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Bright–dark incoherently coupled photovoltaic soliton pair*

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The coupling between two mutually incoherent optical beams that propagate collinearly in open-circuit photovoltaic photorefractive media is investigated. It is shown that an incoherently coupled bright–dark spatial soliton pair can be formed due to photovoltaic effect. The physical properties of such a soliton pair are also discussed.

Keywords: photorefractive effect, spatial soliton, photorefractive material, photovoltaic effect
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1. Introduction

Upon illuminated, a photorefractive material can set up a space-charge field in the crystal, which will induce nonlinear changes in the refractive index of the material by the electro-optic (Pockels) effect. The latter process is then capable of counteracting the effects of diffraction, and thus the non-diffracting optical beam or photorefractive spatial soliton forms. In the last decade, photorefractive spatial solitons have been actively investigated because they can be established at low light intensities, and are potentially useful for all-optical switching, beam steering, and optical interconnects. Thus far, several different types of photorefractive solitons, such as quasi-steady-state solitons,[1−3] screening solitons,[4−7] and photovoltaic solitons[8−12] have been observed. The quasi-steady-state solitons can be formed in a biased photorefractive crystal during the finite time in which an externally applied field is slowly screened by the space charge field. The screening solitons are possible to exist steadily in a biased non-photovoltaic photorefractive crystal. They result from non-uniform screening of the bias field. The photovoltaic solitons can also exist steadily in photovoltaic photorefractive materials. These photovoltaic solitons rely on the photovoltaic effect creating the space charge field.

Soliton interaction is one of the most fascinating aspects in soliton physics, and the soliton pair is an intriguing issue about the interaction between two solitons. In 1996, Christodoulides *et al*[13] investigated the incoherent coupling of two screening solitons. They have shown that incoherently coupled bright-bright, dark-dark, and bright–dark screening soliton pairs are possible to form in biased non-photovoltaic photorefractive crystals under steady-state conditions, provided that the two incident beams have the same polarization, wavelength, and are mutually incoherent. Soon after, Chen *et al*[14−16] observed these incoherently coupled screening soliton pairs in strontium barium niobate (SBN) crystal experimentally. In this paper, we predict that incoherently coupled bright–dark photovoltaic soliton pairs can also be supported in photovoltaic photorefractive materials under the steady-state and open-circuit conditions.

2. Soliton coupling equations

In our model, two optical beams propagate collinearly in a photovoltaic photorefractive crystal along the z axis and are permitted to diffract only along the x direction. The two incident beams have the same polarization, wavelength, and are mutually incoherent. Moreover, the polarizations of the two incident optical beams are assumed to be both parallel to the x axis. As usual, the optical field is expressed in terms of slowly varying envelopes, i.e.

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$\boldsymbol{E}_1 = \hat{x}\phi(x,z)\exp(\mathrm{i}kz)$, and $\boldsymbol{E}_2 = \hat{x}\psi(x,z)\exp(\mathrm{i}kz)$, where $k = k_0 n_\mathrm{e} = (2\pi/\lambda_0)n_\mathrm{e}$, λ_0 is the free-space wavelength, and n_e is the unperturbed extraordinary index of refraction. For this configuration, the two optical beams satisfy the following envelope evolution equations:[6,13]

$$\mathrm{i}\phi_z + \frac{1}{2k}\phi_{xx} - \frac{k_0 n_\mathrm{e}^3 r_{33} E_\mathrm{sc}}{2}\phi = 0, \tag{1a}$$

$$\mathrm{i}\psi_z + \frac{1}{2k}\psi_{xx} - \frac{k_0 n_\mathrm{e}^3 r_{33} E_\mathrm{sc}}{2}\psi = 0, \tag{1b}$$

where $\phi_z = \partial\phi/\partial z$, $\phi_{xx} = \partial^2\phi/\partial x^2$, $\psi_z = \partial\psi/\partial z$, $\psi_{xx} = \partial^2\psi/\partial x^2$, where r_{33} is the electro-optic coefficient of the photorefractive crystal, and $\boldsymbol{E}_\mathrm{sc} = E_\mathrm{sc}\hat{x}$ is the space charge field in the crystal.

