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Adaptive synchronization with nonlinear input

Zhang Xin-Ying(张新莹)†, Guan Xin-Ping(关新平), and Li Hui-Guang(李惠光)

Institute of Electrical Engineering, Yanshan University, Qinhuangdao 066004, *China*

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This paper considers the adaptive synchronization problem of the drive–response type chaotic systems. Using a transmitted scalar signal with an unknown time-delay, a response system is constructed. By appropriately selecting the observer gain and designing the controller, synchronization can be achieved in the presence of the drive system's disturbances and unknown parameters. A well-known chaotic system, Chua's circuit, is considered as an illustrative example to demonstrate the effectiveness of the proposed approach.

Keywords: chaos synchronization, adaptive synchronization, time-delay, output signal
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1. Introduction

In the last decades, chaos synchronization has been widely investigated due to its potential application in many areas such as secure communication, information processing, biological systems, and chemical reactions. A large variety of approaches have been proposed such as backstepping design technique,[1,2] variable structure control,[3,4] delayed feedback control,[5] among many others.[6,7]

Recently, Jiang *et al*[8] addressed a practical issue in chaos synchronization where there is a time-delay in the receiver relative to the transmitter. The approach of unidirectional linear error feedback coupling with time-delay was used. However, it is worth noticing that the system's disturbances are always unavoidable, and some system parameters cannot be exactly known a priori. Under these conditions, the synchronization scheme proposed in Ref.[8] cannot achieve synchronization successfully. Many works have dealt with the unknown parameters: Ref.[9] presents an adaptive synchronization design method via backstepping design for Chua's circuit; Chen and Lu[10] propose a novel control method for a class of chaotic systems dependent linearly on unknown parameters based upon adaptive control; Ref.[11] uses the Lyapunov stability theory to derive an adaptive control law for the Lu system. But all these methods are under the hypothesis that full state variables of the chaotic system are available. However, it is not reasonable to require this to be satisfied in the practical environment. Moreover, in most publications, all is under the assumption of linear input. However, when the controller is realized in practical physical systems, nonlinearity in control does exist due to the physical limitations of actuators. Ignoring its effect may cause serious influence on system performance, and decrease the system response since the chaotic system is very sensitive to any change in system parameters.

In view of the above considerations, the object of this work is to derive an adaptive observer-based response system via a transmitted scalar signal, which has an unknown time-delay to the output signal of the drive system. The layout of this paper is as follows. Section 2 develops an adaptive observer-based response system to synchronize the drive system with the system's disturbances and unknown parameters. Considering the nonlinearity in the practical controller, the synchronization is guaranteed by the Lyapunov stability theory with mild conditions. In Section 3, we present Chua's circuit as an illustrative example to demonstrate the effectiveness of the proposed approach. Finally, we draw conclusions in Section 4.

2. Adaptive synchronization with nonlinear input

Consider a chaotic system of the following form:

$$\dot{x}(t) = Ax(t) + f(y(t)) + B(\varphi^{\mathrm{T}}(y(t)\theta + \delta), \quad (1a)$$

$$y(t) = Cx(t), \quad (1b)$$

†E-mail: zxy@88mail.ysu.edu.cn

where $x(t) \in R^n$ and $y(t) \in R$ are the state vector and the output, respectively. A, B and C are constant matrices of appropriate dimensions. $\theta \in R^p$ represents a constant vector of unknown parameters. f: $R^n \to R^n$ and ϕ: $R \to R^p$ are smooth vector fields. $\delta \in R$ is an unknown bounded disturbance.

We assume that the following assumptions pertain to the drive system (1).

A1 There exists an unknown positive scalar d such that $|\delta| \leq d$.

A2 We can choose a gain matrix L and two symmetric and positive definite matrices P and Q satisfying

$$(A - LC)^{\mathrm{T}}P + P(A - LC) = -Q, \tag{2}$$

$$B^{\mathrm{T}}P = C. \tag{3}$$

Remark 1 According to the Kalman and Yakubovich lemma,[12] the strict positive realness of $H(s) = C(sI - (A - LC))^{-1}B$ assures the existence of matrices P and Q such that Eqs.(2) and (3) are satisfied.

