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# Spontaneous emission spectrum of a four-level atom in a double-band photonic crystal*

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The spontaneous emission spectrum from a four-level atom in a double-band photonic crystal has been investigated. We use the model which assumes three atomic transitions. One of the transitions interacts with the free vacuum modes, and the other two transitions couple to the modes of the isotropic photonic band gap (PBG), the anisotropic PBG and another free vacuum. The effects of the fine structure of the lower levels on the spontaneous emission spectrum of an atom are investigated in detail in the three cases. New features of four (two) transparencies with two (one) spontaneous emission peaks, resulting from the fine structure of the lower levels of an atom, are predicted in the case of isotropic PBG modes.

**Keywords:** photonic band gap, spontaneous emission
**PACC:** 3200, 3270, 4250

## 1. Introduction

The system of vacuum-induced interference or additionally driven by external coherent laser fields is widely studied, and the interference effects in such a system lead to many remarkable phenomena, including very narrow absorption and fluorescence spectra,[1−6] population trapping in a degenerate system,[7,8] fluorescence quenching in the free space,[4,9,10] phase dependent line shapes,[11,12] lasing without inversion,[13] and others.[14−23] It is well known that the spontaneous emission and probe absorption of an atom depends not only on the properties of the excited atomic system but also on the nature of the surrounding environment.[24,25] From the point of view of the surrounding environment of atoms, photonic band gap (PBG) structures have been shown to have different density of states (DOS) compared with a free-space vacuum field.[24−27] The study of quantum and nonlinear optical phenomena for archtypes of atoms embedded in such PBG materials leads to the prediction of many interesting effects: for example, localization of light,[28,29] photon–atom bound states,[24,25,28,30] suppression and even complete cancellation of spontaneous emission,[31,32] enhancement of spontaneous emission,[33−36] electro-magnetically induced transparency and other phenomena.[37−45] In the studies of an atom embedded in a photonic crystal, most papers reported on the population involution and distribution in upper levels[25,28,34] and the spectral components in the spontaneous emission fields[33−36] of an atom. Of particular interest, Knight and his co-workers[37,38] have investigated transparency phenomena of an atom in the Λ-type model where one transition interacts with free vacuum modes and another interacts with modes near the edges of photonic band gap. Focusing on the quantum interference of an atom without external driving coherent laser fields, Zhu [1] and Zhou and Swain[6] independently reported the quantum interference in spontaneous emission spectrum and in probe absorption spectrum of an atom coupled to vacuum space reservoirs. In our previous papers,[46−48] we studied the quantum interference in spontaneous emission spectrum of an atom embedded in a double-band photonic crystal. There is only one lower level in the above studies. From the point of view of practice, the ground state levels of an atom have acquired a superfine energy-level structure, leading to greater versatility. In this paper, we adopt the model of a four-level atom with a superfine energy-level struc-

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ture in a double-band photonic crystal. We assume that the transitions from the only one upper level to the one lower level is coupled to free vacuum modes, and the transitions from upper level to the two lower levels (fine structure) are coupled to the modes which are isotropic PBG, anisotropic PBG and free vacuum. The spontaneous emission spectrum along both the transition coupled to free vacuum modes and that coupled to PBG modes are investigated. In Sec. 2, we derive the equations for describing the spontaneous emission spectrum of the four-level atom in a photonic crystal. The general calculated results and the analysis of them are presented in Sec. 3. The major conclusions are summarized in Sec. 4.

## 2. Equations for the spontaneous emission spectrum

Consider a four-level atom with upper level $|3\rangle$ and lower levels $|2\rangle$, $|0\rangle$ and $|1\rangle$, as shown in Fig.1. We assume that the transition from the upper level $|3\rangle$ to the lower level $|1\rangle$ is coupled by the free vacuum mode ($\omega_\lambda$), and those to the lower levels $|2\rangle$ and $|0\rangle$ are coupled by the double-band isotropic PBG mode, anisotropic PBG mode and free vacuum mode. The

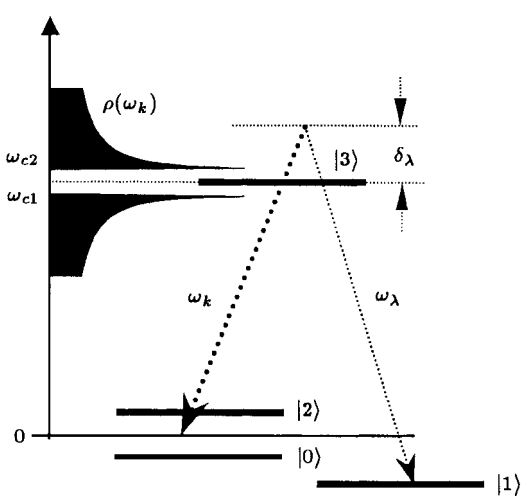



**Fig.1.** Schematic diagram of the four-level atom in isotropic double-photonic-band structure. $\rho(\omega_k)$ denotes the density of state of the PBG modes. $\omega_{c2}$ and $\omega_{c1}$ are the upper and lower frequencies of the forbidden gap, respectively. It is assumed that the transitions from the upper level $|3\rangle$ to the lower levels $|0\rangle$ and $|2\rangle$ are coupled by PBG modes $\omega_k$. The transition from the upper level $|3\rangle$ to the lower level $|1\rangle$ is coupled by free vacuum modes $\omega_\lambda$ . $\delta_\lambda$ is the detuning of frequency of spontaneous emission field from the middle of the forbidden gap. $\Delta$ denotes the split width of the lower state levels.

