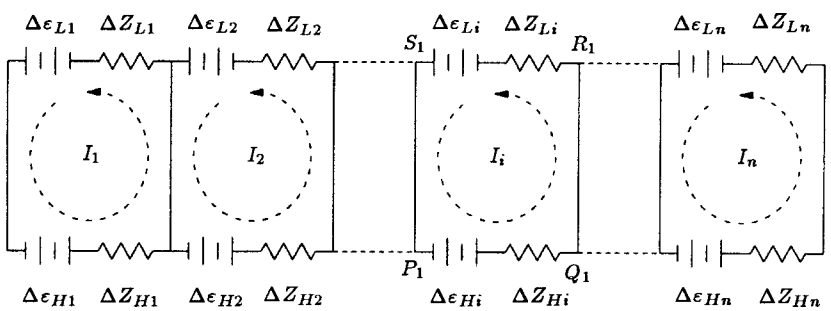


Fig.3. Equivalent circuit for the BZ and MC processes (circuit I).

Similarly, the DL power and torque can be derived by using the equivalent circuit as shown in Fig.4. Henceforth this equivalent circuit is referred to as circuit II, where segments P_2S_2 and Q_2R_2 represent the two adjacent magnetic surfaces consisting of the open field lines in the DL region, and segments P_2Q_2 and R_2S_2 represent the disc surface in the DL region and the load sandwiched by the two adjacent magnetic surfaces, respectively. The quantities ΔZ_A and $\Delta\varepsilon_D$ (the subscript "i" is omitted) are the resistance of the load and EMF due to the rotation of the disc, respectively. The disc load is neglected in calculations on considering the perfect conductivity of plasma.

The following equations are used to derive the DL power in circuit II, which are similar to those given in deriving the BZ and MC powers in Ref.[11].

$$\begin{aligned} \Delta P_{\rm DL} &= I^2_{\rm DL}\Delta Z_A, \\ I_{\rm DL} &= \Delta\varepsilon_D/\Delta Z_A, \\ \Delta\varepsilon_D &= -\,(\Delta\Psi_D/2\pi)\,\Omega_D, \end{aligned} \tag{15}$$

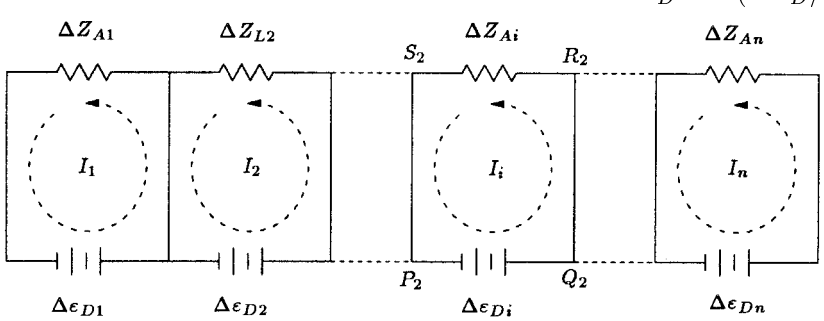


Fig.4. Equivalent circuit for the DL process (circuit II).

where $I_{\rm DL}$ is the current in each loop of circuit II. The minus sign in $\Delta\varepsilon_D$ arises from the direction of the magnetic flux between the two adjacent magnetic surfaces, which is given by

$$\Delta\Psi_D = B_D^{\rm p} 2\pi(\varpi\rho/\sqrt{\Delta})_{\theta=\pi/2}{\rm d}r. \tag{16}$$

The Kerr metric coefficients concerned are given by[28]

$$\begin{cases} \varpi = (\Sigma/\rho)\sin\theta, \\ \Sigma^2 \equiv \left(r^2+a^2\right)^2 - a^2\Delta\sin^2\theta, \\ \rho^2 \equiv r^2 + a^2\cos^2\theta, \\ \Delta \equiv r^2 + a^2 - 2Mr. \end{cases} \tag{17}$$

