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Late-time tails in a stationary axisymmetric EMDA black hole geometry*

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The late-time tails of massless and self-interacting (SI) (massive) scalar fields are investigated analytically in a stationary axisymmetric Einstein–Maxwell dilaton-axion (EMDA) black hole geometry. It is shown that the asymptotic behaviour of massless perturbations is dominated by an inverse power-law decaying tail and the intermediate asymptotic behaviour of SI (massive) perturbations is dominated by an oscillatory one.

Keywords: late-time tail, stationary axisymmetric EMDA black hole, inverse power-law decaying tail

PACC: 0420, 9760L, 0440

1. Introduction

Since Wheeler introduced the no-hair theorem in the early 1970s,[1,2] which states that the external field of a black hole relaxes to a Kerr–Newman field characterized solely by the black-hole's mass, charge and angular momentum, there have been many investigations concerning the dynamical mechanism by which perturbation fields outside a black hole are radiated away. The massless neutral external perturbations were first studied by Price and it was found that the late-time behaviour for a fixed r is dominated by the factor $t^{-(2l+3)}$ for each multiple moment l.[3] The massless charged scalar field was studied in Refs.[4–6] and the conclusion was that a charged hair decay slower than a neutral one, i.e., the charged scalar hair outside a charged black hole is dominated by a $t^{-(2l+2)}$ tail. The massless late-time tail has been considered in the case of a rotating black hole in Refs.[7–9]. Recently the massless Dirac perturbations have been examined in Refs.[10–12].

Although these works are mainly concerned with massless fields, the physical mechanism responsible for the decay of self-interacting (SI) (massive) scalar hair is also important and it has attracted much attention recently. This is qualitatively different from that of massless fields. For instance, it has been shown in Ref.[13] that an oscillatory power-law tail of the form $t^{-(l+3/2)}\cos[\mu t-(l/2+3/4)\pi]$ for SI (massive) scalar fields develops at the intermediate late time characterized by $\mu M \ll 1$ in the Reissner–Nordström background.

Nowadays, it seems that the superstring theories are the most promising candidates for a consistent quantum theory of gravitation. So there have been many investigations concerning the spacetimes of the dilaton black hole. For instance, the asymptotic evolution of a massless charged scalar field in the background of a dilaton black hole was studied in Ref.[14] and it turned out that the charged hair decay is slower than a neutral one as in Refs.[4–6]. Moderski and Rogatko also investigated the late-time tails of SI (massive) scalar fields in the same background and they revealed that the late-time evolution is identical with that in the Schwarzschild background.[15] Obviously they only discussed the power-law tails in the static, spherically symmetric black hole geometry. Thus, it is worthwhile to investigate analytically the late-time behaviour in the background of the dilaton black hole, being the stationary axisymmetric solution of the so-called low-energy string theory.[16,17]

The main purpose of this paper is to examine

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the late-time tails both for massless and SI (massive) scalar fields in the background of a stationary axisymmetric Einstein–Maxwell dilaton–axion (EMDA) black hole, and to see what will affect the late-time evolution of the different scalar fields in the same background. In Section 2 we give a short description of the physical system and formulate the evolution equation. In Section 3 we introduce the black-hole Green function using the spectral decomposition method.[18] In Section 4 we study the late-time behaviour for both massless and SI (massive) scalar fields in the considered black hole geometry. Section 5 is devoted to a summary and conclusion.

2. Description of the system

The stationary axisymmetric EMDA black hole solution is given[16,17] by

$$\begin{aligned} \mathrm{d}s^2 =& -\frac{\Sigma - a^2\sin^2\theta}{\Delta}\mathrm{d}t^2 \\ & - \frac{2a\sin^2\theta}{\Delta}[(r^2-2Dr+a^2)-\Sigma]\mathrm{d}t\mathrm{d}\varphi + \frac{\Delta}{\Sigma}\mathrm{d}r^2 \\ & + \Delta\mathrm{d}\theta^2 + \frac{\sin^2\theta}{\Delta}[(r^2-2Dr+a^2)^2 \\ & - \Sigma a^2\sin^2\theta]\mathrm{d}\varphi^2, \end{aligned} \tag{1}$$

with $\Sigma = r^2 - 2Mr + a^2, \Delta = r^2 - 2Dr + a^2\cos^2\theta$. The ADM mass is $M_{\mathrm{ADM}} = M - D$. Throughout this paper we use $G = c = 1$.