dispersion relations near the photonic band edges are approximated by[24]

$$\begin{aligned}\omega_k &= \omega_{c1} - A_1(k-k_0)^2, \quad k < k_0,\\ \omega_k &= \omega_{c2} + A_2(k-k_0)^2, \quad k > k_0,\end{aligned} \tag{1}$$

$$\begin{aligned}\omega_k &= \omega_{c1} - A_1|(\boldsymbol{k}-\boldsymbol{k}_0)|^2, \quad |\boldsymbol{k}| < |\boldsymbol{k}_0|,\\ \omega_k &= \omega_{c2} + A_2|(\boldsymbol{k}-\boldsymbol{k}_0)|^2, \quad |\boldsymbol{k}| > |\boldsymbol{k}_0|,\end{aligned} \tag{2}$$

for the isotropic dispersion relation and anisotropic dispersion relation, respectively. Here $A_n \approx \omega_{cn}/k_0^2$ $(n = 1, 2)$. $\omega_{c1}$ and $\omega_{c2}$ are the upper and lower frequencies at the edges of the forbidden gap, respectively. The Hamiltonian describing the dynamics of this system in an interaction picture and the rotating wave approximation can be written as

$$\begin{aligned}H_I =& \mathrm{i}\hbar\sum_{\lambda e}(g_{\lambda e}^{31}a_{\lambda e}^{+}b_1^{+}b_3\mathrm{e}^{\mathrm{i}\mu_\lambda^{31}t} - \mathrm{C.C.})\\ &+ \mathrm{i}\hbar\sum_{ke}(g_{ke}^{30}a_{ke}^{+}b_0^{+}b_3\mathrm{e}^{\mathrm{i}\mu_k^{30}t}\\ &+ g_{ke}^{32}a_{ke}^{+}b_2^{+}b_3\mathrm{e}^{\mathrm{i}\mu_k^{32}t} - \mathrm{C.C.}),\end{aligned} \tag{3}$$

where $b_i^+$ and $b_i$ ($i$=0, 1, 2, 3) are respectively creation and annihilation operators for an atom in level i; $a_{\lambda e}^+$ ($a_{\lambda e}$) is the creation (annihilation) operator for the $\lambda$th vacuum mode with frequency $\omega_\lambda$; $a_{ke}^+$ ($a_{ke}$) is the creation (annihilation) operator for the $k$th PBG mode with frequency $\omega_k$; $g_{\lambda e}^{31}$ is the frequency-dependent coupling constant between the atomic transition $|3\rangle - |1\rangle$ and the free vacuum mode $\omega_\lambda$; and $g_{ke}^{3j}$ ($j$=0, 2) is another coupling constant between the atomic transition $|3\rangle - |j\rangle$ and PBG mode $\omega_k$. More precisely, we have

$$\begin{aligned}g_{\lambda e}^{31} =& (\omega_{31}d_{31}/\hbar)(\hbar/(2\varepsilon_0\omega_\lambda V))^{1/2}\boldsymbol{e}_{\lambda e}\cdot\boldsymbol{d}_{31},\\ g_{ke}^{3j} =& (\omega_{3j}d_{3j}/\hbar)(\hbar/(2\varepsilon_0\omega_k V))^{1/2}\boldsymbol{e}_{ke}\cdot\boldsymbol{d}_{3j},\\ &(j=0,2),\end{aligned} \tag{4}$$

$$\begin{aligned}\mu_\lambda^{31} &= \omega_\lambda - \omega_{31},\\ \mu_k^{3j} &= \omega_k - \omega_{3j} \quad (j=0,2),\end{aligned}$$

where $\boldsymbol{d}_{32}$ and $\boldsymbol{d}_{30}$ are the atomic dipole moment unit vectors for the transitions $|3\rangle - |2\rangle$ and $|3\rangle - |0\rangle$, respectively; $\boldsymbol{e}_{\lambda e}$ and $\boldsymbol{e}_{ke}$ ($e$=1, 2) are the polarization unit vectors of the radiation fields from the double V-type transitions, and $V$ is the sample volume. The state vector of the system in the interaction picture can be written as

$$\begin{aligned}|\Psi(t)\rangle =& A_3(t)\mathrm{e}^{-\mathrm{i}\mu_p^{31}t}|3,\{0\}\rangle + \sum_{\lambda e}A_{1\lambda e}|1,\{\lambda e\}\rangle\\ &+ \sum_{ke}A_{0ke}|0,\{ke\}\rangle + \sum_{ke}A_{2ke}|2,\{ke\}\rangle,\end{aligned} \tag{5}$$

where the amplitude $A_iA_i^*$ ($i$=0, 1, 2, 3) represents the probability of an atom appearing in level $i$. Substituting Eqs.(3) and (5) into Schrödinger equation, we obtain

