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Influences of the surface elastic energy term K_{13} on the critical fields of liquid crystal cells and experimental measurement of K_{13}*

Guan Rong-Hua(关荣华)[a)†], Wang Chui-Ru(王翠茹)[b)], Yu Hui(于 慧)[c)], Kang Wen-Xiu(康文秀)[a)], Huai Jun-Xia(淮俊霞)[c)], and Yang Guo-Chen(杨国琛)[c)]

[a)] *Department of Applied Physics, North China Electric Power University, Baoding* 071003, *China*
[b)] *College of Computer Science and Technology, North China Electric Power University, Baoding* 071003, *China*
[c)] *Institute of Physics, Hebei University of Technology, Tianjin* 300130, *China*

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Based on the modified formula of Rapini–Papoular, the equation and boundary condition of the director of weak anchoring nematic liquid crystal (NLC) cell have been obtained in the case of existence of the surface elastic energy term K_{13}. The influences of K_{13} on the threshold field and the saturation field have been studied in detail with the methods of analytical derivation and numerical calculation. A new method of checking whether K_{13} exists or not is given in theory. Combining with the previous work, we also propose an experimental plan to measure the value of K_{13} with a wedge-shaped planar-aligned weak anchoring NLC cell. The numerical experiments are carried out to verify the feasibility of our method.

Keywords: threshold field, saturation field, surface elastic term K_{13}, experimental measurement
PACC: 6130, 6470M

1. Introduction

At present, the study on the surface elastic energy is the focal point in surface physics of liquid crystals, which originates from the question of whether the $(\delta n)^2$ term exists in the expression of total free energy. According to Frank,[1] the free energy density of nematic liquid crystals (NLC) caused by the deformation of the director $\boldsymbol{n}$ can be expressed as

$$f_{\rm F} = \frac{1}{2}K_{11}(\nabla\cdot\boldsymbol{n})^2 + \frac{1}{2}K_{22}(\boldsymbol{n}\cdot\nabla\times\boldsymbol{n})^2 + \frac{1}{2}K_{33}(\boldsymbol{n}\times\nabla\times\boldsymbol{n})^2.$$

But some researchers[2] pointed out that there should be an additional two terms up to the order of $(\delta\boldsymbol{n})^2$ in $f_{\rm F}$. They are

$$f_{24} = \frac{1}{2}(K_{22}+K_{24})\nabla\cdot[(\boldsymbol{n}\cdot\nabla)\boldsymbol{n} - \boldsymbol{n}(\nabla\cdot\boldsymbol{n})], \quad (1)$$

$$f_{13} = K_{13}\nabla\cdot(\boldsymbol{n}\nabla\cdot\boldsymbol{n}). \quad (2)$$

So the total free energy density $f_{\rm d}$ should be

$$f_{\rm d} = f_{\rm F} + f_{24} + f_{13}. \quad (3)$$

The contributions of the two terms, f_{24} and f_{13}, to the total free energy eventually can be written as surface energy. Consequently, we call these two terms surface elastic energy.

In recent years, there have been some controversies[3–12] about whether these two terms exist or not. There are two theories: one considers that the two terms do not exist because of the restriction of elastic theory. The other[13] considers that they do exist and regards that this problem is one of the central problems of the physics of liquid crystals. Recently, Stallinga *et al*[14] have studied the effects of the K_{13} term on the electrically controlled birefringence (ECB) cell, but the conclusion is not gratifying given their experimental inaccuracy. Reference [15] made further investigation in theory about the physical effects induced by the K_{13} surface elastic energy term and pointed out that the existence of the K_{13} term can change the property of the Freedericksz transition. However, no definite conclusion can be drawn from the above studies. As whether these two terms exist or not should be examined by experiments finally, we search for some new experimental methods

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†E-mail: ronghua_guan@sohu.com

to measure the value of K_{13}.

In this paper, we study the influences of the surface elastic energy term K_{13} on the threshold field and saturation field of weak anchoring NLC cells. A new method of checking whether K_{13} exists or not by threshold fields of two different modes is given if both of the Freedericksz transitions in them are second order. Combining the results of Ref.[15] with that of our work, we also provide a feasible experimental method to measure the value of K_{13}. In order to verify the feasibility of our method, numerical experiments are also carried out.

