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Effects of bias on dynamics of an AC-driven two-electron quantum-dot molecule*

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The effects of bias on the dynamical localization of two interacting electrons in a pair of coupled quantum dots driven by external AC fields have been numerically investigated. With an effective two-site model and Floquet formalism, the time-dependent Schrödinger equation is numerically solved and the $P_{\min}$, the minimum of the population evolution of the initial state within a certain time period, is used to quantify the degree of the dynamical localization. Results indicate that the bias can change the energy of the initial state and break the dynamical symmetry of the system with a pure AC field. And the amplitude of the AC field with dynamical localization phenomenon changes with bias. All the numerical results are explained by the perturbation theory and two-level approximation.

Keywords: dynamical localization, quantum dots, quasi-energy, Floquet state
PACC: 7320D, 7335C, 2320

1. Introduction

Quantum state engineering via optical or electrical manipulation over the coherent dynamics of suitable quantum mechanical systems has become a fascinating prospect of modern physics. The pioneering work[1] first noted the dramatic effects on the tunnelling rate arising from crossings and avoided crossings of the Floquet quasi-energies, and the term dynamical localization (DL) was coined to describe the phenomenon in which an AC field appeared to trap the electron within just one well over long time scales. Subsequent analytical and numerical work[2,3] showed that at high frequencies this localization occurred when the ratio of the field strength to the frequency was a root of the Bessel function J_0.

Owning to the fact that a quantum dot (QD) system is suitable in experiment for verifying quantum phenomena, the dynamical properties of two coupled dots containing a single electron or two interacting electrons driven by an AC field have been extensively studied.[4−8] The technique of Floquet analysis[9] has proved to be effective particularly in the study. Similar to the single electron in a double QD, the two-electron quantum-dot molecule (QDM) also shows the DL phenomenon under the interaction with an external AC field, i.e. the two electrons are trapped within just one QD over long time scales.[6,10,11]

In a real experiment,[12] the DC voltage across the source and drain is biased to the quantum-dot molecule. So the effect of the bias on the dynamic localization in a QDM is investigated in this paper. We numerically investigate the dynamics of the two interacting electrons in a closed double quantum dot (DQD) biased by a DC–AC voltage, and find that the DL phenomenon is realized under the certain combination of the amplitudes of the DC and AC fields. Some results are in agreement with those obtained by the perturbation analysis[6] and the two-level approximation.[1,13,14]

2. Theoretical model and methods

We consider a system with two electrons in a double-well potential, a model for the two-electron QDM under the action of a DC–AC external electric field (see Fig.1).

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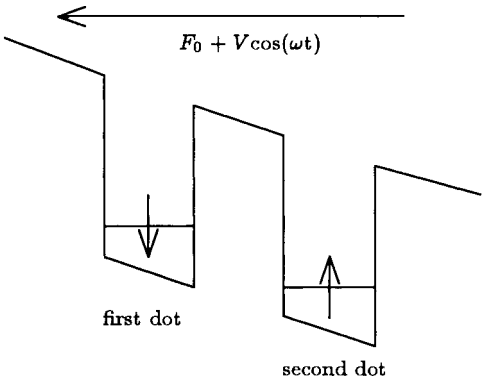


Fig.1. Scheme of the two-electron configuration in a pair of coupled quantum dots biased by a DC–AC electric field.

If the frequency of the external AC field is too low to excite electrons to higher levels, we assume that only one energy level is considered in each QD, i.e. each QD can be replaced by a single site. With the tunnelling between QDs and the interaction between the electrons taken into account, the Hamiltonian of the system in a standard Hubbard approach is[5,11]

$$\begin{aligned} H(t) = & \sum_{\sigma,i=1,2} \varepsilon_i(t) d_{i\sigma}^{+} d_{i\sigma} \\ & + W \sum_{\sigma} \left(d_{1\sigma}^{+} d_{2\sigma} + d_{2\sigma}^{+} d_{1\sigma} \right) \\ & + U_1 \sum_{i} n_{i\uparrow} n_{i\downarrow} + U_2 \sum_{\sigma_1,\sigma_2} n_{1\sigma_1} n_{2\sigma_2}, \end{aligned} \quad (1)$$

where $d_{i\sigma}^{+}\,(d_{i\sigma})$ creates (destroys) an electron with spin σ on the ith QD. W describes the tunnelling between the two QDs, and U_1 and U_2 are the on-site and inter-site Coulomb interactions, respectively. When the quantum-dot molecule is biased by an DC–AC voltage, its energy level in the ith QD becomes $\varepsilon_i(t) = (-1)^{i+1}\left(F_0 + V\cos(\omega t)\right)/2$, where F_0 and V are the amplitudes of the DC and AC voltages respectively, and ω is the driving frequency, which is related to the corresponding period by $T = 2\pi/\omega$.

As there are two electrons in the DQD, the above Hamiltonian (1) can be expanded into

$$H(t) = \begin{bmatrix} U_2 I_{3\times 3} & 0 \\ 0 & H_1'(t) \end{bmatrix}, \quad (2)$$

where $I_{3\times 3}$ is a 3×3 unit matrix and $H_1'(t)$ is given by

$$H_1'(t) = \begin{bmatrix} 2\varepsilon_1(t) + U_1 & \sqrt{2}W & 0 \\ \sqrt{2}W & U_2 & \sqrt{2}W \\ 0 & \sqrt{2}W & 2\varepsilon_2(t) + U_1 \end{bmatrix}, \quad (3)$$

in the two-particle real space with basis-vectors $|1,1\rangle$, $|\bar{1},\bar{1}\rangle$, $\left(|1,\bar{1}\rangle - |\bar{1},1\rangle\right)/\sqrt{2}$, $|2,0\rangle$, $|0,2\rangle$, $\left(|1,\bar{1}\rangle + |\bar{1},1\rangle\right)/\sqrt{2}$. Here $|m,n\rangle$ denotes the state where m electrons are in the first QD and n electrons in the second QD; the values 1 and $\bar{1}$ refer to the up-spin and down-spin, respectively. The six basis-vectors consist of three triplet states and three singlet-states, and there is no mixing between the two subspaces due to the absence of spin-flip terms in Eq.(1). Because the triplet-states are trivial ones and constant occupation number is 1 in each dot due to the Pauli principle, we focus our attention on the reduced spin-singlet Hamiltonian $H_1(t)$ with a constant energy-shift U_2, i.e.

$$H_1(t) = \begin{bmatrix} 2\varepsilon_1(t) + \kappa & \sqrt{2}W & 0 \\ \sqrt{2}W & 0 & \sqrt{2}W \\ 0 & \sqrt{2}W & 2\varepsilon_2(t) + \kappa \end{bmatrix}, \quad (4)$$

where $\kappa = U_1 - U_2$.