We want, for the given drive system modelled by Eq.(1), to design an adaptive observer-based response system on the basis of an available transmitted scalar signal so that these drive and response systems are synchronized.

A response system in the drive–response configuration of chaotic synchronization corresponding to Eq.(1) is given as

$$\dot{\bar{x}}(t) = A\bar{x}(t) + f(y(t-\tau)) + L(y(t-\tau) - \bar{y}(t)) + B\phi(u(t)), \tag{4a}$$

$$\bar{y}(t) = C\bar{x}(t), \tag{4b}$$

where $\bar{x}(t)$ denotes the dynamic estimate of the state $x(t)$. τ is a finite time-delay, which is an unknown constant. $L \in R^n$ is chosen such that $(A - LC)$ is an exponentially stable matrix. $\Phi(u(t))$ is the practical nonlinear control input instead of the linear control input $u(t)$ due to physical limitations of actuators, which satisfies

$$\beta_1 u^2(t) \leq u(t)\phi(u(t)) \leq \beta_2 u^2(t), \tag{5}$$

where β_1 and β_2 are nonzero positive constants. A nonlinear function $\Phi(u(t))$ is illustrated in Fig.1.

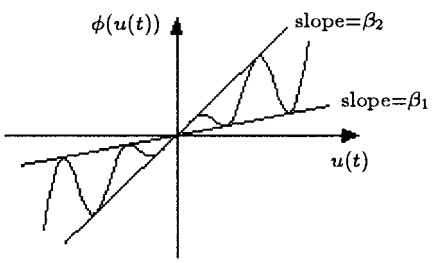


Fig.1. A scalar nonlinear function $\Phi(u(t))$.

The scheme of chaos synchronization using an output signal with an unknown time-delay is shown in Fig.2.

Let $e(t) = \bar{x}(t) - x(t-\tau)$, we get

$$\dot{e}(t) = (A - LC)e(t) - B(\varphi^{\mathrm{T}}(y(t-\tau))\theta + \delta) + B\phi(u(t)). \tag{6}$$

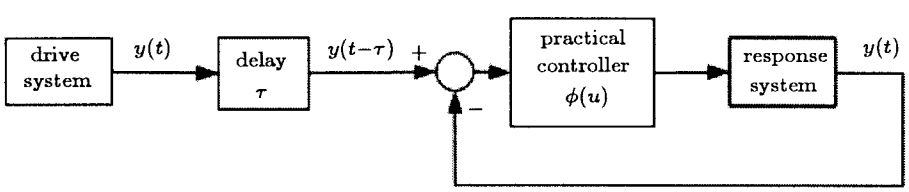


Fig.2. Scheme of chaos synchronization using an output signal with an unknown time-delay τ.

Theorem 1 Consider the drive chaotic system (1) satisfying the assumptions A1 and A2. The observer-based response system (4) globally asymptotically synchronizes the drive system (1) by employing the following control law and adaptation algorithms, i.e. $\|e(t)\| = \|\bar{x}(t) - x(t-\tau)\| \to 0$ as $t \to \infty$ for all initial conditions.

$$u(t) = -\gamma\eta\,\mathrm{sign}(e^{\mathrm{T}}(t)PB), \tag{7}$$

where $\eta = (\hat{d} + |\varphi^{\mathrm{T}}(y(t-\tau))\hat{\theta}|)$ and $\gamma > \dfrac{1}{\beta_1}$.

$$\dot{\hat{\theta}} = -(\bar{y}(t) - y(t-\tau))R\varphi(y(t-\tau)), \quad R > 0, \tag{8}$$

$$\dot{\hat{d}} = l|\bar{y}(t) - y(t-\tau)|, \quad l > 0. \tag{9}$$

Proof Consider a Lyapunov function

$$V = e^{\mathrm{T}}(t)Pe(t) + (\theta - \hat{\theta})^{\mathrm{T}}R^{-1}(\theta - \hat{\theta}) + \frac{1}{l}(d - \hat{d})^2. \tag{10}$$

