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Intrinsic flux noise level of an rf SQUID involving two Josephson junctions connected in series*

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The time-dependent characteristics of a radio frequency superconducting quantum interference device (rf SQUID) involving two Josephson junctions connected in series are studied. First the dynamics of the total flux is given. Then the intrinsic flux noises in both nonhysteretic and hysteretic modes are investigated. The result shows that it is necessary to properly reduce the critical current ratio of the two junctions for low intrinsic noise level. When the critical current ratio is properly reduced, the intrinsic flux noise level of a two-junction rf SQUID is generally close to or slightly larger than that of a single junction rf SQUID, but no more than one order-of-magnitude.

Keywords: rf SQUID, RCSJ model, intrinsic flux noise, hysteretic and nonhysteretic modes
PACC: 0755, 7450

1. Introduction

As a sensitive magnetic field detecting device, the high temperature superconductor (HTS) grain boundary junction (GBJ) radio frequency superconducting quantum interference device (rf SQUID) has many applications. There exist two types of HTS GBJ rf SQUIDs: step edge rf SQUID and bicrystal rf SQUID.[1,2] However, due to the fabrication technology, both of these HTS GBJ rf SQUIDs usually involve two Josephson junctions in series,[1,2] not a single junction as expected. Therefore it is necessary to explore the properties of such a two-junction rf SQUID. In Ref.[3] the equilibrium behaviour of a two-junction rf SQUID has been studied theoretically. To investigate the intrinsic noise we must consider its time-dependent properties.

In this paper, the time-dependent behaviours of a two-junction rf SQUID are investigated for the first time. The intrinsic flux noise levels in the nonhysteretic and hysteretic modes are given. Many results are compared with those of the single-junction rf SQUID.

2. General considerations

In order to study the intrinsic noise, we need to consider the time-dependent behaviours of the junctions. We take the resistively and capacitively shunted junction (RCSJ) model[4] in this paper.

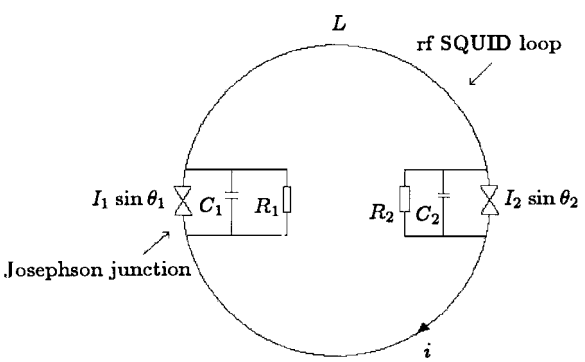


Fig.1. The schematic diagram of an rf SQUID loop closed with two junctions connected in series. Both junctions are resistively and capacitively shunted.

The schematic diagram of a two-junction rf SQUID is drawn in Fig.1. The loop inductance is L. The critical currents of the two junctions are I_1 and I_2 respectively. We assume $I_1 \leq I_2$, and define

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the critical current ratio as $\alpha=I_1/I_2$, $0<\alpha\leq 1$. The Josephson junctions are shunted by the resistances R_1 and R_2, and the capacitances C_1 and C_2. All these parameters are assumed to be constant.

From fluxoid quantization we have

$$\theta_1+\theta_2+\frac{2\pi\phi}{\phi_0}=2k\pi, \tag{1}$$

where ϕ is the total magnetic flux through the loop, θ_1 and θ_2 are the phase differences of the two junctions, ϕ_0 is the flux quantum $h/2e\approx 2.07\times10^{-15}$ Wb and k is an integer.

When the external flux applied through the loop is ϕ_x, we have

$$\phi=\phi_x+Li, \tag{2}$$

where i is the circulating superconducting current flowing along the loop.

The voltages V_1 and V_2 across the junctions are respectively related to the phase differences θ_1 and θ_2 as follows:

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}t}=\frac{2\pi}{\phi_0}V_1,\quad \frac{\mathrm{d}\theta_2}{\mathrm{d}t}=\frac{2\pi}{\phi_0}V_2. \tag{3}$$

According to the RCSJ model,[4] the total current i can be written as

$$\begin{aligned} i=&I_1\sin\theta_1+C_1\frac{\mathrm{d}V_1}{\mathrm{d}t}+\frac{V_1}{R_1}+\tilde{i}\\ =&I_2\sin\theta_2+C_2\frac{\mathrm{d}V_2}{\mathrm{d}t}+\frac{V_2}{R_2}+\tilde{i}, \end{aligned} \tag{4}$$

where $\tilde{i}$ is the thermal noise current.