2. Fundamental equations

Considering a weak anchoring NLC cell the same as that in Ref.[15], two substrates lie at z=0 and $z = l$ in Cartesian coordinates respectively. The ox axis lies in the lower substrate plane. Assuming the two substrates are identical, we see that the easy directions $\boldsymbol{e}$ in both substrates are identical along the ox axis. An external magnetic field $\boldsymbol{H}$ is applied to the LC medium. There are two cases: (1) $\boldsymbol{H}$ is along the oz axis and the director $\boldsymbol{n}$ of the LC deflects from the ox axis in the xz plane; (2) $\boldsymbol{H}$ is along the oy axis and the director deflects from the ox axis in the xy plane. Denote the deflection angle in both cases by θ, the director $\boldsymbol{n}$ is expressed as $\boldsymbol{n} = (\cos\theta, 0, \sin\theta)$ for the first case, and $\boldsymbol{n} = (\cos\theta, \sin\theta, 0)$ for the second case.

Before we write out the Gibbs free energy, two points need to be explained. One is the surface anchoring energy and the other is elastic free energy.

According to Jéôme[16] and Sugimura *et al*[17] the surface density of anchoring energy can be written on the basis of Rapini–Papoular (RP) formula as

$$g_{\rm s} = -\frac{1}{2}A(\boldsymbol{n}\cdot\boldsymbol{e})^2. \tag{4}$$

If we consider the term of $(\boldsymbol{n}\cdot\boldsymbol{e})^4$ as a modification, we have

$$g_{\rm s} = -\frac{1}{2}A(\boldsymbol{n}\cdot\boldsymbol{e})^2[1+\zeta(\boldsymbol{n}\cdot\boldsymbol{e})^2], \tag{5}$$

where ζ is the modified constant. Formula (4) is called the unified formula, and Eq.(5) is a modification of Eq.(4) and equivalent to the following form:[18−24]

$$g_{\rm s} = \frac{1}{2}A\sin^2\theta(1+\zeta\sin^2\theta). \tag{6}$$

On the other hand, if we adopt an ideal interface, the constants K_{13} and K_{24} in Eqs.(1)–(3) should be replaced by the effective elastic constants K_{13}^* and K_{24}^* in Pergamanshchik's theory.[18]

For the first case of external magnetic field $\boldsymbol{H}$ along the oz axis, the total free energy of the system can be written as

$$\begin{aligned} G =& S\Bigg\{\int_0^l {\rm d}z\Bigg[\left(\frac{1}{2}K_{11}\cos^2\theta + \frac{1}{2}K_{33}\sin^2\theta\right)\left(\frac{{\rm d}\theta}{{\rm d}z}\right)^2 - \frac{1}{2}\chi_a H^2\sin^2\theta\Bigg] \\ &- K_{13}^*\cos\theta_0\sin\theta_0\frac{{\rm d}\theta}{{\rm d}z}\bigg|_0 + K_{13}^*\cos\theta_l\sin\theta_l\frac{{\rm d}\theta}{{\rm d}z}\bigg|_l \\ &+ \frac{1}{2}A\sin^2\theta_0(1+\zeta\sin^2\theta_0) + \frac{1}{2}A\sin^2\theta_l(1+\zeta\sin^2\theta_l)\Bigg\}, \end{aligned} \tag{7}$$

where θ_0 and θ_l are the values of θ at z =0 and $z = l$, respectively. Because the free energy G is minimal in a stable state, we can obtain the equation and boundary condition by applying the calculus of variations to Eq.(7):

$$\begin{aligned} &(K_{11}\cos^2\theta + K_{33}\sin^2\theta)\frac{d^2\theta}{{\rm d}z^2} \\ &+ \sin\theta\cos\theta(K_{33}-K_{11})\frac{{\rm d}\theta}{{\rm d}z} \\ &+ \chi_a H^2\sin\theta\cos\theta = 0, \end{aligned} \tag{8}$$