In order to derive expressions for the power and torque of DL process analytically, we assume that the load is axisymmetric and located evenly in a plane $\boldsymbol{P}$ at some height above the disc. Incorporating Eqs.(15)–(17), we derive the expressions for the power and torque of DL process as follows:

$$P_{\rm DL}(a_*,\xi,n)/P_0 = \int_{\xi_S}^{\xi} f(a_*,\xi',n){\rm d}\xi', \quad \xi_S < \xi < \infty, \tag{18}$$

$$T_{\rm DL}(a_*,\xi,n)/T_0 = \int_{\xi_S}^{\xi} g(a_*,\xi',n){\rm d}\xi', \quad \xi_S < \xi < \infty, \tag{19}$$

where ξ_S is determined by criterion (1). The functions $f(a_*,\xi,n)$ and $g(a_*,\xi,n)$ are given by

$$f(a_*,\xi,n) = \frac{(1+q)^2\left(1+a_*^2\chi_{ms}^{-4}\xi^{-2}+2a_*^2\chi_{ms}^{-6}\xi^{-3}\right)\chi_{ms}^4\xi_S\left(\xi/\xi_S\right)^{-2n+3}}{2\left(\xi^{3/2}\chi_{ms}^3+a_*\right)^2\left(1-2\chi_{ms}^{-2}\xi^{-1}+a_*^2\chi_{ms}^{-4}\xi^{-2}\right)}, \tag{20}$$

$$g(a_*,\xi,n) = \frac{(1+q)^2\left(1+a_*^2\chi_{ms}^{-4}\xi^{-2}+2a_*^2\chi_{ms}^{-6}\xi^{-3}\right)\chi_{ms}^4\xi_S\left(\xi/\xi_S\right)^{-2n+3}}{2\left(\xi^{3/2}\chi_{ms}^3+a_*\right)\left(1-2\chi_{ms}^{-2}\xi^{-1}+a_*^2\chi_{ms}^{-4}\xi^{-2}\right)}. \tag{21}$$

Inspecting Eqs.(10)–(19), we find that the powers and torques of the three energy mechanisms are closely related to the boundary angle θ_S. Using Eqs.(10)–(13), we have the relative variation of the powers, as shown in Figs.5 and 6. Henceforth the quantities with superscript "S" are referred to those corresponding to NCMF due to the screw instability.

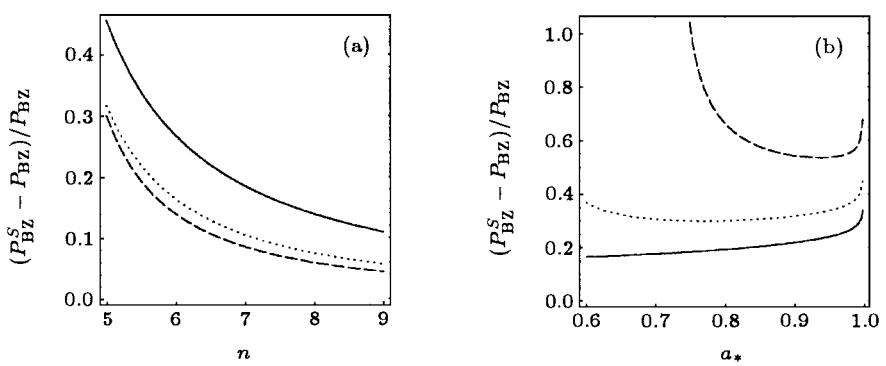


Fig.5. The relative variation of the BZ power, $(P_{\rm BZ}^S - P_{\rm BZ})/P_{\rm BZ}$ (a) with the power-law index n for a_*= 0.8, 0.9 and 0.998 in dashed, dotted and solid lines, respectively; (b) with the BH spin a_* for n = 4.5, 5 and 5.5 in dashed, dotted and solid lines, respectively.