The symmetry $H_1(t) = H_1(t + nT)$ under the discrete time translation $t \to t + nT$ enables the use of the Floquet formalism.[9] So the solution to the time-dependent Schrödinger equation ($\hbar = 1$)

$$\mathrm{i}\frac{\partial}{\partial t}\psi(t) = H_1(t)\,\psi(t) \quad (5)$$

may be written as

$$\psi(t) = \sum_{\alpha} c_\alpha \exp[-\mathrm{i}\varepsilon_\alpha t]\,|u_\alpha(t)\rangle, \quad (6)$$

where $|u_\alpha(t)\rangle$ is a function with the same periodicity as that of the external AC field and is called a Floquet state, and ε_α is termed the quasi-energy. The Floquet state $|u_\alpha(t)\rangle$ can be obtained from the Floquet equation

$$\left(H_1(t) - \mathrm{i}\frac{\partial}{\partial t}\right)|u_\alpha(t)\rangle = \varepsilon_\alpha\,|u_\alpha(t)\rangle, \quad (7)$$

in which $\{\varepsilon_\alpha\}$ and $\{|u_\alpha(0)\rangle\}$ are obtained from diagonalizing $U(T,0)$. And $U(T,0)$ comes from the integration of the motion equation for the time evolution operator $U(t,0)$ from zero to T,

$$\mathrm{i}\frac{\partial}{\partial t}U(t,0) = H_1(t)\,U(t,0). \quad (8)$$

The coefficient c_α in Eq.(6) is obtained by use of the initial state $|\psi(0)\rangle$ of the system

$$c_\alpha = \langle u_\alpha(0)|\,\psi(0)\rangle. \quad (9)$$

So the time evolution of the population $P(t)$ of the initial state $|\psi(0)\rangle$ is

$$\begin{aligned} P(t) = & |\langle\psi(0)|\,\psi(t)|^2 \\ = & \left|\sum_{\alpha,\beta} c_\alpha^{*} c_\beta \exp[-\mathrm{i}\varepsilon_\beta t]\,\langle u_\alpha(0)|\,u_\beta(t)\rangle\right|^2. \end{aligned} \quad (10)$$

Note that the population is related to the initial state, quasi-energy ε_β and its Floquet state $|u_\beta(t)\rangle$. If the quasi-energies $\{\varepsilon_\alpha\}$ are confined to the first Brillouin zone, another index l in $\{\varepsilon_{\alpha,l}\}$ counts the number of photons that have to be subtracted from the eigen-energy level $\{E_\alpha\}$ in order to reach the first Brillouin zone when the driving amplitude of AC voltage is switched off adiabatically, i.e. $V \to 0$. The Floquet states have the following relation[9] with the stationary eigenstates $\{\varphi_\alpha\}$ [11,15]

$$|u_{\alpha,l}(t)\rangle \to |u^0_{\alpha,l}(t)\rangle = \varphi_\alpha \exp(il\omega t). \qquad (11)$$

Under some extreme conditions, the above three-level system reduces to an effective two-level question, which has been solved analytically.[7,14] But the three-level system is generally difficult to deal with; we numerically carry out the above process in the following.

3. Numerical results and discussion

In the following, the initial state of the system is chosen to be $|\psi(0)\rangle = |0,2\rangle$, i.e. both electrons are initially in the second dot (denoted as $|SS\rangle$). The evolution state $|\psi(t)\rangle$ is formed as

$$|\psi(t)\rangle = C_{FF}(t)|FF\rangle + C_{FS}(t)|FS\rangle + C_{SS}(t)|SS\rangle, \qquad (12)$$

in real space with basis-vectors $|2,0\rangle$ (denoted as $|FF\rangle$), $(|1,\bar{1}\rangle + |\bar{1},1\rangle)/\sqrt{2}$ (denoted as $|FS\rangle$), and $|0,2\rangle$ (denoted as $|SS\rangle$). So the time-dependent Schrödinger equation (4) yields a first-order differential equation for the expansion coefficients,

$$\mathrm{i}\frac{\partial}{\partial t}\begin{pmatrix} C_{FF}(t) \\ C_{FS}(t) \\ C_{SS}(t) \end{pmatrix} = H_1(t)\begin{pmatrix} C_{FF}(t) \\ C_{FS}(t) \\ C_{SS}(t) \end{pmatrix}. \qquad (13)$$

This differential equation can be solved by use of the Runge–Kutta method with the initial condition

$$\begin{pmatrix} C_{FF}(0) \\ C_{FS}(0) \\ C_{SS}(0)) \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}. \qquad (14)$$

So the time evolution of the population $P(t)$ of the initial state $|SS\rangle$ is

$$P(t) = |\langle SS|\psi(t)|^2 = |C_{SS}(t)|^2. \qquad (15)$$

In order to investigate the effects of the bias on the dynamical localization of the driven system, we use $P_{\min}$, the minimum of the population $P(t)$ within a longer enough period than T (such as $133T$), to search for the DL phenomenon. Large $P_{\min}$ means the occurrence of the DL phenomenon.

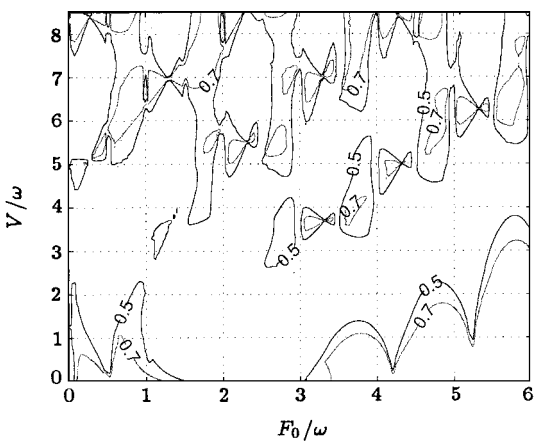


Fig.2. Contour of $P_{\min}$ as a function of the amplitudes of the DC–AC fields with the parameters $\kappa = 2.3\omega$ and $W = 0.25\omega$.[16].

In Fig.2, we present a contour of $P_{\min}$ as a function of the amplitudes of the DC–AC field with parameters $W = 0.25\omega$ and $\kappa = 2.3\omega$.[16] The areas correspond to $P_{\min} > 0.5$, indicating a high probability in which both electrons occupy the second QD over long time scales. Surprisingly, this contour can occur in many other areas besides some discrete points. We note that dynamical localization can be produced over a wide range of field. In the following, we analyse such effects of the bias on the dynamics of the electrons in four aspects.

3.1. Bias-changed the energy of initial state

A clear physical explanation for the islands in a region of small V in Fig.2 is made in the following two steps.

