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Composite particle and field theory in atomic quantum Hall effect*

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In this paper, we explore the composite particle description of the atomic quantum Hall (QH) effect. We further give the Chern–Simon–Gross–Pitaevskii (CSGP) effective theory for the atomic Hall liquid, which is the counterpart of Chern–Simon theory in electron Hall effect. What we obtained is equivalent to the Laughlin wavefunction approach. Our results show that in terms of composite particles, the atomic Hall effect is really the same as the electronic QH effect. The CSGP effective theory would shed new light on the atomic QH effect.

Keywords: quantum Hall effect, composite particle, field theory
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The rotating trapped Bose–Einstein condensates (BECs) have attracted much theoretical and experimental attention in recent years.[1−17] In the rapidly-rotating limit, the strong-correlated bosons which can be regarded as 2-dimensional (2D) gases show a macroscopic quantum phenomenon, atomic quantum Hall (QH) effect, which is the counterpart of the electron QH effect and has been explored thoroughly for nearly two decades.[18,19] These two systems are so similar that many methods dealing with electrons have been exploited for atoms. The incompressible states and the composite particle wavefunctions have been discussed by several groups,[1−4] and the fractional statistics of the quasihole is also explored.[10,11] Many of these papers concentrated on the incompressible wavefunction at different filling factors and they have obtained some interesting results such as the Laughlin variational wavefunction and the Pfaffian state[1,2] and so on. Through thoroughly investigating the similarity between the rapidly-rotating ultra-cold trapped atoms and electrons under strong magnetic field, we find that the atomic QH effect can really be understood as the "bosonic twin of the electronic brother".[17] Some essential issues for understanding the atomic QH effect have been acquired, such as the fractional mass of the Laughlin quasiparticles, macroscopic quantized atomic Hall conductance and the effective Chern–Simon (CS) field theory. These results shed new light on the strong correlated cold atom physics. The edge states and the topological orders of atomic QH liquid, which is another important aspect of QH effect, are discussed on the basis of the CS effective field theory.[20]

Then a question is naturally put forward: what is the composite-particle theory for the atomic Hall effect. This question is interesting as the concept of composite particles is essential for understanding the mechanism of the incompressibility and has induced many new issues for electron QH effects. Several authors have covered this issue for atomic QH effect.[3,4,21,22] However, many aspects, which are important in QH effect, have not been included in their work. The field theory is widely used in the research of condensation physics.[23−25] In this paper, we exploit in our studies the composite particle concepts and the effective field theory based on our previous work. The method introduced in this paper is parallel to that in the papers by Jain and Zhang *et al* to explain the electronic QH liquid.[24,25] We find that the atomic QH effect is nearly the same as the electronic QH ef-

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fect in terms of composite particles. Here a composite particle is an atom attached to Chern–Simon vortex quanta. There exist two equivalent pictures for atomic Hall effects: the composite fermions (an atom bound to $2p-1$ vortex quanta) picture and the composite bosons (an atom bound to $2p$ vortex quanta) picture. The fractional atomic QH effect can be understood as the integer QH effect of composite fermions or the Bose condensate of the composite bosons, similarly to the electron case.[19] One can find that the method is parallel to the one electron Hall effect. This is true since the atomic QH effect is the bosonic twin of its electronic brother.[17]

First let us review the concept of composite particles that was first put forward by Jain in his outstanding paper.[24] In the atomic gases the composite particles are a little different from the ones in the electronic QH liquid. Since the atoms are neutral, they will respond to rotation in the way that the electrons respond to the perpendicular magnetic field, and the vortex quanta replace the magnetic flux quanta. In our previous paper, we have pointed out the equivalence between m and e, and between $\Phi_{\rm D}=\dfrac{2\pi\hbar}{e}$ and $\Phi_{\rm v}=\dfrac{2\pi\hbar}{m}$ in the 2D case.[17] With this equivalence in mind, we can say that the composite particles here are atoms carrying vortex quanta $\Phi_{\rm v}=\dfrac{2\pi\hbar}{m}$, which can be seen clearly throughout the paper. It can be easily seen that an exchange of two atoms carrying vortex $\alpha\Phi_{\rm v}$ produces a phase factor $(-1)^{\alpha}$.[24] When α is even, the composite particle is bosonic; however, when α is odd, the composite particles are fermionic. As is emphasized by Jain, the true atom state is not as rigid as the composite particle state, but the composite particle should provide the correct description of the essential physics at the mean-field level.