with the boundary condition

$$\begin{aligned} &[(K_{11}+K_{13}^*)\cos^2\theta_0 + (K_{33}-K_{13}^*)\sin^2\theta_0]\left.\frac{{\rm d}\theta}{{\rm d}z}\right|_{z=0} \\ &= A\sin\theta_0\cos\theta_0(1+2\zeta\sin^2\theta_0) \\ &\quad - \frac{1}{2}K_{13}^*\sin 2\theta_0\frac{\rm d}{{\rm d}\theta_0}\left(\left.\frac{{\rm d}\theta}{{\rm d}z}\right|_{z=0}\right), \end{aligned} \tag{9}$$

where $\left.\dfrac{{\rm d}\theta}{{\rm d}z}\right|_{z=0}$ is a function of θ_0. This boundary condition is also given in Ref.[14] and meets the requirement in Ref.[13].

Equation (8) with the boundary condition (9) has two trivial solutions:

$$\theta \equiv \frac{\pi}{2},$$

$$\theta \equiv 0.$$

We call $\theta \equiv 0$ the uniform solution and $\theta \equiv \dfrac{\pi}{2}$ the saturation solution. In addition, there is also a nontrivial solution called the disturbed solution. In order to express the equation and the boundary condition of the disturbed solution definitely and make our calculation easy, we introduce a new state parameter u and a new variable v.

Let

$$u = \sin^2\theta_m, \quad v = \frac{\sin^2\theta}{\sin^2\theta_m}, \quad v_0 = \frac{\sin^2\theta_0}{\sin^2\theta_m}, \tag{10}$$

where θ_m is the value of θ at z=$l/2$.

Furthermore, we introduce the reduced quantities simultaneously,

$$\begin{aligned} h =& \frac{H}{H_c^0} = H \Big/ \left(\frac{\pi}{l}\sqrt{\frac{K_{11}}{\chi_a}}\right), \\ \alpha =& \frac{A\cdot l}{2K_{11}}, \quad g = \frac{lG}{2K_{11}S}, \end{aligned} \tag{11}$$

then Eq.(8) can be expressed as

$$\frac{\pi}{2}h = I_1.$$

The boundary condition (9) can be expressed as

$$\begin{aligned} & I_1\left\{1 + \frac{k}{1+k}\frac{v_0(1-uv_0)}{1+\gamma_1 uv_0}\right. \\ & \left.\times\left[\frac{1}{1-v_0}\frac{1-Q}{Q} - \frac{\gamma u}{1+\gamma uv_0}\right]\right\} \\ =& \alpha(1+2\zeta uv_0)\frac{1}{1+k}\frac{1+\gamma uv}{1+\gamma_1 uv_0} \\ & \times\sqrt{\frac{v_0(1-uv_0)}{(1-v_0)(1+\gamma uv_0)}}, \end{aligned} \tag{12}$$

and the reduced free energy can be expressed as

$$\begin{aligned} g =& u\left[I_1^2 - 2I_1I_2 - 2k\sqrt{\frac{v_0(1-v_0)(1-uv_0)}{1+\gamma uv_0}}I_1\right. \\ & \left.+\alpha v_0(1+\zeta uv_0)\right], \end{aligned} \tag{13}$$

where

$$\begin{aligned} k =& \frac{K_{13}^*}{K_{11}}, \\ \gamma =& \frac{K_{33}-K_{11}}{K_{11}}, \\ \gamma_1 =& \frac{K_{33}-K_{11}-2K_{13}^*}{K_{11}+K_{13}^*} \end{aligned}$$

and

$$Q = v_0 + (1+\gamma)u\sqrt{\frac{v_0(1-v_0)(1-uv_0)}{1+\gamma uv_0}}I_3, \tag{14}$$

$$I_1 = \int_{v_0}^1 \frac{1}{2\sqrt{v(1-v)}}\sqrt{\frac{1+\gamma uv}{1-uv}}\mathrm{d}v, \tag{15}$$

$$I_2 = \int_{v_0}^1 \frac{v}{2\sqrt{v(1-v)}}\sqrt{\frac{1+\gamma uv}{1-uv}}\mathrm{d}v, \tag{16}$$

$$\begin{aligned} I_3 =& \int_{v_0}^1 \frac{1}{2\sqrt{v(1-v)}}\sqrt{\frac{1+\gamma uv}{1-uv}} \\ & \cdot\frac{1}{(1+\gamma uv)(1-uv)}\mathrm{d}v. \end{aligned} \tag{17}$$

The derivation of the above equations is cumbersome and thus is omitted here.