The space charge field E_sc in Eq.(1) can be derived from the standard set of rate, current, and Poisson's equations, describing the photorefractive effect of the photovoltaic crystal. In the steady state and under open-circuit conditions these equations become[8]

$$[s(I+I_\mathrm{b})+\beta]\left(N_\mathrm{D}-N_\mathrm{D}^+\right) - \gamma n N_\mathrm{D}^+ = 0, \tag{2}$$

$$\frac{\mathrm{d}E_\mathrm{sc}}{\mathrm{d}x} + \frac{q}{\varepsilon}\left(n + N_\mathrm{A} - N_\mathrm{D}^+\right) = 0, \tag{3}$$

$$J = q\mu n E_\mathrm{sc} + k_\mathrm{B}T\mu\frac{\mathrm{d}n}{\mathrm{d}x} + \kappa s\left(N_\mathrm{D}-N_\mathrm{D}^+\right)(I+I_\mathrm{b}) = 0, \tag{4}$$

where n is the electron density, N_D is the total donor number density, N_D^+ is the density of the ionized donor, N_A is the acceptor density, s is the photoionization cross section, β is the dark generation rate, γ is the carrier recombination rate, μ and q are, respectively, the electron mobility and the charge, ε is the dielectric constant, k_B is the Boltzmann constant, T is the absolute temperature, κ is the photovoltaic constant of the crystal, I is the intensity of the incident optical beam, and I_b is the background irradiance with the same polarization as that of the incident optical beam.

In typical photorefractive crystals, it holds true that $N_\mathrm{D} \gg n$, $N_\mathrm{D}^+ \gg n$ and $N_\mathrm{A} \gg n$. Under these conditions, from Eqs.(2) and (3) one can obtain

$$N_\mathrm{D}^+ = N_\mathrm{A}\left(1 + \frac{\varepsilon}{qN_\mathrm{A}}\frac{\mathrm{d}E_\mathrm{sc}}{\mathrm{d}x}\right), \tag{5}$$

$$n = \frac{\left(N_\mathrm{D}-N_\mathrm{D}^+\right)[s(I+I_\mathrm{b})+\beta]}{\gamma N_\mathrm{A}}\left(1+\frac{\varepsilon}{qN_\mathrm{A}}\frac{\mathrm{d}E_\mathrm{sc}}{\mathrm{d}x}\right)^{-1}. \tag{6}$$

According to Ref.[6], if the optical intensity $I(x,z)$ varies slowly with x in typical photorefractive crystals, the dimensionless term $|(\varepsilon/qN_\mathrm{A})(\mathrm{d}E_\mathrm{sc}/\mathrm{d}x)|$ can be considered to be much less than unity. So, Eqs.(5) and (6) give

$$N_\mathrm{D}^+ \cong N_\mathrm{A}, \tag{7}$$

$$n = \frac{\left(N_\mathrm{D}-N_\mathrm{A}\right)[s(I+I_\mathrm{b})+\beta]}{\gamma N_\mathrm{A}}. \tag{8}$$

Inserting Eqs.(7) and (8) into Eq.(4) yields

$$E_\mathrm{sc} = -E_\mathrm{p}\frac{I+I_\mathrm{b}}{I+I_\mathrm{b}+I_\mathrm{d}} - \frac{k_\mathrm{B}T}{q}\frac{\mathrm{d}I/\mathrm{d}x}{I+I_\mathrm{b}+I_\mathrm{d}}, \tag{9}$$

where $E_\mathrm{p} = \kappa\gamma N_\mathrm{A}/q\mu$ is the photovoltaic field constant. $I_\mathrm{d} = \beta/s$ is the so-called dark irradiance. In photovoltaic photorefractive crystals, the photovoltaic field is dominant. In this case, the term associated with the process of diffusion, i.e. the $k_\mathrm{B}T/q$ term, can be considered to be small and thus can be neglected. So, Eq.(9) can be simplified to

$$E_\mathrm{sc} = -E_\mathrm{p}\frac{I+I_\mathrm{b}}{I+I_\mathrm{b}+I_\mathrm{d}}, \tag{10}$$

where $I(x,z) = I_1(x,z) + I_2(x,z)$ is the total incident intensity in the crystal. By using the Poynting theorem, the total intensity of the incident beams can be expressed as

$$I = (n_\mathrm{e}/2\eta_0)(|\phi|^2 + |\psi|^2), \tag{11}$$

where $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$. Substituting the expressions (10) and (11) into Eq.(1), and adopting the following dimensionless coordinates and variables: $s = x/x_0$, $\xi = z/(kx_0^2)$, $U = [2\eta_0(I_\mathrm{b}+I_\mathrm{d})/n_\mathrm{e}]^{-1/2}\phi$ and $V = [2\eta_0(I_\mathrm{b}+I_\mathrm{d})/n_\mathrm{e}]^{-1/2}\psi$, where x_0 is an arbitrary spatial width for scaling, we can obtain the following coupled equations:

$$\mathrm{i}U_\xi + \frac{1}{2}U_{ss} + \alpha\frac{|U|^2+|V|^2+\delta}{|U|^2+|V|^2+1}U = 0, \tag{12a}$$

$$\mathrm{i}V_\xi + \frac{1}{2}V_{ss} + \alpha\frac{|U|^2+|V|^2+\delta}{|U|^2+|V|^2+1}V = 0, \tag{12b}$$

i.e.