Now consider N bosonic atoms, which are trapped in the x–y plane by an isotropic harmonic potential rotating about $\hat{z}$ at a frequency ω. The atoms can be effectively treated as a 2D system by assuming the confinement to be strong enough in the $\hat{z}$-direction. In the rapidly-rotating limit, the many-body Hamiltonian describing the atomic gases reads

$$H=\sum_{i=1}^{N}\frac{(\boldsymbol{p}_i-m\omega\hat{z}\times\boldsymbol{r}_i)^2}{2m}+\sum_{i<j}^{N}V(\boldsymbol{r}_i-\boldsymbol{r}_j), \quad (1)$$

where $V(\boldsymbol{r})=g\delta(\boldsymbol{r})$ with $g=U_0\sqrt{\dfrac{m\omega_z}{2\pi\hbar}}$ and $U_0=4\pi a\hbar^2/m$.[1,2,10,11] Based on the previous work, one can consider the 2D interacting atoms to be coupled to a gauge velocity field $\boldsymbol{V}=m\omega\hat{z}\times\boldsymbol{r}$, the charge of which is the mass of trapped atoms.[17,20,26] One can derive the composite particle field description directly from the above microscopic Hamiltonian by introducing a statistical transmutation,

$$\phi(\boldsymbol{r})=\mathrm{e}^{-\mathrm{i}\alpha\Theta(\boldsymbol{r})}\psi(\boldsymbol{r}), \quad (2)$$

with $\psi(\boldsymbol{r})$ the atom field operator and $\Theta(\boldsymbol{r})=\int\mathrm{d}^2r'\theta(\boldsymbol{r}-\boldsymbol{r}')\psi^+(\boldsymbol{r}')\psi(\boldsymbol{r}')$, where α is an integer and $\theta(\boldsymbol{r}-\boldsymbol{r}')$ is the angle between the vector $\boldsymbol{r}-\boldsymbol{r}'$ and the x axis. One can then obtain the following second-quantized Hamiltonian:

$$H_{\rm c}=\int\mathrm{d}^2r\phi^+(\boldsymbol{r})\frac{(-\mathrm{i}\hbar\nabla-m\boldsymbol{V}(\boldsymbol{r})-m\boldsymbol{c}(\boldsymbol{r}))^2}{2m}\phi(\boldsymbol{r})+\frac{1}{2}\int\mathrm{d}^2r\mathrm{d}^2r'\phi^+(\boldsymbol{r})\phi^+(\boldsymbol{r}')\boldsymbol{V}(\boldsymbol{r}-\boldsymbol{r}')\phi(\boldsymbol{r}')\phi(\boldsymbol{r}), \quad (3)$$

where $\boldsymbol{V}(\boldsymbol{r})=m\omega\hat{z}\times\boldsymbol{r}$ is the velocity gauge field and $c^i(\boldsymbol{r})=\dfrac{\alpha\hbar}{m}\varepsilon^{ij}\int\mathrm{d}^2r'\dfrac{r_j-r'_j}{|\boldsymbol{r}-\boldsymbol{r}'|}\phi^+(\boldsymbol{r}')\phi(\boldsymbol{r}')$ is the Chern–Simon statistical gauge potential, and the charge of both gauge fields is the mass of trapped atoms (see Refs.[25, 19] for the corresponding electron case). This Hamiltonian describes a system of composite particles with mass m created by the field operator ϕ. One can immediately obtain the following expression for the statistical gauge field:

$$b(\boldsymbol{r})=-\varepsilon^{ij}\partial_i c_j=\frac{2\pi\hbar\alpha}{m}|\phi(\boldsymbol{r})|^2=\alpha\Phi_{\rm v}|\phi(\boldsymbol{r})|^2, \quad (4)$$

which corresponds to associating α units of vortex quanta to each atom. The field operator $\phi(\boldsymbol{r})$ describes composite bosons when $\alpha=2k$ and composite fermions when $\alpha=2k-1$, where k is an arbitrary integer. When $\alpha=2k$, this Hamiltonian is identical with the one in Ref.[25] when we consider the mass of atom m as the charge of the velocity gauge field $\boldsymbol{V}(\boldsymbol{r})$ and the statistical gauge potential $\boldsymbol{c}(\boldsymbol{r})$. A similar Hamiltonian has been discussed thoroughly in the electron QH effect and the induced mean-field Chern–Simon–Landau–Ginzburg (CSGL) theory, which plays an important role in the electron QH effect.[25] When $\alpha=2k-1$, the Hamiltonian describes the Jain composite fermions. In this view, the atomic fractional QH effect is the integer QH effect of composite fermions, and thus will induce the Jain hierarchy for atom fractional QH effect.