3. Dynamics of the total flux

To describe the time-dependent behaviour of a two-junction rf SQUID, we assume in our case that the potential $U(\phi,\phi_x)$ has the form

$$U(\phi,\phi_x)=\frac{(\phi-\phi_x)^2}{2L}-\frac{\phi_0}{2\pi}(I_1\cos\theta_1+I_2\cos\theta_2). \tag{5}$$

Consider $\mathrm{d}U/\mathrm{d}\phi$; when the loop is applied with a constant external field ϕ_x, we have

$$\begin{aligned} \frac{\mathrm{d}U(\phi,\phi_x)}{\mathrm{d}\phi}=&\left(-\frac{\phi_0}{2\pi}\frac{\mathrm{d}\theta_1}{\mathrm{d}\phi}\right)\left[C_1\frac{\mathrm{d}V_1}{\mathrm{d}t}+\frac{V_1}{R_1}\right]\\ &+\left(\frac{\phi_0}{2\pi}\frac{\mathrm{d}\theta_1}{\mathrm{d}\phi}+1\right)\left[C_2\frac{\mathrm{d}V_2}{\mathrm{d}t}+\frac{V_2}{R_2}\right]+\tilde{i}. \end{aligned} \tag{6}$$

From Eq.(3), we obtain

$$\begin{aligned} V_1=&\frac{\phi_0}{2\pi}\frac{\mathrm{d}\theta_1}{\mathrm{d}\phi}\frac{\mathrm{d}\phi}{\mathrm{d}t}=\frac{\phi_0}{2\pi}\frac{\mathrm{d}\theta_1}{\mathrm{d}\phi}\dot{\phi},\\ \frac{\mathrm{d}V_1}{\mathrm{d}t}=&\frac{\phi_0}{2\pi}\frac{\mathrm{d}\theta_1}{\mathrm{d}\phi}\ddot{\phi}+\frac{\phi_0}{2\pi}\frac{\mathrm{d}^2\theta_1}{\mathrm{d}\phi^2}\dot{\phi}^2. \end{aligned}$$

In the case of high damping, $1/(R_1C_1)$ and $1/(R_2C_2)$ are both large, $RC\dot{\phi}/\phi_0\ll 1$, the term of $\dot{\phi}^2$ is negligible.

Here we must emphasize an approximation. When applied with a certain external flux ϕ_x, the total flux $\phi(t)$ will deviate from the equilibrium total flux ϕ_{eq} due to the thermal fluctuations. We assume that the thermal fluctuations are relatively small, i.e. $(\phi(t)-\phi_{\mathrm{eq}})/\phi_{\mathrm{eq}}\ll 1$. The equilibrium behaviour of the two-junction rf SQUID has been studied in Ref.[3]. Some results from Ref.[3] are used here:[1)]

We find

$$\frac{\mathrm{d}\theta_1}{\mathrm{d}\phi}=-\frac{2\pi}{\phi_0}\frac{1+\alpha\cos\dfrac{2\pi\phi}{\phi_0}}{1+\alpha^2+2\alpha\cos\dfrac{2\pi\phi}{\phi_0}}.$$

By arranging Eq.(6), finally we have

$$\frac{\mathrm{d}U(\phi,\phi_x)}{\mathrm{d}\phi}=-\left(C'\ddot{\phi}+\frac{\dot{\phi}}{R'}\right)+\tilde{i}, \tag{7}$$

where

$$C'=C'(\phi)=\frac{C_1\left(1+\alpha\cos\dfrac{2\pi\phi}{\phi_0}\right)^2+C_2\left(\alpha^2+\alpha\cos\dfrac{2\pi\phi}{\phi_0}\right)^2}{\left(1+\alpha^2+2\alpha\cos\dfrac{2\pi\phi}{\phi_0}\right)^2}, \tag{8}$$