First, if there exists no AC field in the system, the problem may be simplified to the one about eigenvalues,[15]

$$\begin{bmatrix} F_0+\kappa-E & \sqrt{2}W & 0 \\ \sqrt{2}W & -E & \sqrt{2}W \\ 0 & \sqrt{2}W & -F_0+\kappa-E \end{bmatrix}\begin{bmatrix} C_{FF}(0) \\ C_{FS}(0) \\ C_{SS}(0) \end{bmatrix} = 0. \qquad (16)$$

With its eigenvalues $\{E_i\}$ and eigenstates $\{C^i_\alpha(0)\}$, where α denotes FF, FS, and SS respectively, the population of the initial state is rewritten as

$$P_{SS} = |C_{SS}(t)|^2 = \left| \sum_i C^{i*}_{SS}(0) C^i_{SS}(0) \exp(-\mathrm{i}E_i t) \right|^2 . \tag{17}$$

Figures 3(a) and (b) show the variations of eigenvalues and minimum of the P_{SS} with bias. Note that $P_{\min}$ is very small when the eigenvalues approach near to $F_0 = 0, \kappa$. The reason is that the initial state $|SS\rangle$ undergoes resonant tunnellings and Rabi oscillations in time respectively with states $|FF\rangle$ (see Fig.3(c)) and $|FS\rangle$ (see Fig.3(e)) at the two bias values.[11] Away from the two values, the increase of the bias enhances the energy difference and the occupation probability in the second QD. So the $P_{\min}$ is large when the bias is away from the two values.

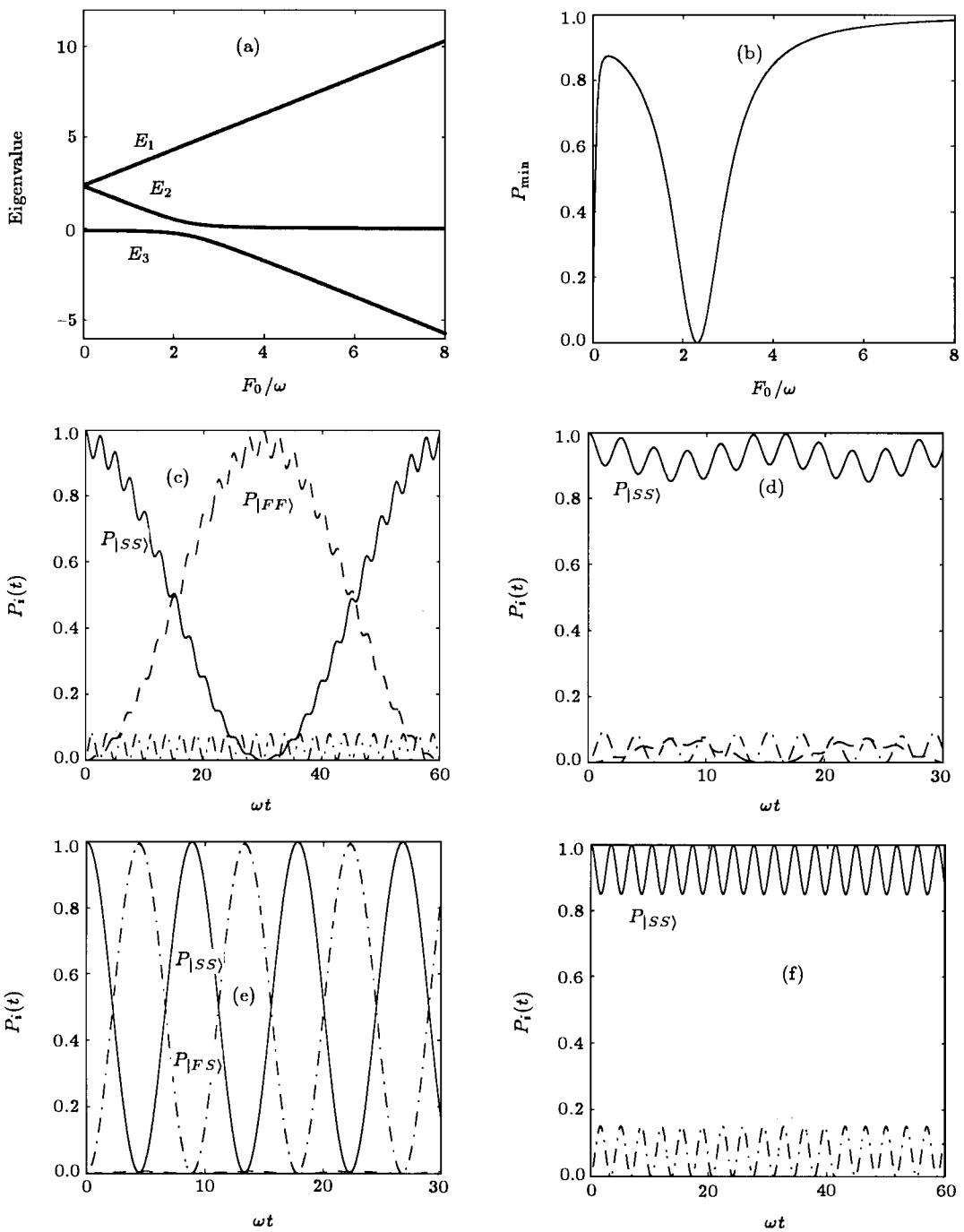


Fig.3. The system with pure DC field (i.e. $V = 0$) with parameters $\kappa = 2.3\omega$, $W = 0.25\omega$: (a) eigenvalues as a function of the DC field strength; (b) the minimum $P_{\min}$ of the population of the initial state as a function of the DC field strength; (c),(d),(e), and (f) are the time evolution populations of the states $|SS\rangle$ (solid curve), $|FF\rangle$ (dashed curve), and $|FS\rangle$ (dot-dashed curve) respectively with $F_0 = 0$, $F_0 = 0.2\omega$, $F_0 = \kappa = 2.3\omega$, and $F_0 = 4\omega$.

Second, if the AC field strength is not equal to zero, the bias changes the quasi-energy and structure of the Floquet state. Figure 4 illustrates the variation of the quasi-energy, the structure of Floquet state, and $P_{\min}$ of the system as a function of the DC field strength with $V = 1.2445\omega$ in Fig.2. Note that the variation of quasi-energy in Fig.4(a) possesses the trends similar to that of the eigen-energy in Fig.3(a) except the avoided crossings. At the avoided crossings, the values of $P_{\min}$ in Fig.4(b) are small, just like those in Fig.3(b). The $P_{\min}$ at $F_0 = 0$ in Fig.4(b) is large because of the exact crossing of the quasi-energies at $F_0 = 0$ in Fig.4(a). Similarly, the small values of $P_{\min}$ near $F_0 \approx 0.5, 1, 1.5$ are due to the avoided crossings of the quasi-energies in Fig.4(a).