Taking the time derivative of V along the trajectories of the resulting error dynamical system (6) leads to

$$\begin{aligned}\dot{V} =&e^{\mathrm{T}}(t)((A-LC)^{\mathrm{T}}P+P(A-LC))e(t)\\ &-2e^{\mathrm{T}}(t)PB(\varphi^{\mathrm{T}}(y(t-\tau))\theta+\delta)\\ &+2e^{\mathrm{T}}(t)PB\phi(u(t))-2(\theta-\hat{\theta})^{\mathrm{T}}R^{-1}\dot{\hat{\theta}}\\ &-\frac{2}{l}(d-\hat{d})\dot{\hat{d}}. \end{aligned} \tag{11}$$

Consider assumption A1 and Eq.(2); we obtain

$$\begin{aligned}\dot{V} \le&-e^{\mathrm{T}}(t)Qe(t)-2e^{\mathrm{T}}(t)PB\varphi^{\mathrm{T}}(y(t-\tau))\theta\\ &+2d|e^{\mathrm{T}}(t)PB|+2e^{\mathrm{T}}(t)PB\phi(u(t))\\ &-2(\theta-\hat{\theta})^{\mathrm{T}}R^{-1}\dot{\hat{\theta}}-\frac{2}{l}(d-\hat{d})\dot{\hat{d}}\\ \le&-e^{\mathrm{T}}(t)Qe(t)+2|e^{\mathrm{T}}(t)PB||\varphi^{\mathrm{T}}(y(t-\tau))\hat{\theta}|\\ &+2\hat{d}|e^{\mathrm{T}}(t)PB|+2e^{\mathrm{T}}(t)PB\phi(u(t))\\ &-2(\theta-\hat{\theta})^{\mathrm{T}}(\varphi(y(t-\tau))B^{\mathrm{T}}Pe(t)+R^{-1}\dot{\hat{\theta}})\\ &+2(d-\hat{d})\left(|e^{\mathrm{T}}(t)PB|-\frac{1}{l}\dot{\hat{d}}\right). \end{aligned} \tag{12}$$

Substitute Eq.(3), Eq.(8) and Eq.(9) into Eq.(12),

$$\begin{aligned}\dot{V} \le&-e^{\mathrm{T}}(t)Qe(t)+2|e^{\mathrm{T}}(t)PB||\varphi^{\mathrm{T}}(y(t-\tau))\hat{\theta}|\\ &+2\hat{d}|e^{\mathrm{T}}(t)PB|+2e^{\mathrm{T}}(t)PB\phi(u(t)). \end{aligned} \tag{13}$$

From Eq.(5) and Eq.(7), we can derive

$$\begin{aligned}u\phi(u) =&-\gamma\eta\mathrm{sign}(e^{\mathrm{T}}(t)PB)\phi(u)\ge\beta_1\gamma^2\eta^2\\ \Rightarrow&\mathrm{sign}(e^{\mathrm{T}}(t)PB)\phi(u)\le-\beta_1\gamma\eta. \end{aligned} \tag{14}$$

Since $(e^{\mathrm{T}}(t)PB)^2\ge 0$, we further have

$$e^{\mathrm{T}}(t)PB\phi(u)\le-\beta_1\gamma\eta|e^{\mathrm{T}}(t)PB|. \tag{15}$$

Consequently, this yields

$$\dot{V}\le-e^{\mathrm{T}}(t)Qe(t)+2\eta|e^{\mathrm{T}}(t)PB|(1-\beta_1\gamma). \tag{16}$$

As $\gamma>\dfrac{1}{\beta_1}$, we can conclude

$$\dot{V}<-e^{\mathrm{T}}(t)Qe(t)<0 \tag{17}$$

for any $e\neq 0$. So the system (6) is globally exponentially stable about zero. This completes the proof.

3. An illustrative example of Chua's circuit

To illustrate the adaptive synchronization scheme proposed herein, a well-known chaotic system is considered and its simulation results are presented.