The above equations are to be used for the discussion of critical fields. It is necessary to point out that all the above equations are appropriate for the first case of external magnetic $\boldsymbol{H}$ applied along the oz axis. As for the second case of $\boldsymbol{H}$ along the oy axis, we can easily prove that the above equations can be obtained only by letting $K_{11} = K_{33} = K_{22}$, K_{13}^*=0, i.e. k=0, $\alpha' = (Al)/(2K_{22})$ in the above equations.

3. Influences of K_{13} on the critical fields

3.1. The threshold field

Suppose that at the threshold and the saturation point, the director changes continuously with the applied field. This is to say that the transitions are of second-order. First we discuss the threshold field $h_{\rm th}$.

For $u = 0$, Eq.(14) becomes $Q = v_0$. Substituting it into Eq.(12) yields

$$I_1\,|_{u=0} = \frac{\alpha}{1+2k}\sqrt{\frac{v_0}{1-v_0}}. \tag{18}$$

From Eqs.(11) and (15), we obtain

$$\frac{\pi}{2}h_{\rm th} = I_1\,|_{u=0}\,, \tag{19}$$

$$I_1|_{u=0} = \int_{v_0}^1 \frac{1}{2\sqrt{v(1-v)}}\mathrm{d}v = \cot^{-1}\sqrt{\frac{v_0}{1-v_0}}. \tag{20}$$

Combining Eq.(19) with Eq.(20) gives

$$\cot\left(\frac{\pi}{2}h_{\rm th}\right) = \sqrt{\frac{v_0}{1-v_0}}. \tag{21}$$

In addition, from Eqs.(18) and (19), we obtain

$$\frac{\pi}{2}h_{\rm th} = \frac{\alpha}{1+2k}\sqrt{\frac{v_0}{1-v_0}}. \tag{21}$$

Eliminating the common term $\sqrt{\frac{v_0}{1-v_0}}$ in Eqs.(21) and (22), we obtain an equation about the threshold field $h_{\rm th}$ ultimately

$$\cot\left(\frac{\pi}{2}h_{\rm th}\right) = \left(\frac{1+2k}{\alpha}\right)\frac{\pi}{2}h_{\rm th}. \tag{23}$$

Equation (23) reflects the influence of surface elastic term $k = \frac{K_{13}^*}{K_{11}}$ on the threshold field $h_{\rm th}$. If letting $k = 0$ in Eq.(23), the equation just becomes the

universal formula for threshold field, which is in the absence of K_{13}.[19] On the other hand, since $h_{\rm th} \geq 0$ always holds, from Eq.(23) we can have the condition of $k > -\dfrac{1}{2}$, which is consistent with the result in Ref.[20].

We now discuss our results using numerical calculations. Figure 1 shows the relation between the threshold field $h_{\rm th}$ and the surface elastic term k for different α. In Fig. 1 we can see that the value of $h_{\rm th}$ will decrease with the increase of k; and for fixed k, $h_{\rm th}$ increases linearly with α.

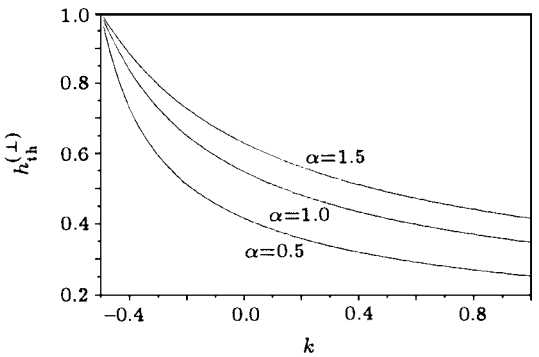


Fig.1. The $h_{\rm th}$ versus k curves for various α.