In the background geometry of this considered black hole, a SI (massive) scalar field evolves according to

$$\frac{1}{\sqrt{-g}}\frac{\partial}{\partial x^\mu}\left(\sqrt{-g}g^{\mu\nu}\frac{\partial\Phi}{\partial x^\nu}\right) - \mu^2\Phi = 0, \tag{2}$$

where μ is the mass of the scalar field which is assumed to be real. So we derive a master equation that governs the evolution of perturbations of the EDMA spacetime

$$\begin{aligned} & \frac{(r^2-2Dr+a^2)^2 - \Sigma a^2\sin^2\theta}{\Sigma}\frac{\partial^2\Phi}{\partial t^2} \\ & - \frac{\partial}{\partial r}\left(\Sigma\frac{\partial\Phi}{\partial r}\right) - \frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Phi}{\partial\theta}\right) \\ & - \frac{\Sigma - a^2\sin^2\theta}{\sin^2\theta\Sigma}\frac{\partial^2\Phi}{\partial\varphi^2} + \frac{4a(M-D)r}{\Sigma}\frac{\partial^2\Phi}{\partial t\partial\varphi} \\ & + \Delta\mu^2\Phi = 0. \end{aligned} \tag{3}$$

Resolving the field in the form

$$\Phi = (r^2-2Dr+a^2)^{-1/2}\sum_{m=-\infty}^{+\infty}\Psi^m\mathrm{e}^{\mathrm{i}m\varphi}, \tag{4}$$

one obtains a wave-equation for each value of m:

$$\begin{aligned} & C_1(r,\theta)\frac{\partial^2\Psi}{\partial t^2} + C_2(r,\theta)\frac{\partial\Psi}{\partial t} + C_3(r,\theta)\Psi - \frac{\partial^2\Psi}{\partial y^2} \\ & - \frac{\Sigma}{(r^2-2Dr+a^2)^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial\Psi}{\partial\theta}\right) = 0, \end{aligned} \tag{5}$$

where the tortoise radial coordinate y is defined by $\mathrm{d}y = (r^2 - 2Dr + a^2)\mathrm{d}r/\Sigma$. The coefficients $C_i(r,\theta)$ are given by

$$C_1(r,\theta) = 1 - \frac{\Sigma a^2\sin^2\theta}{(r^2-2Dr+a^2)^2}, \tag{6}$$

$$C_2(r,\theta) = \frac{4\mathrm{i}am(M-D)r}{(r^2-2Dr+a^2)^2}, \tag{7}$$

$$\begin{aligned} C_3(r,\theta) =& \frac{\Sigma}{(r^2-2Dr+a^2)^2}\left[-m^2\left(\frac{a^2}{\Sigma} - \frac{1}{\sin^2\theta}\right)\right. \\ & + \frac{2(r-M)(r-D)+\Sigma}{r^2-2Dr+a^2} \\ & \left. - \frac{3(r-D)^2\Sigma}{(r^2-2Dr+a^2)^2} + \Delta\mu^2\right]. \end{aligned} \tag{8}$$

3. Formalism

The time evolution of a wave field described by Eq.(5) is given by

$$\begin{aligned} \Psi(x,t) =& 2\pi\int\int_0^\pi\{C_1(x')[G(x,x';t)\Psi_t(x',0) \\ & + G_t(x,x';t)\Psi(x',0)] \\ & + C_2(x')G(x,x';t)\Psi(x',0)\}\sin\theta'\mathrm{d}\theta'\mathrm{d}y' \end{aligned} \tag{9}$$

for $t > 0$, where x stands for (y,θ). The black-hole (retarded) Green function $G(x,x';t)$ is defined by

$$\begin{aligned} & \left[C_1(r,\theta)\frac{\partial^2}{\partial t^2} + C_2(r,\theta)\frac{\partial}{\partial t} + C_3(r,\theta) - \frac{\partial^2}{\partial y^2}\right. \\ & \left. - \frac{\Sigma}{(r^2-2Dr+a^2)^2}\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)\right]G(x,x';t) \\ =& \delta(t)\delta(y-y')\frac{\delta(\theta-\theta')}{2\pi\sin\theta}. \end{aligned} \tag{10}$$