$$\mathrm{i}U_\xi + \frac{1}{2}U_{ss} + \alpha U - \frac{\alpha(1-\delta)U}{|U|^2+|V|^2+1} = 0, \tag{13a}$$

$$\mathrm{i}V_\xi + \frac{1}{2}V_{ss} + \alpha U - \frac{\alpha(1-\delta)V}{|U|^2+|V|^2+1} = 0, \tag{13b}$$

where $U_\xi = \partial U/\partial\xi$, $U_{ss} = \partial^2 U/\partial s^2$, $V_\xi = \partial V/\partial\xi$, $V_{ss} = \partial^2 V/\partial s^2$, $\alpha = (k_0x_0)^2(n_\mathrm{e}^4 r_{33}/2)E_\mathrm{p}$, and $\delta = I_\mathrm{b}/(I_\mathrm{b}+I_\mathrm{d})$. In what follows, the bright–dark soliton solutions of Eq.(13) will be discussed in detail.

3. Bright–dark soliton solutions

For a bright soliton, the optical energy concentrates in the area near to the beam centre, that is, its optical intensity reaches the maximum value at $x = 0$, whereas the dark soliton exhibits a dark notch embedded in a uniform intensity background, i.e. its optical intensity attains asymptotically a constant value at $x \to \infty$. To obtain the bright–dark soliton pair solutions of Eq.(13), we express the normalized envelopes of the two optical beams in terms of $U = r^{1/2} f(s) \exp(\mathrm{i}\mu\xi)$ and $V = \rho^{1/2} g(s) \exp(\mathrm{i}v\xi)$, where $f(s)$ corresponds to the bright beam envelope and $g(s)$ to the dark one, the positive quantity r stands for the ratio of the peak intensity of the bright beam to the sum of the dark irradiance and background irradiance, i.e. $r = I_{1\max}/(I_\mathrm{b} + I_\mathrm{d})$; and ρ represents the ratio of the maximum intensity of the dark beam to the sum of the dark irradiance and background irradiance, namely $\rho = I_{2\max}/(I_\mathrm{b} + I_\mathrm{d})$, where $I_{2\infty} = I_2(x \to \infty, z)$ is a constant. According to the intensity distribution of the bright and dark solitons, we can know that the normalized real functions $f(s)$ and $g(s)$ should satisfy the boundary conditions of $f(0) = 1$, $\dot{f}(0) = 0$, $f(s \to \pm\infty) = 0$, $g(0) = 0$, $g(s \to \pm\infty) = \pm 1$, and moreover, all the derivatives of $f(s)$ and $g(s)$ vanish at infinity.[13] Substituting the expressions of $U = r^{1/2} f(s) \exp(\mathrm{i}\mu\xi)$ and $V = \rho^{1/2} g(s) \exp(\mathrm{i}v\xi)$ into Eq.(13), we obtain

$$\ddot{f} = 2\left[\mu - \alpha + \frac{\alpha(1-\delta)}{1 + r f^2 + \rho g^2}\right] f, \tag{14a}$$

$$\ddot{g} = 2\left[v - \alpha + \frac{\alpha(1-\delta)}{1 + r f^2 + \rho g^2}\right] g, \tag{14b}$$

where $\ddot{f} = \mathrm{d}^2 f/\mathrm{d}s^2$, and $\ddot{g} = \mathrm{d}^2 g/\mathrm{d}s^2$. We now seek particular solutions of Eq.(13) that also satisfy the condition $f^2 + g^2 = 1$. In this case, Eq.(13) can be written as

$$\ddot{f} = 2\left[\mu - \alpha + \frac{\alpha(1-\delta)}{1+\rho}\frac{1}{1+\sigma f^2}\right] f, \tag{15a}$$

$$\ddot{g} = 2\left[v - \alpha + \frac{\alpha(1-\delta)}{1+\rho}\frac{1}{1+\sigma(1-g^2)}\right] g, \tag{15b}$$

where $\sigma = (r-\rho)/(1+\rho)$. Integrating Eq.(15a) once, we can have

$$(\dot{f})^2 = 2(\mu-\alpha)(f^2-1) + \frac{2\alpha(1-\delta)}{\sigma(1+\rho)} \ln\left(\frac{1+\sigma f^2}{1+\sigma}\right). \tag{16}$$