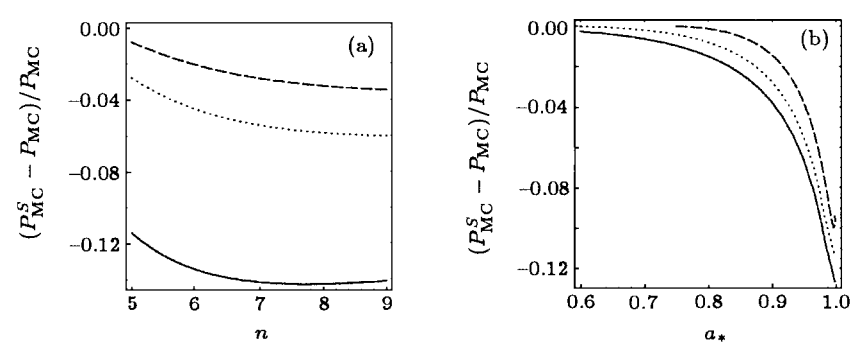


Fig.6. The relative variation of the MC power $(P_{\rm MC}^S - P_{\rm MC})/P_{\rm MC}$ with a_* and n which vary in the same range as in Figs.5(a) and (b).

The total powers of magnetic extraction from the BH-disc system in NCMF and CEBZMC are defined as $P^S_{\rm total}=P^S_{\rm BZ}+P^S_{\rm MC}+P_{\rm DL}$ and $P_{\rm total}=P_{\rm BZ}+P_{\rm MC}$, respectively. Note that the DL process does not exist in the case of CEBZMC. Incorporating Eqs.(10)–(18), we have the curves of $P^S_{\rm total}/P_0$ and $P_{\rm total}/P_0$ for different values of the BH spin and the power-law index, as shown in Fig.7.

The following results are obtained from Figs.5—7.

(1) The BZ power and torque in the NCMF are always greater than the corresponding quantities in CEBZMC, while the MC power and torque in the NCMF are always less than those in CEBZMC.

(2) The total power in the NCMF is always greater than the total power in CEBZMC. This result comes from the fact that: (a) the augment of the BZ power in the NCMF always dominates over the loss of the MC power in the NCMF; (b) the total power is contributed by the DL process in the NCMF.

(3) The total power in the NCMF increases monotonically with increasing BH spin, while the total power in CEBZMC varies nonmonotonically with the BH spin, as shown in Fig.7(b). Both total powers are not sensitive to the variation of the power-law index n, as shown in Fig.7(a).

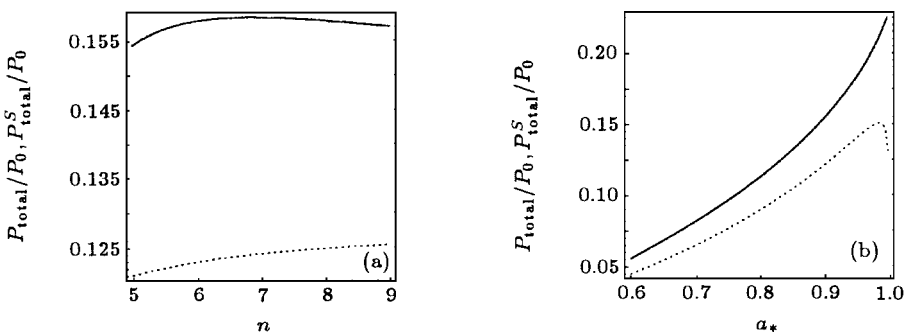


Fig.7. The curves of $P^S_{\rm total}/P_0$ (solid lines) and $P_{\rm total}/P_0$ (dotted lines) versus (a) the power-law index n at $a_*=0.9$ and (b) the BH spin a_* at $n=5$.

3.3. Comparison of efficiency of extracting energy in the NCMF with that in CEBZMC

Now consider the efficiency of extracting energy magnetically from the BH accretion disc for a relativistic quasi-steady thin disc. Based on the conservation of energy in the NCMF, the basic evolution equation of a Kerr BH is written as

$$\mathrm{d}M/\mathrm{d}t=-\left(P^S_{\rm BZ}+P^S_{\rm MC}\right), \tag{22}$$

where a state of suspended accretion is taken into account due to the MC process.[27,29]