The causality condition gives us the initial condition $G(x,x';t) = 0$ for $t \le 0$. In order to get $G(x,x';t)$ we use the Fourier transform

$$\begin{aligned} & \tilde{G}_l(y,y';\omega)Y_l^m(\theta',a\omega) \\ =& 2\pi\int_{0-}^{\infty}\int_0^\pi G(x,x';t)Y_l^m(\theta,a\omega)\sin\theta\mathrm{e}^{\mathrm{i}\omega t}\mathrm{d}\theta\mathrm{d}t. \end{aligned} \tag{11}$$

$G(x,x';t)$ itself is given by the inversion formula

$$\begin{aligned} & G(x,x';t) \\ =& \frac{1}{(2\pi)^2}\sum_l\int_{-\infty+\mathrm{i}c}^{\infty+\mathrm{i}c}\tilde{G}_l(y,y';\omega)Y_l^m(\theta,a\omega) \\ & \times Y_l^m(\theta',a\omega)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}\omega, \end{aligned} \tag{12}$$

where c is some positive constant and $Y_l^m(\theta, a\omega)$ are the spheroidal harmonics which are solutions to the angular equation

$$\left\{\frac{1}{\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right)+\left[a^2(\omega^2-\mu^2)\cos^2\theta-\frac{m^2}{\sin^2\theta}+l(l+1)\right]\right\}Y_l^m(\theta,a\omega)=0. \quad (13)$$

The Fourier transform is analytic in the upper half ω-plane and it satisfies

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}y^2}+\omega^2-R_l(r,\omega)\right]\tilde{G}_l(y,y';\omega)=\delta(y-y'), \quad (14)$$

where

$$\begin{aligned}R_l(r,\omega)=&\frac{\Sigma}{(r^2-2Dr+a^2)^2}\\&\times\left[a^2\omega^2+\frac{4am\omega(M-D)r}{\Sigma}\right.\\&+l(l+1)+\mu^2(r^2-2Dr)\\&-\frac{m^2a^2}{\Sigma}+\frac{2(r-M)(r-D)+\Sigma}{r^2-2Dr+a^2}\\&\left.-\frac{3(r-D)^2\Sigma}{(r^2-2Dr+a^2)^2}\right].\end{aligned} \quad (15)$$

Then we define two auxiliary functions $\tilde{\Psi}_1(y,\omega)$ and $\tilde{\Psi}_2(y,\omega)$ which are (linearly independent) solutions to the homogeneous equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}y^2}+\omega^2-R_l(r,\omega)\right]\tilde{\Psi}_i(y,\omega)=0,\quad i=1,2. \quad (16)$$

Let the Wronskian be

$$W(\omega)=W(\tilde{\Psi}_1,\tilde{\Psi}_2)=\tilde{\Psi}_1\tilde{\Psi}_{2,y}-\tilde{\Psi}_2\tilde{\Psi}_{1,y}, \quad (17)$$

where $W(\omega)$ is y-independent. Thus, using the two solutions $\tilde{\Psi}_1$ and $\tilde{\Psi}_2$, the black-hole Green function can be given by

$$\tilde{G}_l(y,y';\omega)=-\frac{1}{W(\omega)}\begin{cases}\tilde{\Psi}_1(y,\omega)\tilde{\Psi}_2(y',\omega), & y<y',\\ \tilde{\Psi}_1(y',\omega)\tilde{\Psi}_2(y,\omega), & y>y'.\end{cases} \quad (18)$$

To calculate $G(x,x';t)$ using Eq. (12), one needs to close the contour of integration into the lower half of the complex frequency plane. It has been argued that the asymptotic tail is associated with the existence of a branch cut (in $\tilde{\Psi}_2$).[13,18] This tail arises from the integral of $\tilde{G}(y,y';\omega)$ around the branch cut and is denoted by G^c. So our goal is to evaluate $G^c(x,x';t)$.