3.2. The saturation field

Now, we discuss the saturation magnetic field $h_{\rm sat}$. For $u = 1$, Eqs.(11), (12) and (15) become

$$\frac{\pi}{2} h_{\rm sat} = I_1 \,|_{u=1} \,, \tag{24}$$

$$I_1|_{u=1} = \frac{\alpha(1+2\zeta)}{(1+\gamma-2k)} \sqrt{\frac{1+\gamma}{1-v_0}}, \tag{25}$$

$$\begin{aligned} I_1|_{u=1} &= \int_{v_0}^{1} \frac{\sqrt{1+\gamma}}{2v\sqrt{(1-v)}} {\rm d}v \\ &= \sqrt{1+\gamma}\, \tanh^{-1} \sqrt{1-v_0}. \end{aligned} \tag{26}$$

From Eqs.(24), (25), and (26), we can obtain the equation of $h_{\rm sat}$.

$$\coth\left(\frac{1}{\sqrt{1+\gamma}} \frac{\pi}{2} h_{\rm sat}\right) = \frac{1+\gamma-2k}{\alpha(1+2\zeta)} \frac{1}{\sqrt{1+\gamma}} \frac{\pi}{2} h_{\rm sat}. \tag{27}$$

Equation (27) reflects the influence of surface elastic term $k = \dfrac{K_{13}^*}{K_{11}}$ on the saturation field $h_{\rm sat}$. If $k = 0$ in Eq.(27), it just becomes a universal formula of the saturation field, in the absence of K_{13}, and is consistent with the result in Ref.[19].

Next we discuss our results using numerical calculations. In Fig.2, $\gamma = 0.25$ is adopted. We can see that the value of $h_{\rm sat}$ increases with increasing K_{13}. This is just the situation opposite to the situation threshold field $h_{\rm th}$.

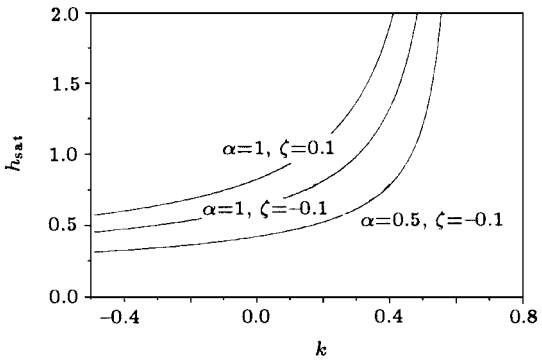


Fig.2. The $h_{\rm sat}$ versus k curves for various α and ζ, $\gamma = 0.25$.

3.3. Measurement of k with two thresholds

The value of k can be determined by two different deformations of liquid crystal cells. One is the splay transition. It corresponds to the first case of applying external magnetic field $\boldsymbol{H}$ to the cell along the oz axis. In this case the threshold field is obtained using Eq.(23) in Section 3.1. The other is the twist transition, which corresponds to the second case of applying external magnetic field $\boldsymbol{H}$ to the cell along the oy axis. In this case the threshold field is denoted by $h'_{\rm th}$ and can be obtained just by setting $K_{11} = K_{33} = K_{22}$, K_{13}^*=0, i.e. k=0, and $\alpha' = (Al)/(2K_{22})$ in Eq.(23),

$$\cot\left(\frac{\pi}{2} h'_{\rm th}\right) = \frac{1}{\alpha'} \frac{\pi}{2} h'_{\rm th}. \tag{28}$$

Combining Eqs.(23) with (28) yields

$$k = \frac{1}{2}\left[\frac{h'_{\rm th}\cot(\pi h_{\rm th}/2)}{h_{\rm th}\cot(\pi h'_{\rm th}/2)} \cdot \frac{\alpha}{\alpha'} - 1\right]. \tag{29}$$

This is an important formula we derived to determine k. If the unified formula of Jérôme[16] and Sugimura *et al*[17] is confirmed, then $\dfrac{\alpha}{\alpha'}$ in Eq.(29) is

$$\frac{\alpha}{\alpha'} = \frac{K_{22}}{K_{11}}. \tag{30}$$

Otherwise, denoting the anchoring strength with A_θ for the case of $\boldsymbol{H}$ along the oz axis and A_φ for the case of $\boldsymbol{H}$ along the oy axis, we obtain

$$\frac{\alpha}{\alpha'} = \frac{A_\theta}{A_\varphi} \frac{K_{22}}{K_{11}}. \tag{31}$$

If the parameters K_{11}, K_{22}, χ_a, l, A_θ, A_φ are known and the threshold field $h_{\rm th}$ and $h'_{\rm th}$ are measured by experiments, the value of k will be determined by Eq.(29). So we can judge whether K_{13} exists or not, based on the calculated results. Examples of curve k versus $h_{\rm th}$ are shown in Fig.3, in which the solid line corresponds to $A_\theta = A_\varphi$.