A typical Chua circuit and its physical meaning can be found in Ref.[13]. After appropriate variable and parameter transformations and taking into account the system's disturbances, the perturbed chaotic system can be expressed by

$$\begin{cases}\dot{x}_1(t)=10(x_2(t)-x_1(t)-g(x_1(t)))+\delta,\\ \dot{x}_2(t)=x_1(t)-x_2(t)+x_3(t),\\ \dot{x}_3(t)=-15x_2(t)-0.0385x_3(t),\end{cases} \tag{18}$$

where δ is an unknown bounded disturbance and

$$g(x_1(t))=ax_1(t)+0.5b(|x_1(t)+1|-|x_1(t)-1|). \tag{19}$$

Here we assume that the parameter a is unknown and b=–0.59 are known. By introducing $y=x_1$, Eq.(18) can be rewritten as

$$\begin{aligned}\dot{x}(t)&=Ax(t)+f(y(t))+B(\varphi^{\mathrm{T}}(y(t)\theta+\delta),\\ y(t)&=Cx(t),\end{aligned} \tag{20}$$

where

$$A=\begin{bmatrix}-10 & 10 & 0\\ 1 & -1 & 1\\ 0 & -15 & -0.0385\end{bmatrix},$$

$$B=\begin{bmatrix}1\\0\\0\end{bmatrix},\quad C=\begin{bmatrix}1 & 0 & 0\end{bmatrix},$$

$$f(y(t))=\begin{bmatrix}2.95(|y(t)+1|-|y(t)-1|)\\ 0\\ 0\end{bmatrix},$$

$\phi^{\mathrm{T}}(y(t))=-10y(t)$, $\theta=a$. It can be verified that the vector $L=[-9\ 1.6883\ 0.6044]^{\mathrm{T}}$ can be found so that the eigenvalues of matrix $A-LC$ are –0.9737 and –0.5324±j4.6519, and that the transfer function

$$\begin{aligned}H(s)=&C(sI-(A-LC))^{-1}B\\ =&\frac{s^2+1.0385s+15.0385}{s^3+2.0385s^2+22.9604s+21.3470}\end{aligned}$$

is strictly positive real. We choose the following symmetric and positive-definite matrices:

$$P = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 15.37 & -0.958 \\ 0 & -0.958 & 1.091 \end{bmatrix},$$

$$Q = \begin{bmatrix} 2 & 0.0002 & 0 \\ 0.0002 & 0 & -0.3318 \\ 0 & -0.3318 & 2.7561 \end{bmatrix},$$

so that assumption A2 is satisfied. An observer-based response system is designed as follows:

$$\dot{\bar{x}}(t) = A\bar{x}(t) + f(y(t-\tau)) + L(y(t-\tau) - \bar{y}(t)) + B\phi(u(t)), \quad (21a)$$

$$\bar{y}(t) = C\bar{x}(t), \quad (21b)$$

where τ is an unknown constant, and the control law is

$$u(t) = -0.25(\hat{d} + 10|y(t-\tau)\hat{\theta}|) \times \mathrm{sign}(\bar{y}(t) - y(t-\tau)).$$

$\hat{\theta}$ and $\hat{d}$ are updated according to the following algorithms:

$$\dot{\hat{\theta}} = (\bar{y}(t) - y(t-\tau))y(t-\tau),$$
$$\dot{\hat{d}} = 0.1|\bar{y}(t) - y(t-\tau)|.$$

For simulation, we suppose $\phi(u(t)) = (5.5 + 0.5\sin(u(t)))u(t)$, then we know β_1=5, β_2=6. Choose a=–0.69, δ=0.1sin(t), τ=0.1, γ=0.25. The initial condition is chosen as $x_1(0)$=0.1, $x_2(0)$=0.2, $x_3(0)$ = –0.1, $\bar{x}_1(0)$=2.1, $\bar{x}_2(0)$=3.1, $\bar{x}_3(0)$=1.1 and $\hat{\theta}(0)$=0.1, $\hat{d}(0)$=0.1. Fig.3 shows the behaviour of the drive system (18).

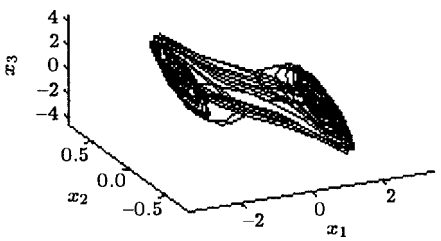


Fig.3. Behaviour of the drive system.

The synchronization behaviours with the existence of a constant unknown time-delay τ and at the same time are shown in Figs.4 and 5, respectively.