4. Late-time behaviour

Now we assume that both the observer and the initial data are situated far away from the black hole so that $r\gg M-D$. One may expand the wave equation (16) for the SI (massive) scalar field as a power series in $\frac{M}{r}$, neglecting terms of order $O\left(\frac{\alpha}{r^{n\geq 2}}\right)$. Thus, one arrives at

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\omega^2-\mu^2+\frac{4(M-D)\omega^2-2(M-D)\mu^2}{r}-\frac{l(l+1)}{r^2}\right]\xi(r,\omega)=0, \quad (19)$$

where $\xi(r,\omega)=[\Sigma/(r^2-2Dr+a^2)]^{1/2}\tilde{\Psi}(y,\omega)$. Equation (19) is also available for the massless scalar field. In this section we shall study analytically the late-time tails for both massless and SI (massive) scalar fields in the background of a stationary axisymmetric EMDA black hole according to Eq.(19).

4.1. Massless scalar late-time tail

It is well known that the backscattering from asymptotically far regions leads to the late-time behaviour of massless perturbations fields.[3,4] Therefore the late-time tails of massless fields are dominated by the low-frequency contribution to the Green function, for only low frequencies will be backscattered by the small spacetime curvature or by the small electromagnetic interaction at these asymptotic regions. Thus, as long as the observer is situated far away from the black hole and the initial data have a considerable support only far away from the black hole, a large-r (or equivalently, a low-ω) approximation is sufficient to discuss the asymptotic late-time behaviour of the field.[19]

For a massless perturbation field, the mass of the scalar field satisfies $\mu=0$. So from Eq.(19) we obtain

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2}+\omega^2+\frac{4(M-D)\omega^2}{r}-\frac{l(l+1)}{r^2}\right]\xi(r,\omega)=0. \quad (20)$$

Let us now introduce a second auxiliary field $\widetilde{\phi}$ defined by

$$\xi(r,\omega)=r^{l+1}\mathrm{e}^{\mathrm{i}\omega r}\widetilde{\phi}(z), \quad (21)$$

where $z=-2\mathrm{i}\omega r$. $\widetilde{\phi}(z)$ satisfies the confluent hypergeometric equation

$$\left\{z\frac{\mathrm{d}^2}{\mathrm{d}z^2}+(2l+2-z)\frac{\mathrm{d}}{\mathrm{d}z}-[l+1-2\mathrm{i}(M-D)\omega]\right\}\widetilde{\phi}(z)=0. \quad (22)$$

It follows that two basic solutions required in order to build the Green function can be written down (for

$r \gg M - D, |a|$):

$$\tilde{\Psi}_1 = Ar^{l+1}\mathrm{e}^{\mathrm{i}\omega r}M(l+1 - 2\mathrm{i}(M-D)\omega, 2l+2, -2\mathrm{i}\omega r), \quad (23)$$

and

$$\tilde{\Psi}_2 = Br^{l+1}\mathrm{e}^{\mathrm{i}\omega r}U(l+1 - 2\mathrm{i}(M-D)\omega, 2l+2, -2\mathrm{i}\omega r), \quad (24)$$

where A and B are normalization constants. $M(a,b,z)$ and $U(a,b,z)$ are the two standard solutions to the confluent hypergeometric equation.[19] The function $U(a,b,z)$ is a many-valued function, i.e. there will be a cut in $\tilde{\Psi}_2$.

Using Eq.(12), we find that the branch cut contribution to the Green function is

$$G^c(x,x';t) = \frac{1}{(2\pi)^2}\sum_l \int_0^{-\mathrm{i}\infty} \tilde{\Psi}_1(y',\omega) \times \left[\frac{\tilde{\Psi}_2(y,\omega\mathrm{e}^{2\pi\mathrm{i}})}{W(\omega\mathrm{e}^{2\pi\mathrm{i}})} - \frac{\tilde{\Psi}_2(y,\omega)}{W(\omega)}\right] \times Y_l(\theta,a\omega)Y_l(\theta',a\omega)\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}\omega. \quad (25)$$

For simplicity, we assume that the initial data have a considerable support only for r-values which are smaller than the observer's location. This, of course, does not change the asymptotic late-time behaviour.