In Eq.(22) $\mathrm{d}M/\mathrm{d}t$ is the rate of change of the BH mass-energy, and the powers $P^S_{\rm BZ}$ and $P^S_{\rm MC}$ are the rates of extracting the BH mass-energy in the BZ and MC processes respectively. In Ref.[11], we expressed the dissipated power $\Delta P_{\rm DSP}$ on the BH horizon as

$$\begin{aligned}\Delta P_{\rm DSP}=&I^2\Delta Z_H=\left(\Delta\Psi/2\pi\right)^2\left(\Omega_H-\Omega_F\right)^2\Big/\Delta Z_H\\ =&\Omega_H\Delta T-\Delta P.\end{aligned} \tag{23}$$

Thus the total dissipated power is expressed by

$$P_{\rm DSP}=\left(P_{\rm DSP}\right)_{\rm BZ}+\left(P_{\rm DSP}\right)_{\rm MC}, \tag{24}$$

where

$$\begin{aligned}\left(P_{\rm DSP}\right)_{\rm BZ}=&\Omega_H T^S_{\rm BZ}-P^S_{\rm BZ}\\ =&2a_*^2P_0\int_0^{\theta_S}\frac{(1-k)^2\sin^3\theta\mathrm{d}\theta}{2-(1-q)\sin^2\theta},\end{aligned} \tag{25}$$

$$\begin{aligned}\left(P_{\rm DSP}\right)_{\rm MC}=&\Omega_H T^S_{\rm MC}-P^S_{\rm MC}\\ =&2a_*^2P_0\int_{\theta_S}^{\theta_L}\frac{(1-\beta)^2\sin^3\theta\mathrm{d}\theta}{2-(1-q)\sin^2\theta}.\end{aligned} \tag{26}$$

So the efficiencies in the BZ and MC processes can be expressed respectively by

$$\eta_{\rm BZ}=\frac{P^S_{\rm BZ}}{P^S_{\rm BZ}+\left(P_{\rm DSP}\right)_{\rm BZ}}=\frac{P^S_{\rm BZ}}{\Omega_H T^S_{\rm BZ}}, \tag{27}$$

$$\eta_{\rm MC}=\frac{P^S_{\rm MC}}{P^S_{\rm MC}+\left(P_{\rm DSP}\right)_{\rm MC}}=\frac{P^S_{\rm MC}}{\Omega_H T^S_{\rm MC}}. \tag{28}$$

Incorporating Eqs.(25) and (26), we express the total efficiency by

$$\begin{aligned}\eta^S_{\rm total}=&\frac{P^S_{\rm BZ}+P^S_{\rm MC}}{P^S_{\rm BZ}+\left(P_{\rm DSP}\right)_{\rm BZ}+P^S_{\rm MC}+\left(P_{\rm DSP}\right)_{\rm MC}}\\ =&\frac{P^S_{\rm BZ}+P^S_{\rm MC}}{\Omega_H\left(T^S_{\rm BZ}+T^S_{\rm MC}\right)}.\end{aligned} \tag{29}$$

It is easy to gain the expression for total efficiency $\eta_{\rm total}$ in CEBZMC as follows:

$$\eta_{\rm total} = \frac{P_{\rm BZ} + P_{\rm MC}}{\Omega_H \left(T_{\rm BZ} + T_{\rm MC}\right)}. \tag{30}$$

It is noticed that the total efficiency $\eta^S_{\rm total}$ is not equal to the sum of $\eta_{\rm BZ}$ and $\eta_{\rm MC}$.

Incorporating Eqs.(10)–(13), (29) and (30), we have the total efficiency versus the BH spin and the power-law index, as shown in Fig.8.

From Fig.8, we conclude that the total efficiency $\eta^S_{\rm total}$ in the NCMF is always greater than the total efficiency $\eta_{\rm total}$ in CEBZMC for a wide range of the BH spin and the power-law index.