$$\begin{aligned}\frac{\mathrm{d}(A_3(t)\mathrm{e}^{-\mathrm{i}\mu_p^{31}t})}{\mathrm{d}t} =&-\mathrm{i}\sum_{\lambda e} g_{\lambda e}^{31}\mathrm{e}^{-\mathrm{i}\mu_\lambda^{31}t}A_{1\lambda e}(t)\\ &-\mathrm{i}\sum_{ke} g_{ke}^{30}\mathrm{e}^{-\mathrm{i}\mu_k^{30}t}A_{0ke}(t)\\ &-\mathrm{i}\sum_{ke} g_{ke}^{32}\mathrm{e}^{-\mathrm{i}\mu_k^{32}t}A_{2ke}(t),\\ \frac{\mathrm{d}A_{1\lambda e}(t)}{\mathrm{d}t} =&-\mathrm{i}g_{\lambda e}^{31}\mathrm{e}^{\mathrm{i}\mu_\lambda^{31}t}A_3(t)\mathrm{e}^{-\mathrm{i}\mu_p^{31}t},\\ \frac{\mathrm{d}A_{0ke}(t)}{\mathrm{d}t} =&-\mathrm{i}g_{ke}^{30}\mathrm{e}^{\mathrm{i}\mu_k^{30}t}A_3(t)\mathrm{e}^{-\mathrm{i}\mu_p^{31}t},\\ \frac{\mathrm{d}A_{2ke}(t)}{\mathrm{d}t} =&-\mathrm{i}g_{ke}^{32}\mathrm{e}^{\mathrm{i}\mu_k^{32}t}A_3(t)\mathrm{e}^{-\mathrm{i}\mu_p^{31}t}.\end{aligned}\tag{6}$$

Formally integrating and eliminating $A_{1\lambda e}(t)$, $A_{0ke}(t)$ and $A_{2ke}(t)$ from Eq.(6), we obtain

$$\begin{aligned}\frac{\mathrm{d}(A_3(t)\mathrm{e}^{-\mathrm{i}\mu_p^{31}t})}{\mathrm{d}t} =&-\int_0^t G_{33}^{\lambda 1}(t-t')\mathrm{e}^{-\mathrm{i}\mu_p^{31}t'}A_3(t')\mathrm{d}t'\\ &-\int_0^t G_{33}^{k0}(t-t')\mathrm{e}^{-\mathrm{i}\mu_p^{31}t'}A_3(t')\mathrm{d}t'\\ &-\int_0^t G_{33}^{k2}(t-t')\mathrm{e}^{-\mathrm{i}\mu_p^{31}t'}A_3(t')\mathrm{d}t',\end{aligned}\tag{7}$$

where

$$\begin{aligned}G_{33}^{\lambda 1}(t-t') =&\sum_{\lambda e} g_{\lambda e}^{31}g_{\lambda e}^{31}\mathrm{e}^{-i\mu_\lambda^{31}(t-t')},\\ G_{33}^{kj}(t-t') =&\sum_{ke} g_{ke}^{3j}g_{ke}^{3j}\mathrm{e}^{-\mathrm{i}\mu_k^{3j}(t-t')},\quad (j=0,2),\end{aligned}\tag{8}$$

are the delayed Green functions which can be derived in the following forms:[24]

$$\begin{aligned}G_{33}^{kj}(t-t') =&\frac{1}{2}\sqrt{\beta_{3j}^{3/2}\beta_{3j}^{3/2}}\left\{\frac{\mathrm{e}^{\mathrm{i}[\delta_{3jc2}(t-t')-\pi/4]}}{\sqrt{\pi(t-t')}}\right.\\ &\left.+\frac{\mathrm{e}^{\mathrm{i}[\delta_{3jc1}(t-t')+\pi/4]}}{\sqrt{\pi(t-t')}}\right\},\\ G_{33}^{kj}(t-t') =&\frac{1}{2}\sqrt{\alpha_{3j}\alpha_{3j}}\left\{\frac{\mathrm{e}^{\mathrm{i}[\delta_{3jc2}(t-t')+\pi/4]}}{\sqrt{4\pi(t-t')^3}}\right.\\ &\left.+\frac{\mathrm{e}^{\mathrm{i}[\delta_{3jc1}(t-t')-\pi/4]}}{\sqrt{4\pi(t-t')^3}}\right\},\end{aligned}\tag{9}$$

$$\begin{aligned}G_{33}^{kj}(t-t') =&\frac{1}{2}\sqrt{\gamma_{3j}\gamma_{3j}}\delta(t-t')\quad (j=0,2)\\ G_{33}^{\lambda 1}(t-t') =&\frac{1}{2}\sqrt{\gamma_{31}\gamma_{31}}\delta(t-t'),\end{aligned}$$

for isotropic PBG, anisotropic PBG and free space reservoirs, respectively. $\delta_{3jc1}$ ($\delta_{3jc2}$) ($j$=0,2) is the difference between the upper level and the lower (upper) edge of the forbidden gap. The definitions of the parameters are

$$\begin{aligned}\beta_{3j}^{3/2} =&\frac{1}{2\pi\varepsilon_0}\frac{\omega_{3j}^2 d_{3j}^2\omega_{c2}^{3/2}}{3\hbar c^3},\\ \alpha_{3j} =&\frac{1}{2\pi\varepsilon_0}\frac{\omega_{3j}^2 d_{3j}^2\omega_{c2}^{1/2}}{3\hbar c^3},\\ \gamma_{3j} =&\frac{2}{\pi\varepsilon_0}\frac{\omega_{3j}^3 d_{3j}^2}{3\hbar c^3}\quad (j=0,2),\\ \gamma_{31} =&\frac{2}{\pi\varepsilon_0}\frac{\omega_{31}^3 d_{31}^2}{3\hbar c^3}.\end{aligned}\tag{10}$$