Since $W(\omega)$ is r-independent, using the large-r (or equivalently, the low-ω) approximation, we can show that

$$W(\omega) = (-1)^{l+1}\mathrm{i}AB\frac{(2l+1)!(2\omega)^{-2l-1}}{\Gamma(l+1-2\mathrm{i}(M-D)\omega)}, \quad (26)$$

and since there is no cut in $\tilde{\Psi}_1$, we find

$$W(\omega\mathrm{e}^{2\pi\mathrm{i}}) = W(\omega). \quad (27)$$

Thus, we arrive at (for $\omega \to 0$)

$$G^c(x,x';t) = \sum_l \frac{\mathrm{i}(M-D)(-1)^{2l}2^{2l+1}(l!)^2}{\pi A^2[(2l+1)!]^2}Y_l(\theta)Y_l^*(\theta') \times \int_0^{-\mathrm{i}\infty} \tilde{\Psi}_1(y,\omega)\,\tilde{\Psi}_1(y',\omega)\omega^{2l+2}\mathrm{e}^{-\mathrm{i}\omega t}\mathrm{d}\omega. \quad (28)$$

First we consider the asymptotic behaviour of the fields at timelike infinity i_+. Using Eq.(13.5.5) from Ref.[19] we find (for $|\omega| = 0(1/t)$)

$$\tilde{\Psi}_1(y,\omega) \simeq Ay^{l+1}, \quad \tilde{\Psi}_1(y',\omega) \simeq Ay'^{l+1}. \quad (29)$$

Substituting them in Eq.(28) and performing the integration, we obtain the asymptotic behaviour of the massless scalar field at timelike infinity:

$$G^c(x,x';t) = \sum_l \frac{(M-D)(-1)^{l+1}2^{2l+1}(l!)^2(2l+2)!}{\pi[(2l+1)!]^2} \times Y_l(\theta)Y_l^*(\theta')(yy')^{l+1}t^{-(2l+3)}. \quad (30)$$

Then we consider the asymptotic behaviour of the fields at the EMDA black-hole outer horizon r_+. As $y \to -\infty$ the wave Eq.(16) can be approximated by the equation

$$\frac{\mathrm{d}^2\tilde{\Psi}(y,\omega)}{dy^2} + \eta^2\tilde{\Psi}(y,\omega) = 0, \quad (31)$$

where $\eta = \omega - m\omega_+$ and $\omega_+ = a/[2(M-D)r_+]$. So we take

$$\tilde{\Psi}_1(y,\omega) = C(\omega)\mathrm{e}^{-\mathrm{i}\eta y}, \quad (32)$$

and we use Eq.(29) for $\tilde{\Psi}_1(y',\omega)$. We can take $C(\omega)$ to be ω-independent.[4,9] And in this case we must replace the roles of y' and y in Eq.(28). Thus, the asymptotic behaviour at the EMDA black-hole outer horizon is dominated by

$$G^c(x,x';t) = \sum_l \Gamma_0\frac{(M-D)(-1)^{l+1}2^{2l+1}(l!)^2(2l+2)!}{\pi[(2l+1)!]^2} \times Y_l(\theta)Y_l^*(\theta')y'^{l+1}\mathrm{e}^{\mathrm{i}m\omega_+ y}v^{-(2l+3)}, \quad (33)$$

where Γ_0 is a constant.

Obviously they are dominated by the anticipated inverse power-law decaying tail $t^{-(2l+3)}$ at late times for each multiple moment l.

4.2. SI (massive) scalar late-time tail

It has been argued that the asymptotic SI (massive) tail is associated with the existence of a branch cut (in $\tilde{\Psi}_2$) placed along the interval $-\mu \le \omega \le \mu$.[13,18] As will be shown in this paper, this tail arises from the integral of $\tilde{G}(y,y';\omega)$ around the branch cut (denoted by G^c) which gives rise to an oscillatory inverse power-law behaviour of the field.

It is by now well known that SI (massive) tails exist even in a flat spacetime.[20] This phenomenon is related to the fact that different frequencies forming a massive wave packet have different phase velocities. One will see that at intermediate times the backscattering from asymptotically far regions (which dominates the tails of massless fields) is negligible compared to the flat spacetime SI (massive) tails that appear here. So the intermediate asymptotic behaviour

of a SI (massive) scalar field on the EMDA background is dominated by flat spacetime effects.