Why are the total power and efficiency in the NCMF always greater than those in CEBZMC? The reason lies in the variation of the boundary angle. When the screw instability occurs, the boundary angle has the following change $\theta_M \to \theta_S > \theta_M$. Thus, the total power and efficiency both increase. The ultimate reason for the result is the two assumptions: the magnetic field on the horizon is constant, but on the disc surface varies as a power law with the radial coordinate of the disc in our model.[11−12]

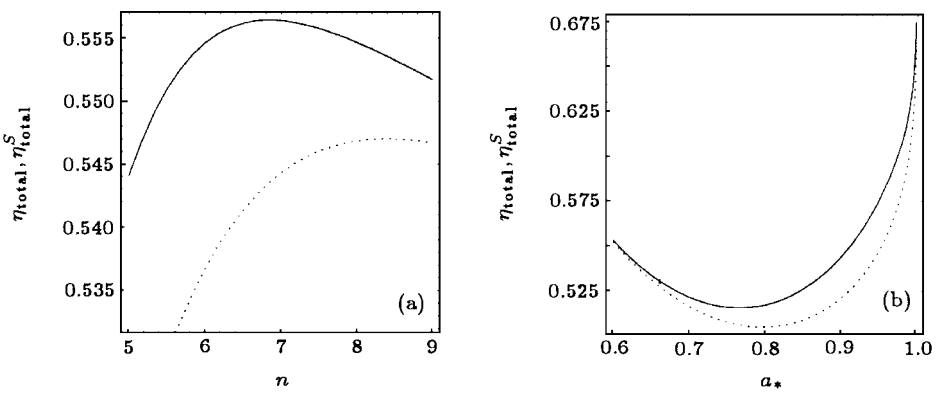


Fig.8. The total efficiency $\eta_{\rm total}$ (dotted lines) and $\eta^S_{\rm total}$(solid lines) (a) versus the power-law index n with $a_* = 0.9$; (b) versus the BH spin a_* with $n = 5$.

4. NCMF and the emissivity index

Radiation from a relativistic quasi-steady thin disc around a Kerr BH has been discussed by some authors, based on the three conservation laws of mass, energy and angular momentum.[25,30] By combining the MC effects with the above conservation laws and the "no-torque boundary condition", the radiation from a relativistic quasi-steady thin disc is derived:[10]

$$F^{\rm total}_{\rm MC} = F^I_{\rm DA} + F_{\rm MC}, \tag{31}$$

where $F^I_{\rm DA}$ and $F_{\rm MC}$ are the radiation fluxes due to disc accretion and the MC effects, respectively. Considering the state of suspended accretion due to the MC effects,[29] we have

$$F^{\rm total}_{\rm MC} = F_{\rm MC} = \frac{(-\partial\Omega_D/\partial r)}{r\left(E^\dagger - \Omega_D L^\dagger\right)^2} \times \int_{r_S}^{r} \left(E^\dagger - \Omega_D L^\dagger\right) r H_{\rm MC} {\rm d}r. \tag{32}$$

In Eq.(32) $H_{\rm MC}$ is the flux of angular momentum transferred between the BH and the disc by the magnetic field. In Ref.[12] the flux of angular momentum $H_{\rm MC}$ is expressed by

$$H_{\rm MC}/H_0 = A\left(a_*, \xi\right)\xi^{-n}, \quad 1 < \xi < \xi_S, \tag{33}$$

where

$$H_0 = \left(B^{\rm p}_H\right)^2 M = 1.48 \times 10^7 \times B^2_4 \left(M/M_\odot\right) {\rm T}\cdot{\rm s}^{-2}, \tag{34}$$

$$A(a_*, \xi) = \frac{a_*(1-\beta)(1+q)}{2\pi\chi^2_{ms}[2\csc^2\theta - (1-q)]} F_H(a_*, \xi),$$
$$F_H(a_*, \xi) = \frac{\sqrt{1 + a^2_*\chi^{-4}_{ms}\xi^{-2} + 2a^2_*\chi^{-6}_{ms}\xi^{-3}}}{\sqrt{(1 + a^2_*\chi^{-4}_{ms} + 2a^2_*\chi^{-6}_{ms})(1 - 2\chi^{-2}_{ms}\xi^{-1} + a^2_*\chi^{-4}_{ms}\xi^{-2}}}. \tag{35}$$