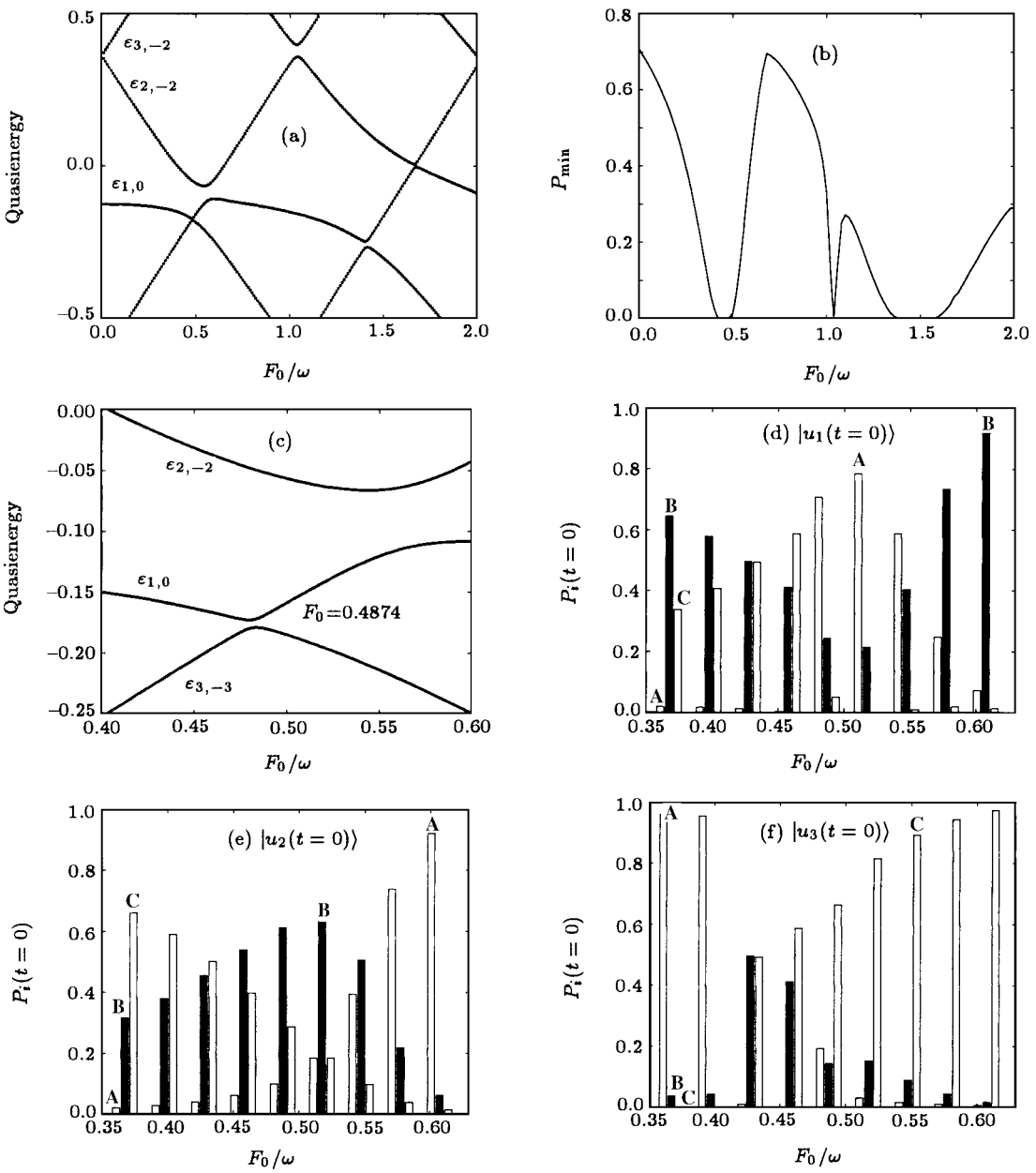


Fig.4. The system with parameters $\kappa = 2.3\omega$ and $W = 0.25\omega$ under an AC field of $V = 1.2445\omega$: (a) the quasi-energy spectrum as a function of DC field amplitude F_0; (b) the minimum $P_{\min}$ of the population of the initial state as a function of the DC field strength; (c) the enlargement of the avoided crossing point $F_0 = 0.4874\omega$; (d), (e), and (f) are the structures (occupation probabilities) of the Floquet states $|u_{1,0}(0)\rangle$, $|u_{2,-2}(0)\rangle$, and $|u_{3,-3}(0)\rangle$ in the two-particle states A, B and C (see text) for different values of the amplitude F_0 shown in (c).

At the avoided crossing, the change of the Floquet state structure occurs.[16] The enlargement of the avoided crossing at $F_0 = 0.4874\omega$ is given in Fig.4(c), and the changes of the Floquet state structure near this avoided crossing are given in Figs.4(d)–(f), which present the occupation probabilities of the projections of all three Floquet states in the two-electron states $|FF\rangle$ (denoted by A), $|FS\rangle$ (denoted by B), and $|SS\rangle$ (denoted by C) for nine values of F_0 near the avoided crossing point $F_0 = 0.4874\omega$. Note that, with the increasing of the DC field strength F_0 to the avoided crossing, the Floquet state structure of $|u_3(0)\rangle$ changes from prominent-point A to prominent-point C (from $|FF\rangle$ to $|SS\rangle$), and the corresponding quasi-energy $\epsilon_{3,-3}$ changes from increasingly to decreasingly. The Floquet states $|u_2(0)\rangle$ and $|u_1(0)\rangle$ change their structures twice at the two avoided crossings. So the initial localized state $|SS\rangle$ undergoes resonant tunnelling and Rabi oscillation at the avoided crossing just as in the case of the above DC system, and the values of $P_{\min}$ are small.

According to the above analysis, we know that the bias changes the energy of the initial state $|SS\rangle$, which results in the initial state undergoing Rabi oscillation with $|FF\rangle$ state or $|FS\rangle$ state. Such effects of the bias may apply to the changes of the three states with a rectangle voltage pulse.

3.2. Bias breaking the dynamical symmetry of the system with a pure AC field

Similarly, the islands in a region of small F_0 may be physically explained in the following.

First, the Hamiltonian with a pure AC field ($F_0 = 0$ in Eq.(4)) is invariant under the generalized parity operation $x \to -x$, $t \to T/2$.[11] A consequence is that the Floquet states also possess this symmetry, one being even and the other being odd. The two different parity quasi-energies can cross as the AC field strength V is varied (see Fig.5(a)) according to the von Neumann–Wigner theorem.[17] At an exact crossing point, the DL phenomenon occurs (see Fig.5(b)). So the degeneracy of quasi-energies can be used to check the occurrence of the DL phenomenon in the pure AC system.