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1) $\sin\theta_1=\dfrac{-\sin\left(2\pi\dfrac{\phi}{\phi_0}\right)}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)\right]^{1/2}}$, $\cos\theta_1=\dfrac{\alpha+\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)\right]^{1/2}}$;

$\sin\theta_2=\dfrac{-\alpha\sin\left(2\pi\dfrac{\phi}{\phi_0}\right)}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)\right]^{1/2}}$, $\cos\theta_2=\dfrac{1+\alpha\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\dfrac{\phi}{\phi_0}\right)\right]^{1/2}}$.

$$\frac{1}{R'} = \frac{1}{R'(\phi)} = \frac{\frac{1}{R_1}\left(1+\alpha\cos\frac{2\pi\phi}{\phi_0}\right)^2 + \frac{1}{R_2}\left(\alpha^2+\alpha\cos\frac{2\pi\phi}{\phi_0}\right)^2}{\left(1+\alpha^2+2\alpha\cos\frac{2\pi\phi}{\phi_0}\right)^2}. \tag{9}$$

Equation (7) can be rearranged as

$$\ddot{\phi} = -\frac{1}{C'}\frac{\mathrm{d}U}{\mathrm{d}\phi} - \frac{1}{R'C'}\dot{\phi} + \frac{\tilde{i}}{C'}. \tag{10}$$

Approximating $C'(\phi)$ and $R'(\phi)$ with $C'(\phi_{\mathrm{eq}})$, and $R'(\phi_{\mathrm{eq}})$ respectively, we obtain a standard equation of Brownian motion for the total flux $\phi(t)$ in an equivalent potential field U/C'. From Eq.(10), we notice that a two-junction rf SQUID is equivalent to a single-junction rf SQUID, with the junction shunted by the effective capacitance $C'(\phi)$ and the effective resistance $R'(\phi)$.

4. Intrinsic noise level

Define the hysteresis parameter $\beta{=}2\pi LI_1/\phi_0$. As the study in Ref.[3] shows, when $\beta < 1-\alpha$, the rf SQUID will operate in the nonhysteretic mode and when $\beta > 1-\alpha$, it will operate in the hysteretic mode. The system must operate in the hysteretic mode when the critical current ratio $\alpha = 1$.

In the nonhysteretic mode, the thermal noise current $\tilde{i}$ is generated in the effective resistance $R'(\phi)$. The noise current spectral density $S_{\tilde{i}}$ is proportional to $k_{\mathrm{B}}T/R'$.[5] So we obtain the intrinsic white flux noise $\langle\delta\phi_i^2\rangle^{1/2}/\mathrm{Hz}^{1/2} = (L^2S_{\tilde{i}})^{1/2} \propto L\left(\frac{T}{R'}\right)^{1/2}$, where R' is given by Eq.(9). Considering the expression of the effective resistance R' in Eq.(9), we find

$$\frac{1}{R'(\phi)} \in \left[\frac{1}{(1+\alpha)^2R_1} + \frac{\alpha^2}{(1+\alpha)^2R_2}, \frac{1}{(1-\alpha)^2R_1} + \frac{\alpha^2}{(1-\alpha)^2R_2}\right].$$

To avoid large intrinsic flux noise in the nonhysteretic mode, the critical current ratio α should be properly reduced. As was reported by Herrmann *et al*, for the step edge rf SQUID of large step angle, the critical current ratio is typically 1/3–1/5.[1] If $R_2 \approx R_1$, then $R' \to R_1$. However, in the case of bicrystal rf SQUID,[2] the value of critical current ratio depends mainly on the layout design and can range from 0 to 1. So we must be cautious to increase the critical current ratio when we fabricate the bicrystal rf SQUID. The bicrystal rf SQUID with about 1/4 asymmetric junction width ratio was reported by Fardmanesh *et al.*[2]

In general, when the critical current ratio α is properly reduced, the intrinsic flux noise level of a two-junction rf SQUID will be close to that of a single junction rf SQUID, which has the same shunt resistance R_1.

In the hysteretic mode, $\beta > 1-\alpha$, the above thermal noise also exists. However, the thermal fluctuations will cause the flux transitions in the hysteretic mode. The flux transitions will generate an extra noise, which is much greater than the thermal noise normally. In the following part we will study such an extra noise. Kurkijarvi studied the corresponding extra noise of a single-junction rf SQUID.[6] Some methods used in Ref.[6] is also applicable here.