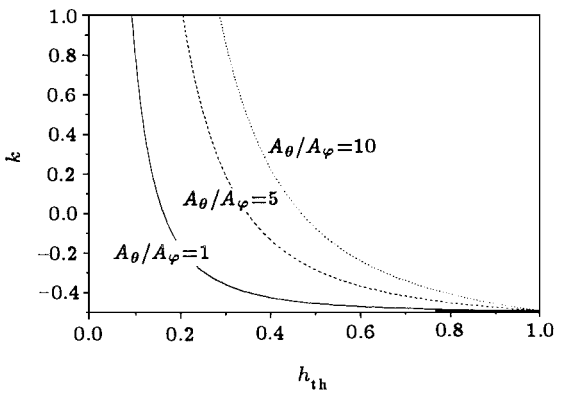


Fig.3. The reduced threshold field $h_{\rm th}$ versus k for different A_θ/A_φ, with $K_{22}/K_{11} \approx 0.67$, α'=0.1.

4. Experimental measurement of K_{13}

Nowadays, there are some methods to measure the surface elastic term K_{13}. However, there is a large discrepancy among values obtained by different authors.[20,21] We now propose a feasible experimental procedure to measure the value of K_{13}.

We adopt a wedge-shaped planar-aligned weak anchoring NLC cell (see Fig.4). There is a small angle φ between the two substrates. φ is so small that the two substrates can be treated as parallel with each other in a small region and the formulae we obtained in the previous section are still valid in this small region. But, for the whole liquid-crystal cell, the upper substrate is not parallel to the lower substrate. The thickness l of the liquid-crystal medium is variable. That is to say, the reduced anchoring strength α $\left(\alpha = \dfrac{A \cdot l}{2K_{11}}\right)$ is variable.

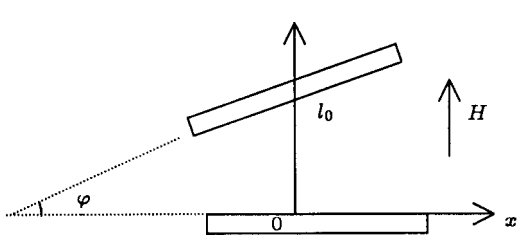


Fig.4. A liquid-crystal cell with two unparallel substrates.

Equation (23) and Ref.[15] show that both the threshold field and the property of the Freedericksz transition are connected with α and the first-order Freedericksz transition will occur when α is small enough. So, for our liquid-crystal cell, the position corresponding to the transition on the curve shifts successively with the increase of the external field $\boldsymbol{H}$, and there is a first-order transition appearing at the place where l is small.

At the threshold point, we have free energy $g = 0$. From Eq.(13) we know that this requires

(i) $u = 0$. This means that a second-order transition occurs in the cell. The threshold field can be obtained using Eq.(23). That is:

$$\cot\left(\frac{l}{2}\sqrt{\frac{\chi_a}{K_{11}}}H_{\rm th}\right) = \left(\frac{1+2k}{A}\right)\sqrt{\chi_a K_{11}}H_{\rm th}. \quad (32)$$

(ii) $u \neq 0$, but

$$I_1^2 - 2I_1 I_2 - 2k\sqrt{\frac{v_0(1-v_0)(1-uv_0)}{1+\gamma uv_0}}I_1 + \alpha v_0(1+\zeta uv_0) = 0. \quad (33)$$