The spontaneous spectrum $S(\omega_\lambda)$ for the $\lambda$th mode of spontaneous emission field is the Fourier transform of[1]

$$\begin{aligned}&\langle E^-(t+\tau)E^+(t)\rangle_{t\to\infty}\\ =&\langle\Psi(t)|\sum_{\lambda,\lambda'} a_{\lambda e}^+ a_{\lambda' e}\mathrm{e}^{\mathrm{i}\omega_\lambda(t+\tau)}\mathrm{e}^{-\mathrm{i}\omega_{\lambda'}t}|\Psi(t)\rangle_{t\to\infty}.\end{aligned}\tag{11}$$

Substituting Eq.(5) into Eq.(11), we have

$$\begin{aligned}&\langle E^-(t+\tau)E^+(t)\rangle_{t\to\infty}\\ =&\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}\omega_\lambda\tau}\mathrm{d}\omega_\lambda D(\omega_\lambda)\\ &\times\int\sum_{e=1}^{2}[A_{1\lambda e}^*(\infty)A_{1\lambda e}(\infty)]\mathrm{d}\Omega,\end{aligned}\tag{12}$$

$$\begin{aligned}&\langle E^-(t+\tau)E^+(t)\rangle_{t\to\infty}\\ =&\int_{-\infty}^{+\infty}\mathrm{e}^{\mathrm{i}\omega_k\tau}\mathrm{d}\omega_k D(\omega_k)\\ &\times\int\sum_{e=1}^{2}[A_{0ke}^*(\infty)A_{0ke}(\infty)+A_{2ke}^*(\infty)A_{2ke}(\infty)]\mathrm{d}\Omega,\end{aligned}\tag{13}$$

for the spontaneous emissions along the transitions from the upper level $|3\rangle$ to the lower level $|1\rangle$, and to the lower levels $|2\rangle$ and $|0\rangle$, respectively. Here $D(\omega_k)$ ($D(\omega_\lambda)$) is the DOS of the radiation field, which can be derived as[24]

$$\begin{aligned}D(\omega_k) \sim&\frac{1.0}{\sqrt{\omega_k-\omega_{c2}}}\Theta(\omega_k-\omega_{c2})\\ &+\frac{1.0}{\sqrt{\omega_{c1}-\omega_k}}\Theta(\omega_{c1}-\omega_k),\\ D(\omega_k) \sim&\sqrt{\omega_k-\omega_{c2}}\,\Theta(\omega_k-\omega_{c2})\\ &+\sqrt{\omega_{c1}-\omega_k}\,\Theta(\omega_{c1}-\omega_k),\\ D(\omega_k) \sim&1.0,\\ D(\omega_\lambda) \sim&1.0,\end{aligned}\tag{14}$$

for isotropic PBG, anisotropic PBG and free space reservoirs, respectively, with $\Theta$ being the Heaviside

step function. From Eqs.(6), (7), (12) and (13), we have

$$\begin{aligned} S_(\omega_\lambda) =& D(\omega_\lambda)[P_{31}\widetilde{A}_3^*(s\to -\mathrm{i}\mu_\lambda^{31})\widetilde{A}_3(s\to -\mathrm{i}\mu_\lambda^{31})], \\ S_(\omega_k) =& D(\omega_k)[P_{30}\widetilde{A}_3^*(s\to -\mathrm{i}\mu_k^{30})\widetilde{A}_3(s\to -\mathrm{i}\mu_k^{30}) \\ &+ P_{32}\widetilde{A}_3^*(s\to -\mathrm{i}\mu_k^{32})\widetilde{A}_3(s\to -\mathrm{i}\mu_k^{32})], \end{aligned} \quad (15)$$

for the spontaneous emission spectra along the transitions from the upper level $|3\rangle$ to the lower level $|1\rangle$, and to the lower levels $|2\rangle$ and $|0\rangle$, respectively. Here the definitions of $P_{3j}$ ($j$=0, 1, 2) are

$$\begin{aligned} P_{3j} =& \beta_{3j}^{3/2}, \\ P_{3j} =& \alpha_{3j}, \\ P_{3j} =& \gamma_{3j}, \end{aligned} \quad (16)$$

for isotropic PBG, anisotropic PBG and free vacuum modes respectively. $\widetilde{A}_3(s)$ is the Laplace transform of $A_3(t)$. Taking the Laplace transform of Eq.(7), we obtain

$$\widetilde{A}_3(s) = \frac{A_3(0)}{s+\widetilde{G}_{33}^{\lambda 1}(s)+\widetilde{G}_{33}^{k0}(s)+\widetilde{G}_{33}^{k2}(s)}. \quad (17)$$