From Eq.(5), rewrite $U(\phi,\phi_x)$ in the approximation mentioned before:[1)]

$$U(\phi,\phi_x) = \frac{(\phi-\phi_x)^2}{2L} - \frac{\phi_0 I_1}{2\pi\alpha}\left(1+\alpha^2+2\alpha\cos\frac{2\pi\phi}{\phi_0}\right)^{1/2}. \tag{11}$$

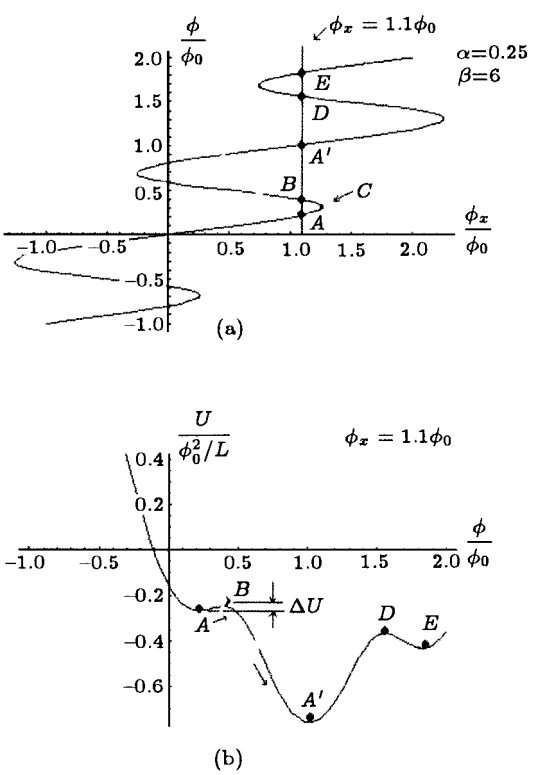


Fig.2. (a) The equilibrium ϕ-ϕ_x curve with $\phi_x = 1.1\phi_0$, $\alpha = 0.25$, $\beta = 6$; (b) the corresponding U vs ϕ curve.

[1)] See p302, 1).

Figure 2 is an example with certain values of ϕ_x, α and β. Figure 2(a) shows the equilibrium ϕ-ϕ_x curve, which is defined by the equation

$$\phi_x = \phi + \frac{LI_1 \sin\left(2\pi\frac{\phi}{\phi_0}\right)^{[3]}}{\left[1+\alpha^2+2\alpha\cos\left(2\pi\frac{\phi}{\phi_0}\right)\right]^{1/2}}.$$

The U–ϕ curve is plotted in Fig.2(b). Consider two states (ϕ_{xA}, ϕ_A) and (ϕ_{xB}, ϕ_B), $\phi_{xA} = \phi_{xB} = \phi_x$, and define $\Delta\phi_x = \phi_{xC} - \phi_x$. ϕ_{xC} corresponding to the state (ϕ_{xC}, ϕ_C) in Fig.2(a), where $\frac{\mathrm{d}\phi_\mathrm{x}}{\mathrm{d}\phi}|_{\phi=\phi_c}$=0, i.e.

$$\beta\frac{\left(1+\alpha\cos\frac{2\pi\phi_C}{\phi_0}\right)\left(\alpha+\cos\frac{2\pi\phi_C}{\phi_0}\right)}{\left(1+\alpha^2+2\alpha\cos\frac{2\pi\phi_C}{\phi_0}\right)^{3/2}}+1=0.$$

where

$$\gamma=\gamma(\alpha,\beta)=\beta\sin\frac{2\pi\phi_C}{\phi_0}\left\{\frac{1}{\left(1+\alpha^2+2\alpha\cos\frac{2\pi\phi_C}{\phi_0}\right)^{1/2}}+\frac{3\alpha}{\beta}\frac{1}{1+\alpha^2+2\alpha\cos\frac{2\pi\phi_C}{\phi_0}}\right\},$$
$$\phi_C=\phi_C(\alpha,\beta). \tag{14}$$