However, the bias ($F_0 \neq 0$) breaks the above generalized parity, thereby the exact crossings change into avoided crossings (see Fig.5(c)). At these avoided crossings, the time evolution of the population $P(t)$ (see Fig.5(d)) shows that the DL phenomenon still occurs due to the mismatch of the energies.[18] So it fails to use the exact crossings of quasi-energies to search for the DL phenomenon in the DC–AC field system, which is the reason for the use of $P_{\min}$ in this paper. The enlargements of the islands with small bias are shown in Figs.5(e) and (f)), which will be discussed further in the next part.

Another result of such a symmetry breakdown is that the exact crossing and avoided crossing occur simultaneously between two quasi-energies, which is illustrated by the system with $F_0 = 3.33\omega$ in Fig.6. Figure 6(a) shows the quasi-energy spectrum of the system, in which there is an exact crossing at $V = 3.6939\omega$ and an avoided crossing at $V = 6.9895\omega$. Their time evolution populations of the initial state $|SS\rangle$ are respectively shown in Figs.6(b) and (c), which show the occurrence of the DL phenomenon. According to Fig.6(c), the quantum beats occur at the avoided crossing.[16] The variation of $P_{\min}$ near the exact crossing in Fig.6(a) is given in Fig.6(d), which will be discussed in the next section.

3.3. Bias-changed AC field strength with DL phenomenon

In order to give a clear physical explanation for the island shapes in Figs.5(f) and 6(d), we treat the Hamiltonian (4) with a form of perturbation theory introduced by Holthaus[2] in the following.

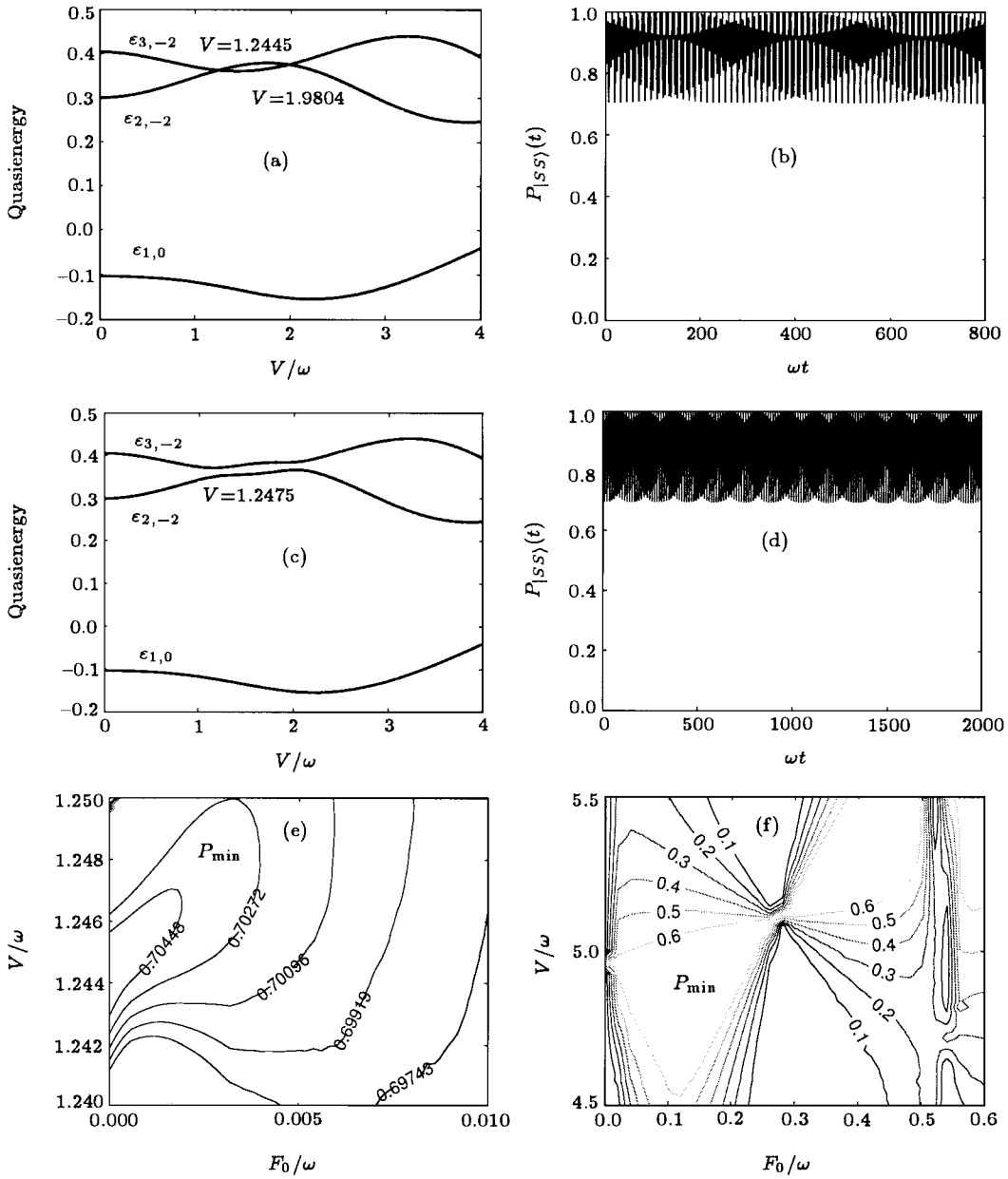


Fig.5. The Floquet spectra as a function of the amplitude of the AC electric field are shown respectively in (a) with $F_0 = 0$ and in (c) with $F_0 = 0.01\omega$. (b) and (d) are the time evolution of the probability $P_{SS}(t)$ respectively for the first crossing in (a) and for the avoided crossing in (c). (e) and (f) are the enlargements of the $P_{\min}$ islands near $V = 1.2445\omega, 4.9\omega$ as a function of amplitude of the DC–AC field with a small DC field strength.