Here, $\widetilde{G}_{33}^{k(\lambda)j}(s)$ ($j$=0, 1, 2) is the Laplace transform of the delayed Green function in Eq.(9), which can be derived in the following forms:

$$\begin{aligned} \widetilde{G}_{33}^{kj}(s) =& \frac{1}{2}\sqrt{\beta_{3j}^{3/2}\beta_{3j}^{3/2}}\left[\frac{\mathrm{i}}{\sqrt{\mathrm{i}s+\delta_{3jc1}}}+\frac{1.0}{\sqrt{\mathrm{i}s+\delta_{3jc2}}}\right], \\ \widetilde{G}_{33}^{kj}(s) =& \frac{1}{2}\sqrt{\alpha_{3j}\alpha_{3j}}[-\mathrm{i}\sqrt{\mathrm{i}s+\delta_{3jc1}}+\sqrt{\mathrm{i}s+\delta_{3jc2}}], \\ \widetilde{G}_{33}^{kj}(s) =& \frac{1}{2}\sqrt{\gamma_{3j}\gamma_{3j}}, \quad (j=0,2), \\ \widetilde{G}_{3l}^{\lambda 1}(s) =& \frac{1}{2}\sqrt{\gamma_{31}\gamma_{31}}, \end{aligned} \quad (18)$$

for isotropic PBG, anisotropic PBG and free space reservoirs, respectively. The definitions of the detuning parameters are

$$\begin{aligned} \delta_{30c1} =& \delta_{3c1}-\frac{1}{2}\Delta, \\ \delta_{32c1} =& \delta_{3c1}+\frac{1}{2}\Delta, \\ \delta_{30c2} =& \delta_{30c1}-\delta_{c2c1}, \\ \delta_{32c2} =& \delta_{32c1}-\delta_{c2c1}, \end{aligned} \quad (19)$$

where $\delta_{3jcn}$ ($j$=0, 2, $n$=1, 2) is the difference between the upper level and the lower edge ($n=1$) or the upper edge ($n=2$) of the forbidden gap. The calculation of a spontaneous emission spectrum can be performed based on Eqs.(15), (17) and (18).

## 3. Results and discussion

In order to investigate the effects of the splitting of lower levels on the spontaneous emission spectra in the PBG and free vacuum reservoirs, we plot the spontaneous emission spectra as functions of detuning $\delta_\lambda$ in the three cases, as shown in Fig.2. From Figs.2(b) and (c), we see that the spontaneous emission peak corresponds to the resonant transition from the upper level $|3\rangle$ to the lower level $|1\rangle$. This is to be expected and understandable, as the splitting will change the transitions coupled to the anisotropic PBG and free vacuum reservoirs. However, for the case of isotropic

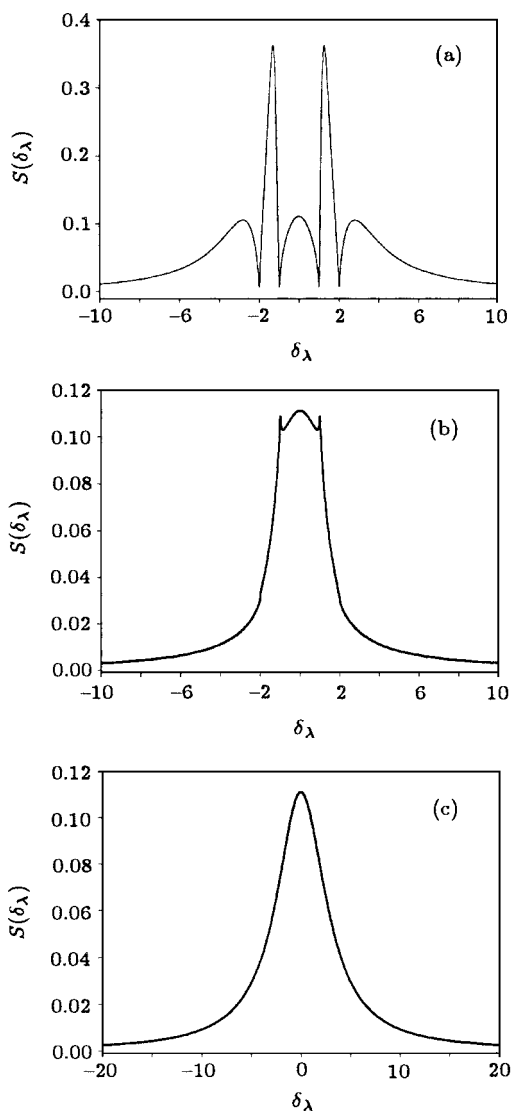



**Fig.2.** The absorption spectrum $S(\delta_\lambda)$ (in arbitrary units) for the transition from the upper level $|3\rangle$ to the lower level $|1\rangle$ as a function of detuning $\delta_\lambda$ of spontaneous emission field (a) for double-band isotropic PBG reservoir with $\beta_{30}=\beta_{32}=1.0$, (b) for double-band anisotropic PBG reservoir with $\alpha_{30}^2=\alpha_{32}^2=1.0$, (c) for free vacuum reservoirs with $\gamma_{30}=\gamma_{32}=1.0$. The other parameters used are $\delta_{c2c1}=1.0$, $\delta_{3c1}=0.5$, $\Delta=3.0$, $A_3(0)=1.0$. All parameters in this paper are in units of $\gamma_{31}$.

PBG reservoir (Fig.2(a)), the effect of the splitting width $\Delta$ on the spontaneous emission spectrum is quite different. It is seen that additional and unexpected peaks appear besides the peaks corresponding to the resonant transition. In order to study what parameters have effect on this new feature, we plot in Figs.3(a), (b) and (c) the spontaneous emission spectra for different splitting width $\Delta$ in the cases of isotropic PBG modes, anisotropic PBG modes and free vacuum modes.