First let us consider the composite boson description when $\alpha=2k$. As discussed in the electron QH

effect, one can obtain the mean-field theory, Chern–Simon–Gross–Pitaevskii (CSGP) description of the QH liquid, which dominates the long-distance effects of the quantum Hall liquids.[25] The CSGP Hamiltonian for the atomic QH effect reads

$$H = \frac{1}{2m^*}\int \mathrm{d}^2 r \phi^+(\boldsymbol{r})(-\mathrm{i}\hbar\nabla - m\boldsymbol{V}(\boldsymbol{r}) - m\boldsymbol{c}(\boldsymbol{r}))^2\phi(\boldsymbol{r}) + \frac{g}{2}\int \mathrm{d}^2 r(|\phi|^2 - n)^2, \quad (5)$$

where m^* is the effective mass of the composite bosons (but not the mass of the atoms), and n is the atom density. The Lagrangian is given by

$$\begin{aligned} L =& \phi^+(\mathrm{i}\hbar\partial_t + mV_0 + mc_0)\phi \\ & - \frac{1}{2m^*}\phi^+(-\mathrm{i}\hbar\nabla - m\boldsymbol{V} - m\boldsymbol{c})^2\phi \\ & - \frac{g}{2}(|\phi|^2 - n)^2 - \frac{m^2}{4\pi\alpha\hbar}\varepsilon^{\mu\nu\lambda}c_\mu\partial_\nu c_\lambda, \end{aligned} \quad (6)$$

where the last term is called the Chern–Simon term,[25,27–29] and the scalar field V_0 and c_0 are defined as in Ref.[17].

Then consider the case that ω=const and $V_0 = 0$. It is obvious that there is a trivial constant solution $\phi = \sqrt{n}$, $\boldsymbol{c} = -\boldsymbol{V}$, $a_0 = 0$. From Eq.(4) this solution exists only in the case $\nu = \dfrac{1}{2k}$. The physical meaning is clear: that is the Chern–Simon gauge field cancels the velocity gauge field, and the composite bosons condensate at T=0. The atomic Hall conductance can be naturally obtained when we consider the atomic mass m as the charge of the velocity gauge field. The induced mass current now is $j^i_{\mathrm{ind}} = \nu\dfrac{m^2}{\hbar}\varepsilon^{ij} = \sigma^{(m)}_{ij}g^j$, where $g^i = \partial_i V_0$ (we call it g for the fact that it is coupled to the atomic mass m) and $\sigma^{(m)}$ is defined in Ref.[17]. The ground state is superfluid and there are static non-uniform, finite-energy vortex excitations. The centre of the vortex is at r=0, and the $r \to \infty$ behaviour is $\phi(r,\varphi) = \sqrt{n}\mathrm{e}^{\pm\mathrm{i}\varphi}$, $\boldsymbol{c}(r,\varphi) = \pm\dfrac{\hat{\varphi}}{mr}$, with $a_0 = 0$. The total mass carried by the vortex $m_{\mathrm{v}} = \dfrac{1}{2k}m$, which is the same as what we acquired in Ref.[17]. It is clear that the vortex quasiparticles obey fractional statistics with $\theta = \pi/2k$.[30] The above discussion is equivalent to what we have obtained in Ref.[17], but it is more powerful due to the success of the CSGL theory in dealing with the other problems in the electron QH effect, such as the global phase diagram and the anomaly filling factor.[27–29] We hope that some new concepts for the current atom case can be developed by using this theoretical insight in future.