If all tunnelling components in Eq.(4) act as the perturbation, the Hamiltonian is divided into two parts, i.e.

$$H_1 = H_a + H_t = \begin{pmatrix} F_0 + V\cos(\omega t) + \kappa & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -F_0 - V\cos(\omega t) + \kappa \end{pmatrix} + \begin{pmatrix} 0 & \sqrt{2}W & 0 \\ \sqrt{2}W & 0 & \sqrt{2}W \\ 0 & \sqrt{2}W & 0 \end{pmatrix}, \quad (18)$$

where H_a contains all the interaction terms and is solved exactly. The interaction term H_a is diagonal. And it is straightforward to obtain an orthogonal set of eigenvectors of Eq.(7)

$$\begin{cases} |u_1(t)\rangle = \left(\exp\left[-\mathrm{i}t(\kappa + F_0 - \varepsilon_1) - \mathrm{i}\dfrac{V}{\omega}\sin(\omega t)\right], 0, 0\right), \\ |u_2(t)\rangle = (0, \exp[\mathrm{i}t\varepsilon_2], 0), \\ |u_3(t)\rangle = \left(0, 0, \exp\left[-\mathrm{i}t(\kappa - F_0 - \varepsilon_3) + \mathrm{i}\dfrac{V}{\omega}\sin(\omega t)\right]\right). \end{cases} \tag{19}$$

Imposing T-periodic boundary conditions $u_\alpha(t+T) = u_\alpha(t)$ reveals the corresponding eigenvalues to be $\varepsilon_1 = \kappa + F_0$, $\varepsilon_2 = 0$, $\varepsilon_3 = \kappa - F_0$. So the solutions to H_a are

$$\begin{cases} \psi_1(t) = \left(\exp\left[-\mathrm{i}t(\kappa + F_0) - \mathrm{i}\dfrac{V}{\omega}\sin(\omega t)\right], 0, 0\right), \\ \psi_2(t) = (0, 1, 0), \\ \psi_3(t) = \left(0, 0, \exp\left[-\mathrm{i}t(\kappa - F_0) + \mathrm{i}\dfrac{V}{\omega}\sin(\omega t)\right]\right). \end{cases} \tag{20}$$

These states are stationary states in the real space. So the initial localized state $\psi_3(0) = |SS\rangle$ remains unchanged in time evolution if the perturbation H_t is not considered.

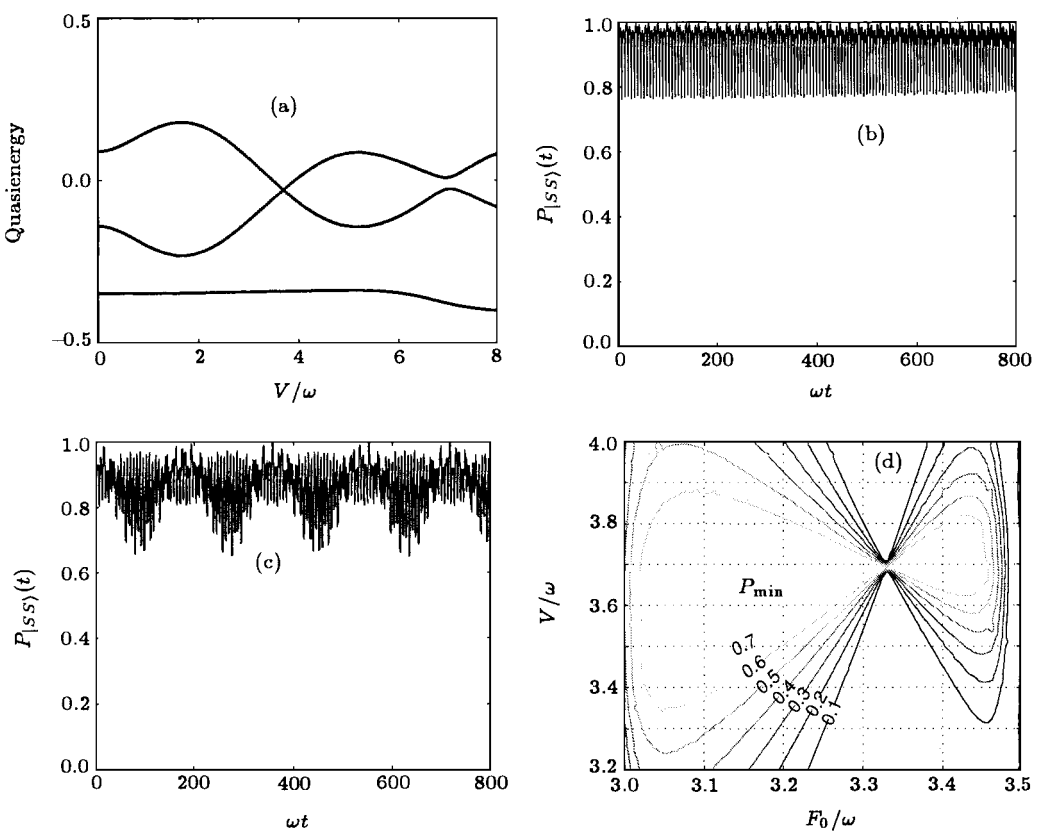


Fig.6. Exact crossing and avoided crossing occurring between two quasi-energies for the system with a DC field strength of $F_0 = 3.33\omega$. (a) quasi-energy spectrum; (b) and (c) are the time evolution populations of the initial state $|SS\rangle$ (solid curve) respectively at the exact crossing $V = 3.6939\omega$ and avoided crossing $V = 6.9895\omega$; (d) the variation of $P_{\min}$ as a function of the DC–AC field strengths around the exact crossing in (a).

When the perturbation H_t is considered, quantum transition is possible among these states. The transition probability is related to the perturbing operator H_t as follows:

$$(H_t)_{i,j} = \langle\langle \psi_i(t)|H_t|\psi_j(t)\rangle\rangle = \frac{1}{T}\int_0^T \mathrm{d}t\,\langle\psi_i(t)|H_t|\psi_j(t)\rangle\,, \qquad (21)$$

where $\langle\langle\cdots\rangle\rangle$ denotes the inner product in an extended Hilbert space of T-periodic functions.[2,6] Using the well-known identity

$$\exp[-\mathrm{i}\beta\sin\omega t] = \sum_{m=-\infty}^{\infty} J_m(\beta)\exp[-\mathrm{i}m\omega t] \qquad (22)$$

to rewrite the form of $|\psi_i(t)\rangle$, we can straightforwardly obtain

$$(H_t)_{2,3} = \begin{cases} \sqrt{2}WJ_m\,(V/\omega)\,, & \text{if} \quad \kappa - F_0 = m\omega, \\ 0, & \text{else}, \end{cases} \qquad (23)$$

and

$$(H_t)_{1,3} = 0, \qquad (24)$$

where $J_m(V/\omega)$ is an m-order Bessel function. Such results imply that the transitions from the initial localized state $|SS\rangle$ to the other states under the perturbation are obviously forbidden for any AC field strength if $F_0 \neq \kappa - m\omega$, and the DL phenomenon occurs. When $F_0 = \kappa - m\omega$, the transition from the localized state $|SS\rangle$ to the delocalized state $|FS\rangle$ is allowable except for V/ω being the roots of $\sqrt{2}WJ_m\,(V/\omega) = 0$. So the corresponding constraint conditions of the DL phenomenon in the parameter space (F_0, V, ω) are

$$\kappa - F_0 = m\omega, \qquad (25)$$

and

$$J_m\,(V/\omega) = 0. \qquad (26)$$

The above conditions are responsible for the formation of the island shapes in Figs.5(f) and 6(d). According to the conditions (25) and (26), we present the simplified diagram of the DL phenomenon in the $F_0 - V$ plane (see Fig.7(a)), which is in accord with that in Fig.2 except for the point $(2.3, 2.4)$ in a weak field region. Such agreement is verified further by the $J_m(V)$ and $P_{\min}(V)$ with F_0=2.3ω, 3.3ω in Figs.7(b) and (c).