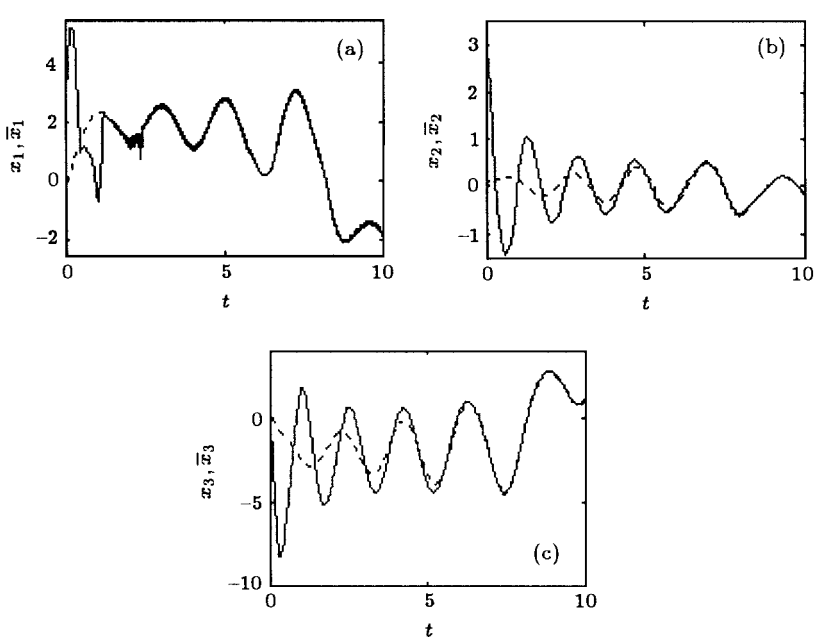


Fig.4. Synchronization behaviour with the existence of unknown time-delay τ (τ=0.1).

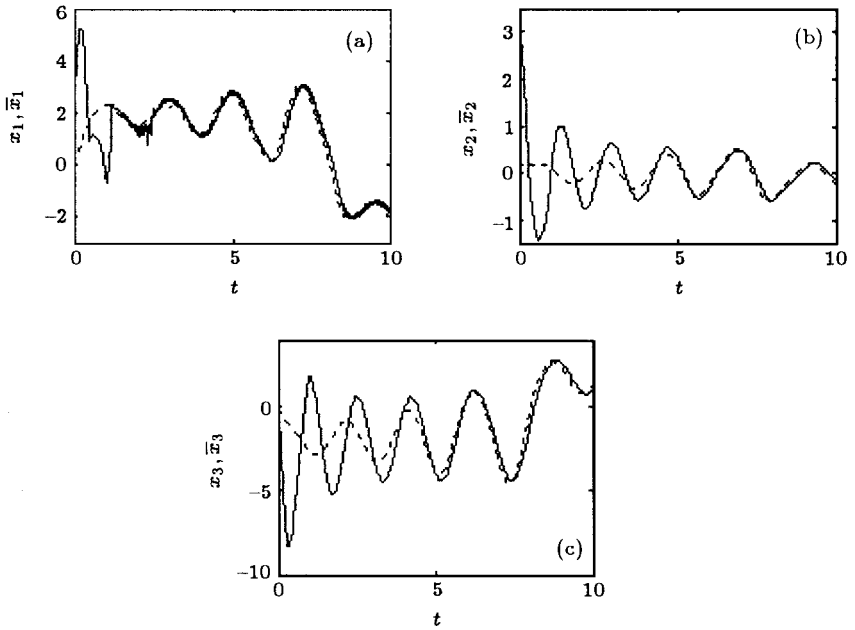


Fig.5. Synchronization behaviour at the same time (τ=0.1).

When the time-delay increases to τ=1 and other conditions are the same, the synchronization behaviours with the existence of a constant unknown time-delay τ and at the same time are shown in Figs.6 and 7.