According to Eq.(19), if one further assumes that the observer and the initial data are in the region where $M - D \ll r \ll (M - D)/[\mu(M - D)]^2$ and one shall be interested in the intermediate asymptotic behaviour of a SI (massive) scalar field $r \ll t \ll (M - D)/[\mu(M - D)]^2$, we obtain

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}r^2} + \omega^2 - \mu^2 - \frac{l(l+1)}{r^2}\right]\xi(r,\omega) = 0. \qquad (34)$$

In this case we introduce the auxiliary field $\widetilde{\phi}$ defined by

$$\xi(r,\omega) = r^{l+1}\mathrm{e}^{\mathrm{i}\sqrt{\omega^2-\mu^2}r}\widetilde{\phi}(z), \qquad (35)$$

where $z = -2\mathrm{i}\sqrt{\omega^2 - \mu^2}r$. $\widetilde{\phi}(z)$ satisfies the confluent hypergeometric equation

$$\left[z\frac{\mathrm{d}^2}{\mathrm{d}z^2} + (2l + 2 - z)\frac{d}{\mathrm{d}z} - (l+1)\right]\widetilde{\phi}(z) = 0. \qquad (36)$$

Thus, the procedure of getting the solution of Eq.(36) is similar to that described in the massless scalar field. We only write the final form of the $G^c(x, x'; t)$ dependence. The intermediate asymptotic behaviour of the SI (massive) scalar field at a fixed radius has the form

$$\begin{aligned} G^c(x,x';t) = &\sum_l \frac{1}{(2\pi^3)^{1/2}}\frac{(-1)^{l+1}\mu^{l+1/2}}{(2l+1)!!} \\ &\times Y_l(\theta)Y_l^*(\theta')(yy')^{l+1}t^{-(l+3/2)} \\ &\times \cos\left[\mu t - \left(\frac{1}{2}l + \frac{3}{4}\right)\pi\right], \end{aligned} \qquad (37)$$

while the intermediate behaviour at the EMDA black-hole outer horizon r_+ is dominated by

$$\begin{aligned} G^c(x,x';t) = &\sum_l \rho_0\frac{1}{(2\pi^3)^{1/2}}\frac{(-1)^{l+1}\mu^{l+1/2}}{(2l+1)!!} \\ &\times Y_l(\theta)Y_l^*(\theta')y'^{l+1}\mathrm{e}^{\mathrm{i}m\omega_+ y}v^{-(l+3/2)} \\ &\times \cos\left[\mu v - \left(\frac{1}{2}l + \frac{3}{4}\right)\pi\right], \end{aligned} \qquad (38)$$

where ρ_0 is a constant.

Hence at intermediate times they are dominated by an oscillatory inverse power-law decaying tail $t^{-(l+3/2)}\cos[\mu t - (l/2 + 3/4)\pi]$ for each multiple moment l.

It is clear from Eqs.(37) and (38) that SI (massive) fields decay slower than massless ones, and waves with peculiar frequency ω quite close to μ can contribute to SI (massive) tails, while low frequency waves contribute to massless tails.

5. Conclusions

We have studied analytically the late-time behaviour of a massless scalar field and the intermediate asymptotic behaviour of a SI (massive) scalar field in the background of a stationary axisymmetric EMDA dilaton black hole. The main results and their physical implications are the following.

In the massless perturbations fields around a fixed background of this EMDA black hole we have shown that the asymptotic solutions develop the anticipated inverse power-law decaying tails $\Phi \sim t^{-(2l+3)}$ at time-like infinity and at the black-hole outer horizon. Obviously the dumping exponents are independent of the rotation parameter a of the black hole and very similar to the ones in the static black hole geometry.

In the SI (massive) perturbations fields the oscillatory inverse power-law decaying tails $\Phi \sim t^{-(l+3/2)}\cos[\mu t - (l/2 + 3/4)\pi]$ dominate at intermediate times at a fixed radius and at the black-hole outer horizon. The intermediate asymptotic behaviour of these fields depends on the field's mass μ. The dumping exponents, describing the fall-off of a SI (massive) field at intermediate times and also independent of the rotation parameter a of the black hole, are smaller compared with those of massless neutral perturbations. This is due to the fact that one may neglect the backscattering from asymptotically far regions at intermediate times in dealing with SI (massive) perturbations.

Recently the very late-time tails of the SI (massive) fields in the Schwarzschild[21] or Reissner–Nordström[22] background were studied, and it has been shown that the SI (massive) perturbation fields decay at very late times slower than any power law. So we hope to return to the problem of the late-time behaviour of these fields in a stationary axisymmetric EMDA black hole geometry elsewhere.

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