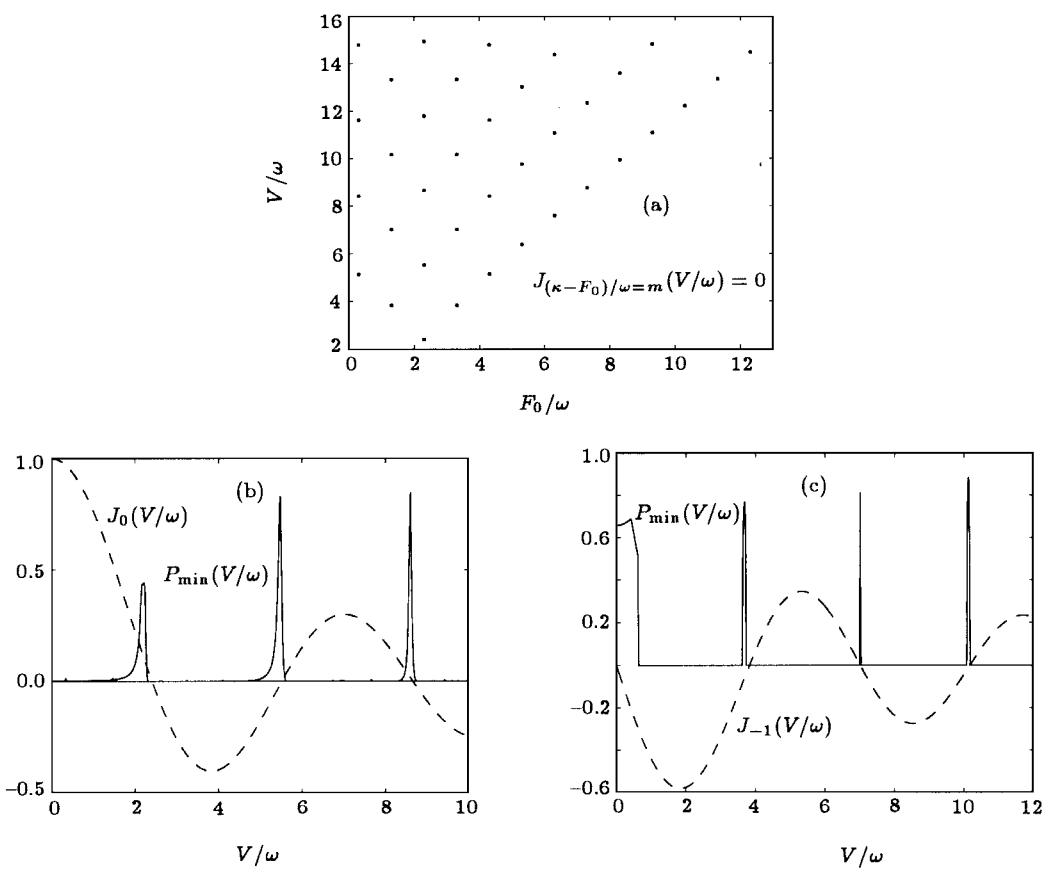


Fig.7. (a) The perturbation results of the simplified DL diagram in the $F_0 - V$ plane; (b) the $J_0(V)$ and $P_{\min}$ as a function of the amplitude of the AC field with $\kappa - F_0 = 2.3\omega - 2.3\omega = 0$; (c) the $J_{-1}(V)$ and $P_{\min}$ as a function of the amplitude of the AC field with $\kappa - F_0 = 2.3\omega - 3.3\omega = -\omega$.

In fact, the constraint conditions (25) and (26) describe both roles of Coulomb interaction and bias in the quantum tunnelling. When $F_0 = \kappa - m\omega$, one of the two electrons can tunnel between the two QDs if the strength V of the AC field is not the root of Bessel function $J_m(V/\omega)$. If both electrons synchronously tunnel between QDs, i.e. entanglement of the two electrons, the Coulomb interaction does not change after the two electrons have tunnelled. So the transition between the localized state $|SS\rangle$ and the localized state $|FF\rangle$ can be treated as a single electron transition by the driven two-level model.[1]

The constraint conditions of the DL phenomenon obtained by the driven two-level model are[13,19]

$$2F_0 = n\omega, \qquad J_n(2V/\omega) = 0, \tag{27}$$

or[7,14]

$$F_0 = n\omega, \qquad J_n(V/\omega) = 0. \tag{28}$$

Such constraint conditions can explain the small $P_{\min}$ values when the amplitude of the bias is an integer- or half-integer-folded frequency of the AC field in Fig.2. But accuracy of the approximation is poor for our system according to the comparison in Fig.8. However, such conditions can give a clear physical explanation. With the decreasing of W or the increasing of κ,[19,20] the accuracy of the above constraint conditions is improved, and the borderline of the DL areas in the DC–AC field strength plane becomes clear.