This means that a first-order transition occurs in the cell. In this case, besides Eq.(33), there should be a boundary condition

$$I_1\left\{1 + \frac{k}{1+k}\frac{v_0(1-uv_0)}{1+\gamma_1 uv_0} \times \left[\frac{1}{1-v_0}\frac{1-Q}{Q} - \frac{\gamma u}{1+\gamma uv_0}\right]\right\} = \alpha(1+2\zeta uv_0)\frac{1}{1+k}\frac{1+\gamma uv}{1+\gamma_1 uv_0} \times \sqrt{\frac{v_0(1-uv_0)}{(1-v_0)(1+\gamma uv_0)}}, \quad (34)$$

and the threshold field is

$$H_{\rm th} = I_1 \cdot \frac{2}{l}\sqrt{\frac{K_{11}}{\chi_a}}. \quad (35)$$

The value of k can be gained by substituting the values of the threshold field $H_{\rm th}$ and the corresponding experimental transition position l into Eqs.(32)–(35).

Before carrying out the numerical experiment, we first discuss the shift direction of the transition boundary. Supposing that the transitions are of second-order at the threshold point, from Eq.(32) we can

obtain the relation between $H_{\rm th}$ and l. The results are illustrated in Fig.5, in which parameters are $A \approx 2 \times 10^{-7}{\rm Jm}^{-2}$,[22] $\chi_{\rm a} = 0.97 \times 10^{-7}$cgs,[23] $K_{11} = 5.8 \times 10^{-12}$N,[24] $k = -0.1$.

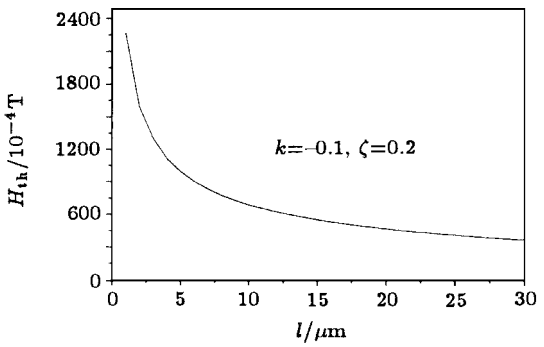


Fig.5. The curve of $H_{\rm th}(l)$ versus l with $A \approx 2 \times 10^{-7}{\rm Jm}^{-2}$, $\chi_{\rm a} = 0.97 \times 10^{-7}$cgs, $K_{11} = 5.8 \times 10^{-12}$N, $k = -0.1$, $\zeta = 0.2$.

From Fig.5, one can see that the position of transition l is inversely proportional to the threshold field $H_{\rm th}$. That is to say, with the increase of $H_{\rm th}$, the transition position will shift to the place with smaller l. So, we can conclude that with the decrease of l, a second-order transition occurs, followed by a first-order transition.

Generally, the values of parameters K_{11}, K_{33}, $\chi_{\rm a}$ appearing in the above equations can be found easily by consulting reference books; $H_{\rm th}$ and corresponding l can be obtained by experimental measurement. But A and ζ are unknown quantities for they are related to measuring techniques, and k is just the quantity we want to determine.

In order to eliminate A and ζ and obtain the value of k, at least three measurements must be performed. With the increase of external magnetic field, we choose the three measurements as follows.

(1) The position $l^{(1)}$ of the second-order transition boundary and the corresponding magnetic field $H_{\rm th}^{(1)}$. (2) The first position $l^{(2)}$ of the first-order transition boundary and the corresponding magnetic field $H_{\rm th}^{(2)}$. (3) The second position $l^{(3)}$ of the first-order transition boundary and the corresponding magnetic field $H_{\rm th}^{(3)}$. The shift process of the transition boundary with the increase of magnetic field is shown in Figs.6(a)–(c).