Eq.(33) is derived using the expression for $T_{\rm MC}$ expressed by Eq.(13). In Eq.(32) $E^{\dagger}$ and $L^{\dagger}$ are the specific energy and angular momentum of a particle in the disc, respectively, and expressed as[25]

$$E^{\dagger}=\frac{1-2\chi^{-2}+a_{*}\chi^{-3}}{(1-3\chi^{-2}+2a_{*}\chi^{-3})^{1/2}}, \tag{36}$$

$$L^{\dagger}=\frac{M\chi(1-2a_{*}\chi^{-3}+a_{*}^{2}\chi^{-4})}{(1-3\chi^{-2}+2a_{*}\chi^{-3})^{1/2}}. \tag{37}$$

The magnetic coupling between a BH and a disc can produce a very steep emissivity with index $\alpha=4.3-5.0$,[12−14] which is consistent with the recent *XMM-Newton* observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15. However, such a steep emissivity is very difficult to explain by a standard accretion. The emissivity index is defined as

$$\alpha\equiv-\mathrm{d}\ln F/\mathrm{d}\ln r, \tag{38}$$

which mimics locally the relation between the radiation flux and the disc radius, i.e. $F\propto r^{-\alpha}$. So the steep emissivity implies that the radiation flux F decreases very fast with increasing radius r.

In Ref.[12], we discussed the emissivity index in CEBZMC. Using Eqs.(32) and (38), we obtain the curves of the emissivity index α versus $\lg(r/M)$ in NCMF and in CEBZMC, as shown in Fig.9.

Inspecting Fig.9, we find that the emissivity indices in both NCMF and CEBZMC vary very steeply with $\lg(r/M)$. It is shown by the curves in the shaded region that the emissivity index α produced in NCMF is well fitted with the observations as well as that produced in CEBZMC, provided that the BH spin and the power-law index are high enough. The difference between the two cases lies in that the emissivity index in NCMF only emerges at a smaller range compared with that in CEBZMC. This result arises from the effects of the screw instability.

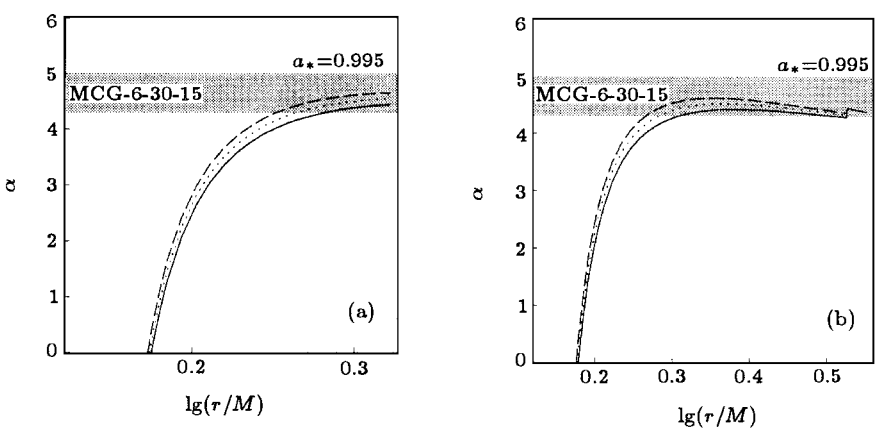


Fig.9. The emissivity index α versus $\lg(r/M)$ for $a_{*}=0.995$ and $n=6.5$, 7, 7.5 in solid, dotted and dashed lines, respectively. (a) The emissivity index in NCMF; (b) the emissivity index in CEBZMC. The emissivity index of the Seyfert 1 galaxy MCG-6-30-15 inferred from the observation of *XMM-Newton* is shown by the shaded region.