The potential difference between points A and B is also given as a function of $\Delta\phi_x$:

$$\begin{aligned}\Delta U =& U_B - U_A\\ =&\frac{4}{3L}\left(\frac{2}{\gamma}\right)^{1/2}\left(\frac{\phi_0}{2\pi}\right)^2\left(\frac{2\pi}{\phi_0}\Delta\phi_x\right)^{3/2}\\ =&U_0(\frac{2\pi}{\phi_0}\Delta\phi_x)^{3/2},\end{aligned} \tag{15}$$

where $U_0=\frac{4}{3L}\left(\frac{2}{\gamma}\right)^{1/2}\left(\frac{\phi_0}{2\pi}\right)^2$.

We also find

$$\left.\frac{\mathrm{d}^2U}{\mathrm{d}\phi^2}\right|_{A,B}\propto\pm\left(\frac{2\pi}{\phi_0}\Delta\phi_x\right)^{1/2},$$

$$\omega_{A,B}=\sqrt{\left|\frac{d^2U}{d\phi^2}\right|/C'}\Big|_{A,B}=\omega_{0\ A,B}\left(\frac{2\pi}{\phi_0}\Delta\phi_x\right)^{1/4},$$

where $\omega_{0\ A,B}$ is a prefactor.

Comparing the above results of two-junction rf SQUID with those of single-junction rf SQUID,[6] we obtain almost the same expression of the rate P as a function of $\Delta\phi_x$. The difference between them lies in the different coefficients such as U_0, $\omega_{0A,B}$, η and γ. We also find that when $\gamma(\alpha,\beta)=\gamma(0,\beta)$, the coefficient U_0 is turned into $\frac{4}{3L}\left[2\beta^{-1}\left(1-\frac{1}{\beta^2}\right)^{-1/2}\right]^{1/2}\left(\frac{\phi_0}{2\pi}\right)^2$, i.e. the expression of U_0 in the case of single-junction rf SQUID. We still assume that the external flux is swept at a constant but relatively slow rate i.e. $\frac{\dot{\phi}_x}{\phi_0}\ll\frac{R'}{L}$ in order to leave enough time for the transitions. In evaluating the level of the extra intrinsic noise, we care about the width of the probability distribution in terms of ϕ_x, which is given by

$$\sigma_{\phi_x}=\left\langle\left(\phi_x-\bar{\phi}_x\right)^2\right\rangle^{1/2}=\sigma_0\frac{\phi_0}{2\pi}\left(\frac{k_\mathrm{B}T}{U_0}\right)^{2/3}. \tag{16}$$

It is found[8] that the spectral density of the extra intrinsic noise is

$$\left\langle\delta\phi_i^2\right\rangle/\mathrm{Hz}\cong\frac{1}{2}\left(\frac{\omega_\mathrm{rf}}{2\pi}\right)^{-1}\sigma_{\phi_x}^2,$$

where $\omega_\mathrm{rf}/2\pi$ is the bias frequency. Therefore the corresponding intrinsic noise is

$$\begin{aligned}\left\langle\delta\phi_i^2\right\rangle^{1/2}/\mathrm{Hz}^{1/2}\cong&\left(\frac{\pi}{\omega_\mathrm{rf}}\right)^{1/2}\sigma_0\frac{\phi_0}{2\pi}\left(\frac{k_\mathrm{B}T}{U_0}\right)^{2/3}\\ =&\sigma_0\pi^{1/2}\left(\frac{2\pi}{\phi_0}\right)^{1/3}\left(\frac{3}{4\sqrt{2}}\right)^{2/3}\\ &\times\frac{(k_\mathrm{B}T)^{2/3}}{\omega_\mathrm{rf}^{1/2}}L^{2/3}\gamma^{1/3}.\end{aligned} \tag{17}$$

Consider a transition from A to A' through B, as indicated in Fig.2. Stochastic theory [7] shows in the case of high damping, where $1/(R'C')$ is large, the transition rate P at which the system transits from A to A' through B per unit time is given by

$$P=\frac{\omega_\mathrm{A}\omega_B}{2\pi\eta}\exp\left(-\frac{U_B-U_A}{k_\mathrm{B}T}\right), \tag{12}$$

where $\eta=\frac{1}{R'C'}$, $\omega_{A,B}=\sqrt{\left|\frac{\mathrm{d}^2U}{\mathrm{d}\phi^2}\right|/C'}\Big|_{A,B}$, k_B is the Boltzmann constant, T is the temperature. η can be approximated with $1/(R'(\phi_C)C'(\phi_C))$.