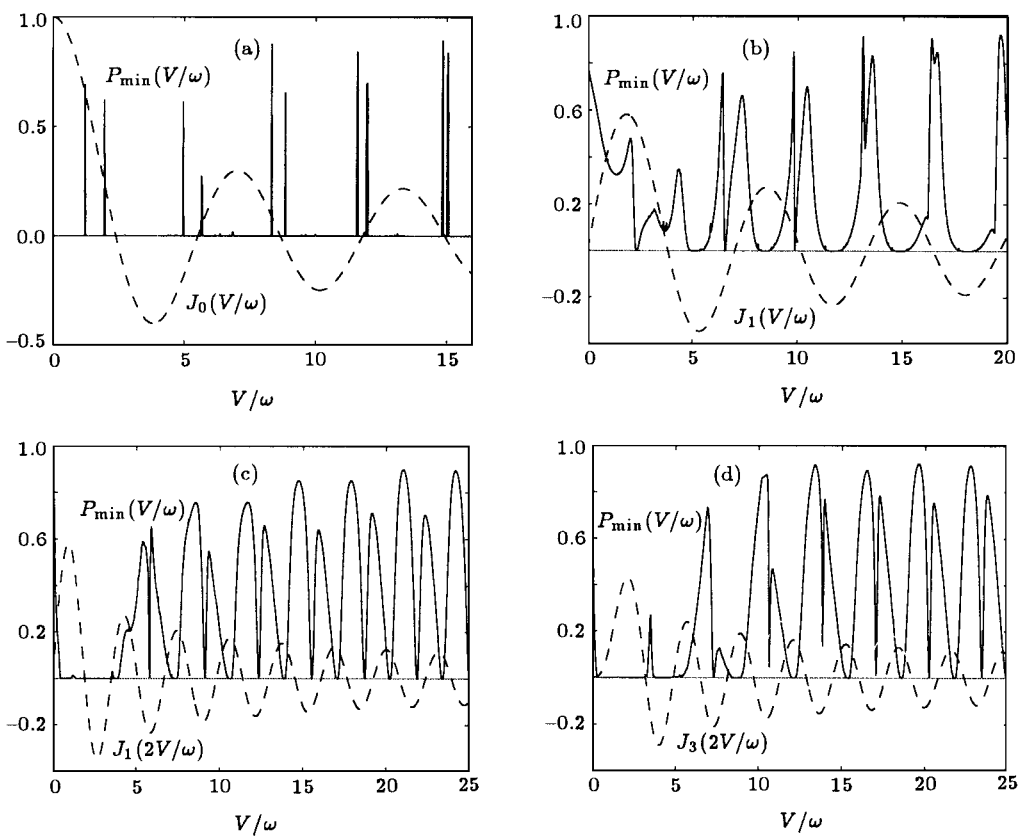


Fig.8. The comparison between the conditions given by the driven two-level model and the numerical simulation for different DC field strengths: (a) $J_0(V)$ and $P_{\min}(V)$ with $F_0 = 0$; (b) $J_1(V)$ and $P_{\min}(V)$ with $F_0 = \omega$; (c) $J_1(2V)$ and $P_{\min}(V)$ with $F_0 = 0.5\omega$; (d) $J_3(2V)$ and $P_{\min}(V)$ with $F_0 = 1.5\omega$.

3.4. Bias-weakened dynamical localization

According to the constraint conditions (25) and (26), the amplitudes of the AC field producing the DL are the same for different amplitudes of the bias satisfying $\kappa - F_0 = F_0' - \kappa$. So we can compare the degrees of the dynamical localization for the same amplitude of AC field with different amplitudes of bias. Fig.9 illustrates such a comparison for $V = 7.0\omega$ respectively with $F_0 = 1.3\omega$ and $F_0 = 3.3\omega$. Note that the $P_{\min}$ with a weak bias is larger than with a strong one, which implies that the bias can weaken, rather than strengthen, the dynamical localization.

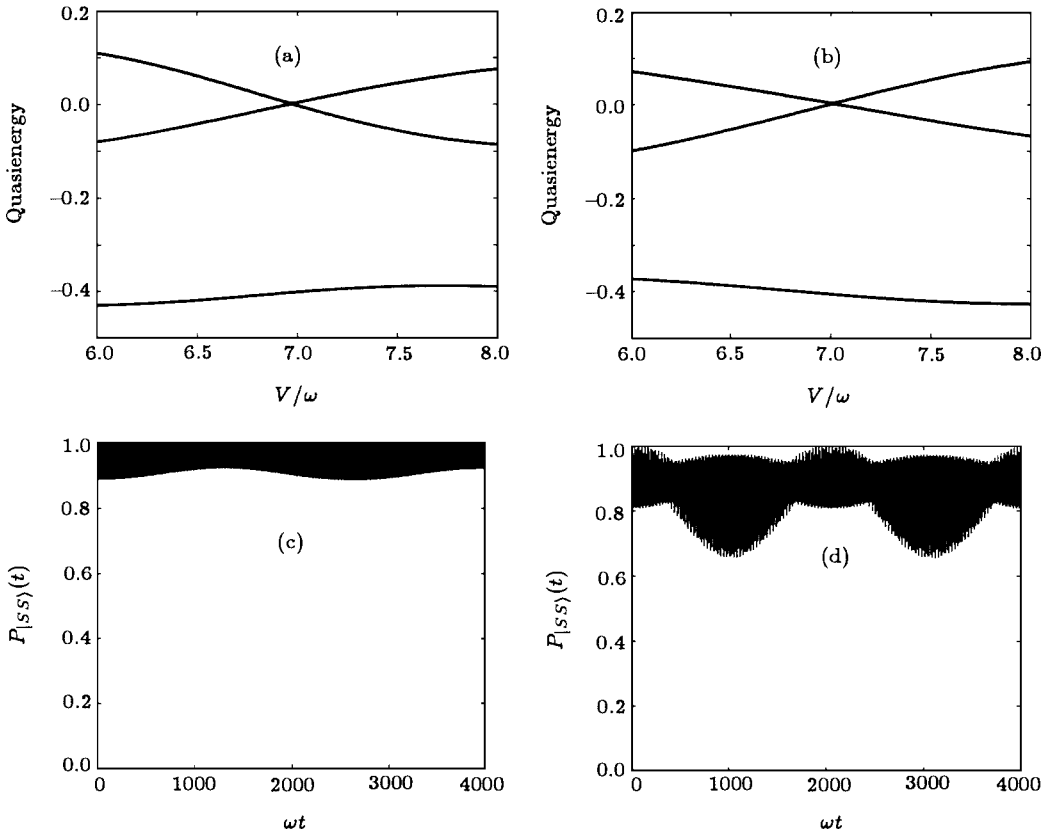


Fig.9. The comparison of the degrees of the dynamical localization with the same amplitude of AC field but different DC field strengths: (a) and (b) are the quasi-energy spectra near $V = 7.0\omega$ respectively with $F_0 = 1.3\omega$ and $F_0 = 3.3\omega$; (c) and (d) are the time evolutions of initial population $P_{|SS\rangle}(t)$ at the crossings respectively in (a) and (b).

4. Conclusions

With Floquet theory, we have numerically investigated the effects of the bias on the DL phenomenon of the AC field driven DQD. According to the results and analysis, we conclude that the DC field changes the energy of the initial state, and makes it oscillate with different states. The bias can break the dynamical symmetry of the system with a pure AC field, and make the exact crossings of the quasi-energies with a pure AC field turn into avoided crossings. The DL phenomenon still occurs at some avoided crossings. The bias extends the range of the AC field strength to such an extent that the DL phenomenon occurs at except $\kappa - F_0 = n\omega$ and $F_0 = m\omega/2$. When $\kappa - F_0 = n\omega$, the values of the AC field strength with DL phenomenon are roots of the Bessel function $J_n(V/\omega) = 0$. While when $F_0 = m\omega/2$, the values of the AC field strength with DL phenomenon are roots of the Bessel function $J_{2m}(2V/\omega) = 0$ or $J_m(V/\omega) = 0$ with m being even integer. The above relations among the amplitudes of the DC–AC field and Coulomb interaction κ are obvious with small tunnel W or strong Coulomb interaction κ. Under a certain combination of amplitudes of the DC–AC field, the DL phenomena can be weakened by a stronger bias. The difference between the DL phenomenon and the stationary localization phenomenon is remarkable in Fig.2. We believe that the results presented here are useful and may be applied to experiments.

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