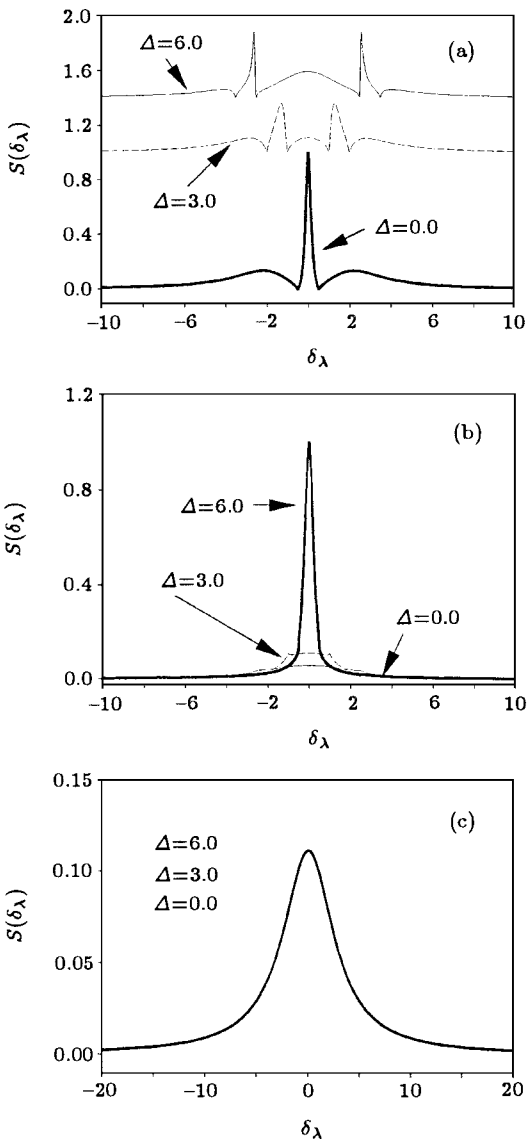



**Fig.3.** The absorption spectrum $S(\delta_\lambda)$ (in arbitrary units) for the transition from the upper level $|3\rangle$ to the lower level $|1\rangle$ as a function of detuning $\delta_\lambda$ of spontaneous emission field for different splitting width of the lower state levels; (a) for double-band isotropic PBG reservoir, (b) for double-band anisotropic PBG reservoir, (c) for free vacuum reservoir. $\Delta = 0.0$ (thick solid lines), $\Delta = 3.0$ (dashed lines), $\Delta = 6.0$ (thin solid lines). The other parameters used are the same as those in Fig.2.

From Fig.3 we see that the splitting of the lower levels has no effect on the spontaneous emission spectrum in the case of the free vacuum modes. The splitting expands the width of spontaneous emission spectrum as the splitting width becomes larger in the case of anisotropic PBG modes, and has a complicated effect on spontaneous emission in the case of isotropic PBG modes. In order to investigate the effect of splitting width of lower levels on the spontaneous emission in the cases of isotropic and anisotropic PBG modes, we plot the spontaneous emission spectra for different widths of forbidden gap as shown in Fig.4, and the different detuning of the upper level from the lower edge of the forbidden gap, as shown in Fig.5. From Figs.2(a), 3(a), 4(a) and 5(a), it can be seen that the detunings for the additional peaks in the case of isotropic PBG modes are covered in two ranges: from $\frac{1}{2}(\Delta - \delta_{c2c1})$ to $\frac{1}{2}(\Delta + \delta_{c2c1})$ and from $-\frac{1}{2}(\Delta - \delta_{c2c1})$ to $-\frac{1}{2}(\Delta + \delta_{c2c1})$, and are unconcerned with other parameters. As a result, the four (two) transparencies with two (one) spontaneous emission peaks appear in the case of isotropic PBG modes. The origin for this feature can be traced back to Eqs.(15), (17) and (18). From these equations, the forms of Laplace transform of the delayed Green function involved in the spontaneous emission spectrum include the following items:

$$\left( \frac{\mathrm{i}}{\sqrt{\delta_k + \frac{1}{2}(\delta_{c2c1} - \Delta)}} + \frac{1.0}{\sqrt{\delta_k - \frac{1}{2}(\delta_{c2c1} + \Delta)}} \right)$$

and

$$\left( \frac{\mathrm{i}}{\sqrt{\delta_k - \frac{1}{2}(\delta_{c2c1} - \Delta)}} + \frac{1.0}{\sqrt{\delta_k + \frac{1}{2}(\delta_{c2c1} + \Delta)}} \right).$$

The above items show clearly that the splitting of the lower levels induces additional singularities of the Laplace transform of the delayed Green function for isotropic PBG modes. These additional singularities occur at $\frac{1}{2}(\Delta - \delta_{c2c1})$, $\frac{1}{2}(\Delta + \delta_{c2c1})$, $-\frac{1}{2}(\Delta - \delta_{c2c1})$ and $-\frac{1}{2}(\Delta + \delta_{c2c1})$. This shows conclusively that the spontaneous emission peaks resulting from the splitting of the atomic ground state stem from the contribution of the splitting to the Laplace transform of the delayed Green function of the isotropic PBG modes.