According to the boundary condition of $f(s)$ at infinity, we can find from Eq.(16) that

$$\mu = \alpha - \frac{\alpha(1-\delta)}{\sigma(1+\rho)} \ln(1+\sigma). \tag{17}$$

The insertion of the boundary conditions of $g(s)$ into Eq.(15b) yields

$$v = \alpha\left(\frac{\delta+\rho}{1+\rho}\right). \tag{18}$$

When $|\sigma| \ll 1$, which implies that the peak intensities of the two beams are nearly equal, Eq.(15) can then be solved approximately. In this case, by employing Taylor expansion, μ can be given by

$$\mu \cong \alpha - \frac{\alpha(1-\delta)}{1+\rho}\left(1 - \frac{\sigma}{2}\right). \tag{19}$$

Using the expressions (18) and (19) and $|\sigma| \ll 1$, Eq.(15) can be simplified as

$$\ddot{f} = \frac{\alpha\sigma(1-\delta)}{1+\rho}\left(f - 2f^3\right), \tag{20a}$$

$$\ddot{g} = -\frac{2\alpha\sigma(1-\delta)}{1+\rho}\left(g - g^3\right). \tag{20b}$$

The bright–dark soliton pair solution of Eq.(20) can be readily obtained and are given by

$$f(s) = \mathrm{sech}\left\{\left[\frac{\alpha\sigma(1-\delta)}{1+\rho}\right]^{1/2} s\right\}, \tag{21a}$$

$$g(s) = \tanh\left\{\left[\frac{\alpha\sigma(1-\delta)}{1+\rho}\right]^{1/2} s\right\}. \tag{21b}$$

So the dimensionless amplitudes of the optical field for the soliton pair can be expressed as

$$U(s,\xi) = r^{1/2}\mathrm{sech}\left\{\left[\frac{\alpha\sigma(1-\delta)}{1+\rho}\right]^{1/2} s\right\} \times \exp\left\{\mathrm{i}\alpha\left[1 - \frac{1-\delta}{1+\rho}\left(1-\frac{\sigma}{2}\right)\right]\xi\right\}, \tag{22a}$$

$$V(s,\xi) = \rho^{1/2}\tanh\left\{\left[\frac{\alpha\sigma(1-\delta)}{1+\rho}\right]^{1/2} s\right\} \times \exp\left[\mathrm{i}\alpha\left(\frac{\delta+\rho}{1+\rho}\right)\xi\right]. \tag{22b}$$

According to the expressions (22a) and (22b) the intensity full width at half maximum (FWHM) of the bright–dark soliton pair can be obtained directly and given by

$$\Delta s = 2\ln\left(1+\sqrt{2}\right)\left[\frac{1+\rho}{\alpha\sigma(1-\delta)}\right]^{1/2}, \tag{23}$$

or

$$\Delta x = \frac{2\sqrt{2}\ln\left(1+\sqrt{2}\right)}{k_0 n_\mathrm{e}^2}\left[\frac{1+\rho}{\sigma(1-\delta) r_{33} E_\mathrm{p}}\right]^{1/2}. \tag{24}$$

4. Discussion and summary

In summary, the coupled equations for two mutually incoherent optical beams propagating collinearly in open-circuit photovoltaic photorefractive media are presented, and the bright–dark soliton pair solutions are obtained. It is shown that the formation of bright–dark spatial soliton pair is possible due to the photovoltaic effect. The expressions (21) and (22) show that the bright–dark soliton pair solutions are possible only when $\alpha\,(r-\rho) > 0$. So we can conclude that the photovoltaic photorefractive crystals with a positive photovoltaic field (i.e. $E_{\mathrm{p}} > 0$) can support bright–dark soliton pairs with the peak intensity of the bright component being slightly larger than that of the dark one, whereas, the photovoltaic photorefractive crystals with a negative photovoltaic field (i.e. $E_{\mathrm{p}} < 0$) are able to support bright–dark soliton pairs with the peak intensity of the bright component being slightly smaller than that of the dark one. Here, it needs to be explained that in our derivation the photovoltaic field can be taken to be positive for photovoltaic crystals with positive changes in refractive index, and for the photovoltaic crystals with negative changes in refractive index the photovoltaic field can be considered to be negative. By changing the intensities of the two original incident beams the intensities of the bright and dark soliton components can be adjusted, so that one can make the peak intensity of the bright component slightly larger (or smaller) than that of the dark one to establish the bright–dark soliton pairs. Besides the above mentioned conclusions, from the expressions (23) and (24) we can also find that the intensity full width at half maximum (FWHM) of such bright–dark soliton pairs is inversely proportional to the square root of the absolute value of the photovoltaic field.

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