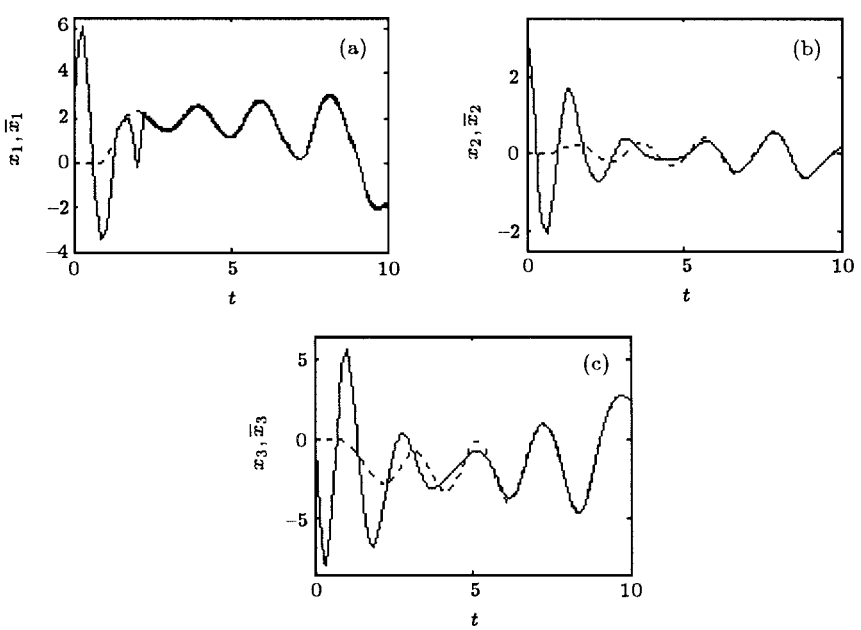


Fig.6. The synchronization behaviour with the existence of unknown time-delay τ (τ=1).

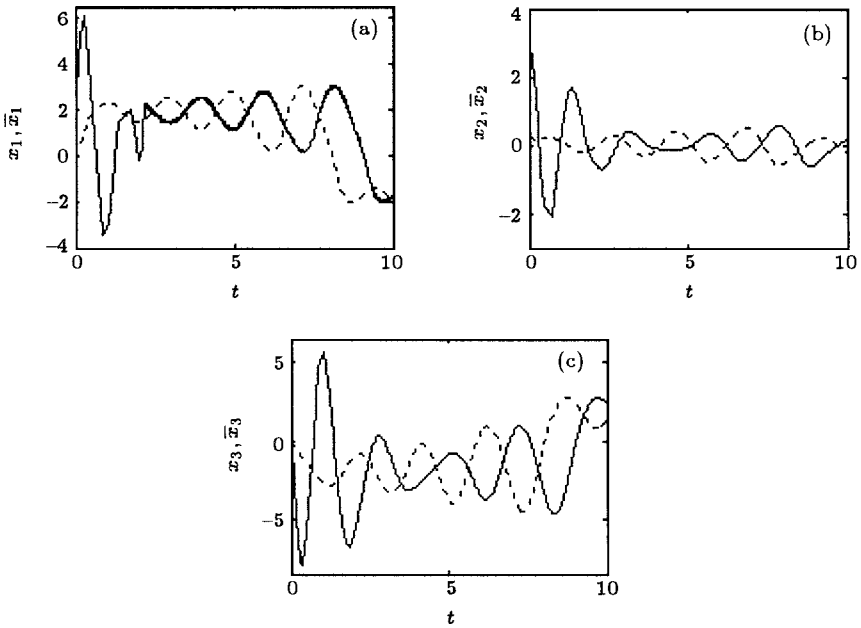


Fig.7. The synchronization behaviour at the same time (τ=1).

So we can see that for a larger τ, the synchronization can also be achieved quickly using the controller designed in this paper.

4. Conclusions

In this paper, an adaptive observer-based response system is constructed so that it synchronizes the drive system in the presence of the system's disturbances and unknown parameters. Unknown time-delay in the channel has been considered. The practical nonlinear input $\Phi(u)$ is applied to the response system instead of $u(t)$, considering the physical limitations of actuators. By the Lyapunov stability theory, we can use a transmitted scalar time-delay signal without requiring any knowledge of the constant unknown time-delay to achieve the synchronization. The chaotic Chua circuit has been simulated successfully for demonstration.

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