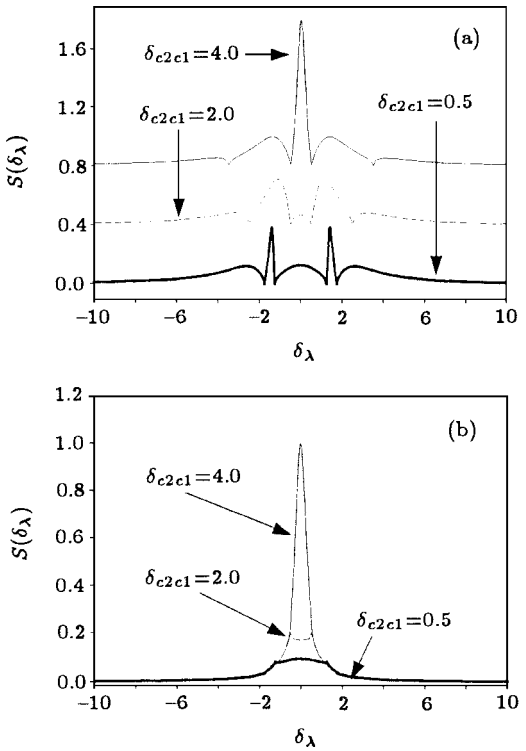



**Fig.4.** The absorption spectrum $S(\delta_\lambda)$ (in arbitrary units) for the transition from the upper level $|3\rangle$ to the lower level $|1\rangle$ as a function of detuning $\delta_\lambda$ of spontaneous emission field for different forbidden gap widths; (a) for double-band isotropic PBG reservoir, (b) for double-band anisotropic PBG reservoir. $\delta_{2c1} = \frac{1}{2}\delta_{c2c1}$, $\delta_{c2c1} = 0.5$ (thick solid lines), $\delta_{c2c1} = 2.0$ (dashed lines), $\delta_{c2c1} = 4.0$ (thin solid lines). The other parameters used are the same as those in Fig.2.

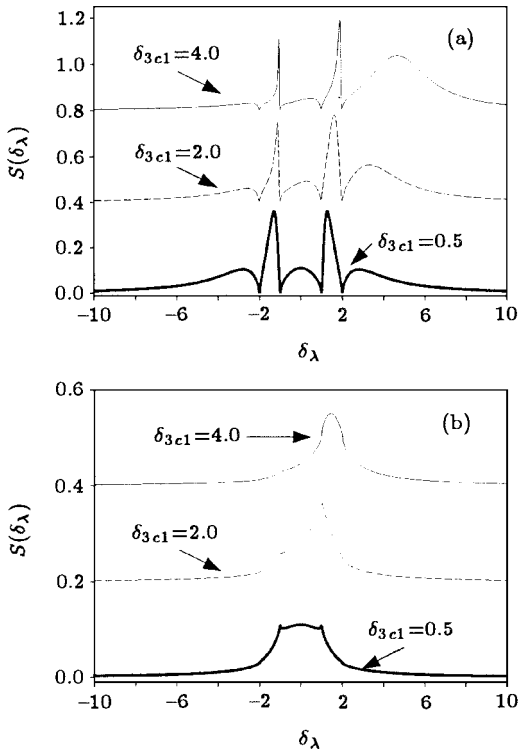



**Fig.5.** The absorption spectrum $S(\delta_\lambda)$ (in arbitrary units) for the transition from the upper level $|3\rangle$ to the lower level $|1\rangle$ as a function of detuning $\delta_\lambda$ of spontaneous emission field for different detunings of the upper level $|3\rangle$ from the lower edge of the forbidden gap; (a) for double-band isotropic PBG reservoir, (b) for double-band anisotropic PBG reservoir. $\delta_{3c1} = 0.5$ (thick solid lines), $\delta_{3c1} = 2.0$ (dashed lines), $\delta_{3c1} = 4.0$ (thin solid lines). The other parameters used are the same as those in Fig.2.

The spontaneous emission spectra discussed above is for the transition from the upper level $|3\rangle$ to the lower level $|1\rangle$. In order to compare it with those for the transitions from the upper level $|3\rangle$ to the lower levels $|2\rangle$ and $|0\rangle$, we plot the spontaneous emission spectra for these transitions, as shown in Fig.6, which shows that the detuning for the additional peaks in the cases that the isotropic PBG modes are different from that discussed above. They cover the ranges from $\Delta - \frac{1}{2}\delta_{c2c1}$ to $\Delta + \frac{1}{2}\delta_{c2c1}$ and from $-\Delta - \frac{1}{2}\delta_{c2c1}$ to $-\Delta + \frac{1}{2}\delta_{c2c1}$. The origin for this feature can be traced back to Eqs.(15), (17) and (18). But, the forms of Laplace transform of the delayed Green function involved in the spontaneous emission spectrum are given as

$$\left(\frac{\mathrm{i}}{\sqrt{\delta_k + \frac{1}{2}\delta_{c2c1} - \Delta}} + \frac{1.0}{\sqrt{\delta_k - \frac{1}{2}\delta_{c2c1} - \Delta}}\right)$$

and

$$\left(\frac{\mathrm{i}}{\sqrt{\delta_k + \frac{1}{2}\delta_{c2c1} + \Delta}} + \frac{1.0}{\sqrt{\delta_k - \frac{1}{2}\delta_{c2c1} + \Delta}}\right).$$