Next, we turn to the composite fermions description with $\alpha = 2k-1$. The composite fermions description has been investigated in Ref.[22] by numerical calculation; they found that the ground state of Jain sequence is impressible state. A composite fermion is an atom bound to $2k-1$ vortices. We briefly comment on some aspects of the integer Hall effect of a neutral fermion. Suppose we rotate non-interacting neutral fermions, which are described by Eq.(1) except that the two-body interaction is zero. One can easily obtain the Landau level for the free fermions. The Landau levels are broadened by impurity effects in actual samples. This is the same as the non-interacting electrons under a magnetic field, which is essential to explain the electronic integer Hall effect. The neutral fermions will show integer Hall effect when the Fermi energy lies between two neighboring Landau levels. But for bosonic atoms, there is not the simple integer filling factor, but only a fractional filling factor. In the viewpoint of Jain,[24] one can say that the atomic fractional QH effect is the integer QH effect of composite fermion under a reduced velocity gauge field $\boldsymbol{V}_{\mathrm{eff}}(\boldsymbol{r})$ with $2\omega_{\mathrm{eff}} = 2\omega - (2p-1)\Phi_{\mathrm{v}}n$. In this case the role of interaction is assumed to generate composite fermions and the bosonic atom gas is equivalent to non-interacting composite fermions.[24] The composite fermions under a gauge field $\boldsymbol{V}_{\mathrm{eff}}(\boldsymbol{r}) = m\omega_{\mathrm{eff}}\hat{z}\times\boldsymbol{r}$ show an integer QH effect at a filling factor $\nu_{\mathrm{eff}} = p$ with p being an integer at low temperatures. Then the atomic Hall resistance is related to the mass transported from one edge to the other as one vortex is pierced through the centre adiabatically.[17] At the integer filling factor p, p composite fermions are transported across the sample. For $\alpha = 2k-1$, when a composite fermion is transported in this process, one has to supply $(2k-1)p$ additional vortex quanta $\Phi_{\mathrm{v}} = \dfrac{2\pi\hbar}{m}$. This process will give the filling factor for atoms at $\nu = \dfrac{p}{(2k-1)p+1}$, which is the Jain hierarchy for atomic QH effects.[22] The impurities in the sample create localized states, which ensure the Hall plateau when the Fermi level lies between two Landau levels. The wavefunction related to the composite fermions has been explored in Refs.[3,4], and what we discussed above shows a more clearly physical picture.

The deep connection between the rapidly-rotating ultra-cold atoms and the 2D electron gas under a magnetic field has been discussed by several groups.[17,26,31] The connection is clearer in the quantum Hall regime. The present paper further demon-

strates this correspondence in terms of composite particles. The two systems are nearly the same, though one is neutral while the other is charged. The profound similarity is summarized as follows:

$$2\omega \leftrightarrow B, m \leftrightarrow e$$

$$\boldsymbol{V} \leftrightarrow \boldsymbol{A}, \boldsymbol{c} \leftrightarrow \boldsymbol{a}$$

$$\Phi_{\rm v} \leftrightarrow \Phi_{\rm D}, CP_{\rm v} \leftrightarrow CP_{\rm D}$$

with $CP_{\rm v}$ denoting the particle bound to $\Phi_{\rm v}$ while the other is bound to $\Phi_{\rm D}$.

In conclusion, we have derived the composite particle theory and the CSGP theory for the atomic fractional QH effect. Both are essential for the understanding of the interesting and exciting macroscopic quantum phenomenon in atomic physics. Many interesting ideas (e.g., the Hall insulator, the global phase of the Hall sample and even the anomaly filling factor) have been exploited in the electron QH system on the basis of the CSGP theory and the concept of composite particles. We expect that similar phenomena might occur in atomic strong-correlated systems. For $\nu = 1$ Wilkin *et al* obtain the wavefunction in Refs.[1, 2], which can be understood more clearly and shows some exotic phenomena like the electronic QH effect at $\nu = \frac{1}{2}$ (explainable in the framework of the CSGP theory as discussed in Refs.[32, 33]). Though the Hall regime experiment is hard to reach with current technology, several groups have made important progress to achieve the goal. We hope our work may stimulate the experimental efforts to observe some new macroscopic quantum phenomena.

Note added: Recently, a similar CSGP effective theory is proposed in Ref.[34] to explain the novel phase transition in experiment.[35] They derived the effective theory from the energy function.

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