5. Summary and discussion

In this paper we have discussed the magnetic field configuration restricted by the screw instability in the BH magnetosphere, and the expressions for the BZ, MC and DL powers of extracting energy from the BH-disc system are derived by using two kinds of equivalent circuits. Our calculations show that the total power and efficiency of extracting energy in NCMF are always greater than those in CEBZMC. These results arise from the new magnetic field configuration itself, which is based on the two assumptions: (1) the magnetic field on the horizon is constant; (2) the magnetic field on the disc surface varies in a power law with the radial coordinate of the disc. Very recently, the assumption (1) is supported by solving the Grad–Shafranov equation.[31] In addition, we find that a very steep emissivity can be produced in NCMF as well as in CEBZMC. In both cases the magnetic coupling of a rapidly rotating BH with its surrounding disc is consistent with the recent *XMM-Newton* observation of the nearby bright Seyfert 1 galaxy MCG-6-30-15. As is well known, an ordered poloidal magnetic field threading a rotating BH and its surrounding accretion disc can produce powerful outflows or jets. Recently, there has been increasing interest in the possibility

that a system of a stellar rotating BH-magnetized accretion disc can be the central engine for powering gamma-ray bursts (GRBs).[6,7] We shall discuss these high energy radiations from the BH-disc system with NCMF in our future work.

References

[1] Blandford R D and Znajek R L 1977 *Mon. Not. R. Astron. Soc.* **179** 433

[2] Rees M J, Begelman M C, Blandford R D and Phinney E S 1982 *Nature* **295** 17

[3] Begelman M C, Blandford R D and Rees M J 1984 *Rev. Mod. Phys.* **56** 255

[4] Rees M J 1984 *Ann. Rev. Astron. Astrophys.* **22** 471

[5] Ferrari, A. 1998 *Ann. Rev. Astron. Astrophys.* **36** 539

[6] Lee H K, Wijers R A M J and Brown G E, 2000 *Phys. Rep.* **325** 83

[7] Wang D X, Lei W H and Xiao K 2002 *Astrophys. J.* **580** 358

[8] Blandford R D 1999 *Astrophysical Discs* ed Sellwood J A and Goodman J (Astronomical Society of the Pacific Conference Series) **160** 265 (astro-ph/9902001)

[9] Li L X and Paczynski B 2000 *Astrophys. J.* **534** L197 (astro-ph/0003187)

[10] Li L X 2002 *Astrophys. J.* **567** 463

[11] Wang D X, Xiao K and Lei W H 2002 *Mon. Not. R. Astron. Soc.* **335** 655

[12] Wang D X, Lei W H and Ma R Y 2003 *Mon. Not. R. Astron. Soc.* **342** 851

[13] Li L X 2002 *Astron. Astrophys.* **392** 469

[14] Wilms J et al 2001 *Mon. Not. R. Astron. Soc.* **328** L27

[15] Wang D X, Ma R Y, Lei W H and Yao G Z 2003 *Astrophys. J.* **595** 109

[16] Wang D X, Lei W H and Ma R Y 2004 *Astrophys. J.* **601** 1031

[17] Macdonald D and Thorne K S 1982 *Mon. Not. R. Astron. Soc.* **198** 345

[18] Kadomtsev B B 1966 *Rev. Plasma Phys.* **2** 153

[19] Freidberg J P 1987 Ideal Magnetohydrodynamics (New York: Plenum Press)

[20] Bateman G 1978 *MHD Instabilities* (Cambridge: The MIT Press)

[21] Gruzinov A 1999 astro-ph/9908101

[22] Li L X 2000 *Astrophys. J.* **531** L111

[23] Tomimatse A, Matsuoka T and Takahashi M 2001 *Phys. Rev.* D **64** 123003

[24] Wang D X 1999 *Acta Phys. Sin.* **48** 1552 (in Chinese)

[25] Novikov I D and Thorne K S 1973 *Black Holes*, ed. Dewitt C (New York: Gordon and Breach) p345

[26] Blandford R D 1976 *Mon. Not. R. Astron. Soc.* **176** 465

[27] Wang D X, Ye Y C and Gong X L 2004 *New Astron.* **9** 585

[28] Thorne K S, Price R H and Macdonald D A 1986 *Black Holes: The Membrane Paradigm.* (New Haven, CT: Yale University Press) chap 2 p36

[29] van Putten M H P M and Levison A 2003 *Astrophys. J.* **584** 937

[30] Page D N and Thorne K S 1974 *Astrophys. J.* **191** 499

[31] Uzdensky D A 2004 *Astrophys. J.* **603** 652