Therefore, these additional singularities occur at $\Delta - \frac{1}{2}\delta_{c2c1}$, $\Delta + \frac{1}{2}\delta_{c2c1}$, $-\Delta - \frac{1}{2}\delta_{c2c1}$ and $-\Delta + \frac{1}{2}\delta_{c2c1}$. The spontaneous emission peaks resulting from the splitting of the atomic ground state stem from the contribution of the splitting to the Laplace transform of the delayed Green function of the isotropic PBG modes. The physical origin is that the DOS of the isotropic PBG radiation field has a sudden change at the edges of the forbidden gap. Furthermore, comparing the solid and dotted lines in Fig.6, we see that the transition from the upper level $|3\rangle$ to the lower level $|1\rangle$ reduces the intensity of spontaneous emission in the transitions from the upper level $|3\rangle$ to the lower levels $|0\rangle$ and $|2\rangle$.

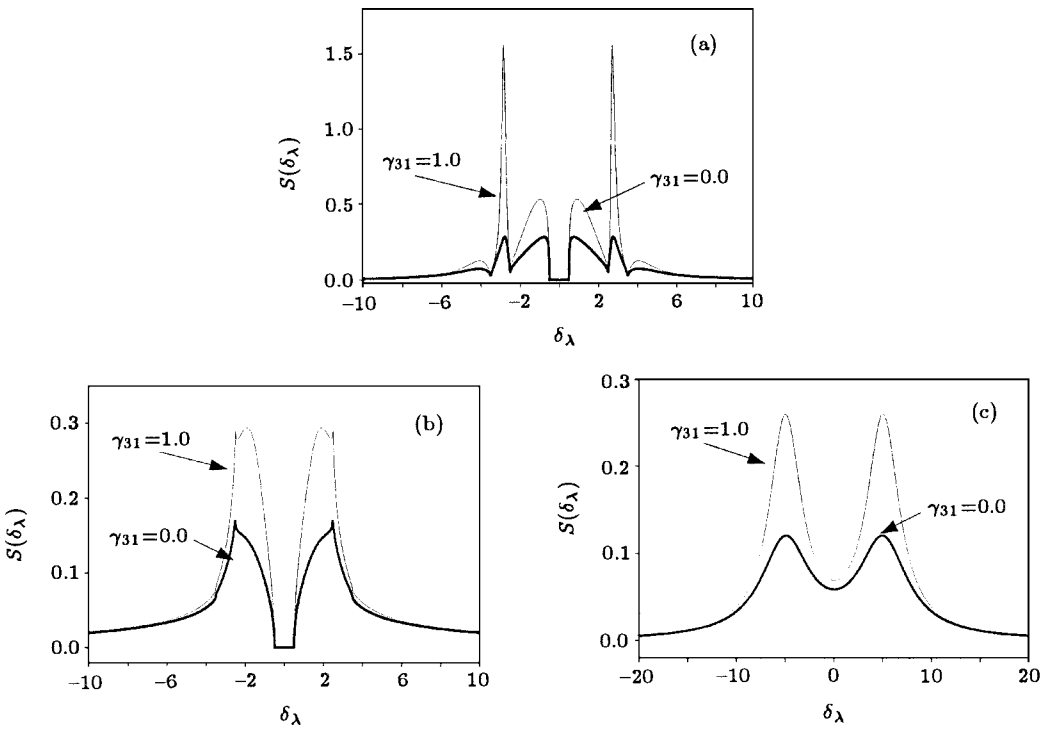



**Fig.6.** The absorption spectra $S(\delta_\lambda)$ (in arbitrary units) for transitions from the upper level $|3\rangle$ to the lower levels $|2\rangle$ and $|0\rangle$ as functions of detuning $\delta_\lambda$ of spontaneous emission field; (a) for double-band isotropic PBG reservoir, (b) for double-band anisotropic PBG reservoir, (c) for free vacuum reservoir and $\Delta = 10.0$. $\gamma_{31} = 0.0$ (thin solid lines), $\gamma_{31} = 1.0$ (thick solid lines). The other parameters used are the same as that in Fig.2.

# 4. Conclusion

The spontaneous emission spectra of a four-level atom embedded in a double-band photonic crystal have been discussed. A transition from the one upper level to one of the three lower levels was assumed to interact with free vacuum modes, and the two transitions from the upper level to the two other lower levels were assumed to interact separately with isotropic PBG modes, anisotropic PBG modes and free vacuum modes. The spontaneous emission spectra for the transitions coupled to the free vacuum modes and the PBG modes are compared with each other. The effect of splittings of the two lower levels on the spontaneous emission spectra of an atom are investigated in detail in three cases. In the case of free vacuum modes, the splitting width has not any effect on the spontaneous emission spectra. The splitting of the lower levels can expand the width of spontaneous emission spectra as the splitting width is large in the case of anisotropic PBG modes. It induces the four (two) transparencies with two (one) spontaneous emission peaks in the case of isotropic PBG modes. This new features result from the DOS of the isotropic PBG radiation field having a sudden change at the edges of the forbidden gap.

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