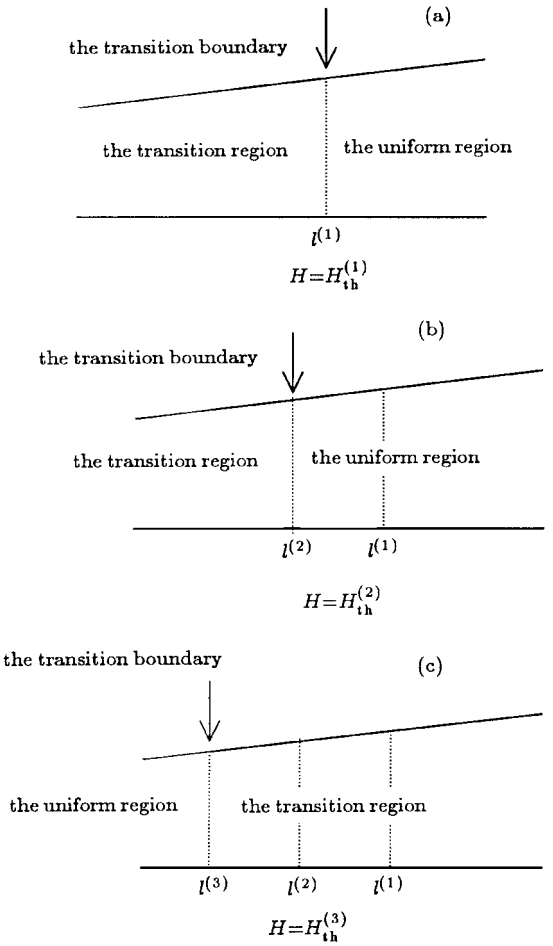


Fig.6. (a) The Freedericksz transition at $l^{(1)}$ under threshold field $H_{\rm th}^{(1)}$ for the first measurement. (b) The Freedericksz transition at $l^{(2)}$ under threshold field $H_{\rm th}^{(2)}$ for the second measurement. (c) The Freedericksz transition at $l^{(3)}$ under threshold field $H_{\rm th}^{(3)}$ for the third measurement.

There are seven unknown parameters in Eqs.(32)–(35): they are k, A, ζ, u_1, v_{01}, u_2, v_{02}. On the other hand, among the three measurements, $l^{(1)}$ and $H_{\rm th}^{(1)}$ must satisfy Eq.(32) because the transition is of second-order. $l^{(2)}$, $H_{\rm th}^{(2)}$ and $l^{(3)}$, $H_{\rm th}^{(3)}$ must satisfy Eqs.(33), (34) and (35) because the transitions are of first-order. Thus seven equations can be obtained. Thus we can determine the seven unknowns.

We carry out numerical experiments by choosing the first- and second-order transition parameters from Fig.6 given in Ref.[15]. The results are given in Table 1.

Table 1. Numerical result for k with the parameters $\chi_a = 0.97 \times 10^{-7}$cgs, $K_{11} = 5.8 \times 10^{-12}$N, $K_{33}/K_{11} = 1.25$.

The first measurement (The second order transition)		The second measurement (The first order transition)		The third measurement (The first order transition)	
$l^{(1)}/10^{-6}$m	$H_{th}^{(1)}/10^{-4}$T	$l^{(2)}/10^{-6}$m	$H_{th}^{(2)}/10^{-4}$T	$l^{(3)}/10^{-6}$m	$H_{th}^{(3)}/10^{-4}$T
29	382.634	11.6	639.685	5.8	917.439
$A \approx 2 \times 10^{-7}$Jm^{-2}		$\zeta = 0.2$		$k = -0.1$	

5. Conclusions

The equilibrium equation and boundary condition for the director are obtained taking into consideration K_{13}. The influence of K_{13} on the critical fields of the weak anchoring NLC cell have been discussed in detail. A new method of detecting K_{13} is given in theory and a feasible experimental procedure to measure the value of K_{13} is also proposed, which is verified by numerical calculation. Conclusions can be drawn as follows.

1. Both the threshold field H_{th} and the saturation field H_{sat} are seriously affected by the surface elastic term K_{13}. With the increase of K_{13}, the value of H_{sat} increases, but H_{th} decreases.

2. The surface elastic term $k = K_{13}^*/K_{11}$ can be determined by two threshold fields if both of the Freedericksz transitions at the threshold points are of second-order. One threshold field corresponds to the application of an external magnetic field perpendicular to the substrates, and the other corresponds to the field applied parallel to substrates.

3. Adopting a wedge-shaped planar-aligned weak anchoring NLC cell, we can determine the value of k experimentally by measuring the position of the transition on the curve and the corresponding threshold field.

It is a basic problem in liquid crystal physics whether K_{13} surface elastic energy term exists or not, which has not been solved yet. We need to investigate it thoroughly by using different methods and test the results experimentally.

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