Expand ϕ as a function of $\Delta\phi_x$ around ϕ_{xC} to the lowest order in $\Delta\phi_x$:

$$\phi_{A,B}=\phi_C\mp\left(\frac{\phi_0}{\pi}\right)^{1/2}\gamma^{-1/2}(\Delta\phi_x)^{1/2}, \tag{13}$$

The prefactor σ_0 is related to many parameters such as U_0, $\omega_{0\ A,B}$, η, T and the sweep rate. However, σ_0 depends very weakly on these parameters and is typically 0.4.[6]

Finally, we have

$$\langle \delta\phi_i^2 \rangle^{1/2} / \mathrm{Hz}^{1/2} \propto \frac{T^{2/3}}{\omega_{\mathrm{rf}}^{1/2}} L^{2/3} \gamma^{1/3}, \qquad (18)$$

where the parameter $\gamma(\alpha, \beta)$ is defined by Eq.(14). The extra intrinsic noise is related to the loop inductance L and the critical currents of the junctions I_1 and I_2.

It is interesting to compare the extra intrinsic noise level of a two-junction rf SQUID with that of a single junction rf SQUID. Assume that both systems have the same operating temperature T, bias frequency $\omega_{\mathrm{rf}}/2\pi$, loop inductance L and junction critical current I_1, i.e. the same hysteresis parameter β. From Eq.(18) the ratio of the extra intrinsic noise of two-junction rf SQUID to that of single-junction rf SQUID is given by $\left[\frac{\gamma(\alpha,\beta)}{\gamma(0,\beta)}\right]^{1/3}$, as is plotted in Fig.3. The hysteresis parameter β is chosen to start from 1 because a single-junction rf SQUID will operate in the hysteretic mode only when $\beta > 1$. For a certain hysteresis parameter β, the noise ratio becomes greater when the critical current ratio α is increased. Here we find that properly reducing the critical current ratio will be useful to obtain low intrinsic noise. We also notice that the extra intrinsic noise of two-junction rf SQUID is generally greater than that of single-junction rf SQUID, but no more than one order-of-magnitude.

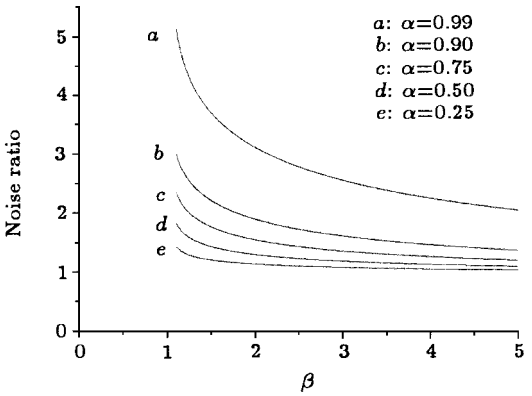


Fig.3. In the hysteretic mode, ratio of the extra intrinsic noise of two-junction rf SQUID to that of single-junction rf SQUID vs. hysteresis parameter $\beta(> 1)$, for different critical currents α=0.99, 0.9, 0.75, 0.5 and 0.25.

5. Conclusions

In this paper, we have investigated the time-dependent characteristics of a two-junction rf SQUID. The dynamics of the total flux is studied and the intrinsic noise levels in both nonhysteretic and hysteretic modes are given. The results show that the intrinsic noise level depends on many parameters such as the critical current ratio α, the shunt resistances of junctions R_1 and R_2 and the hysteresis parameter β. We find that in both nonhysteretic and hysteretic modes, properly reducing the critical current ratio will make the intrinsic noise lower. This conclusion is very useful for the experimental work. We also compare two-junction rf SQUID with single-junction rf SQUID. In general, when the critical current ratio is properly reduced, the intrinsic flux noise level of a two-junction rf SQUID is close to or greater than that of a single-junction rf SQUID, but no more than one order-of-magnitude.

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