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Nonthermal effect of dilatonic black holes*

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The quantum nonthermal effect of the spherically symmetric and rotating dilatonic black holes is studied. A crossing of the positive and negative Dirac energy of particles occurs near dilatonic black holes. We find that the dilaton coupling parameter α affects the energy of spontaneous radiant particles. The energy of particles decreases when the coupling parameter α increases.

Keywords: black holes, dilaton fields, nonthermal effects
PACC: 0420, 0440, 0470

1. Introduction

In general relativity, the stationary black holes coupled to electromagnetism are described by the Kerr–Newman solutions. If one considers that gravity couples to a Maxwell field and a dilaton field, the action is[1]

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\left[R - 2(\nabla\Phi)^2 - \mathrm{e}^{-2\alpha\Phi}F^2\right], \quad (1)$$

where Φ and $F_{\mu\nu}$ are a dilaton field and a Maxwell field, respectively, α is a free parameter which governs the strength of the coupling of the dilaton to the Maxwell field. When $\alpha = 0$, the action reduces to the usual Einstein–Maxwell theory. The black hole solution for this case is the well-known Kerr–Newman solution. When $\alpha = 1$, the action is part of the low-energy action of string theory. The case of $\alpha = \sqrt{3}$ corresponds to the four-dimensional effective model reduced from the five-dimensional Kaluza–Klein theory. Furthermore, we may also include an axion field $H_{\mu\nu\rho}$. The low-energy action is then

$$S = \frac{1}{16\pi}\int \mathrm{d}^4x\sqrt{-g}\Big[R - 2(\nabla\Phi)^2 - \mathrm{e}^{-2\Phi}F^2 - \frac{1}{12}e^{-4\Phi}H^2\Big]. \quad (2)$$

In the model (1), nonrotating charged dilaton black hole solutions with arbitrary coupling constant α have been found.[2,3] Explicit solutions describing rotating charged black holes for arbitrary α have not been found yet. However, for $\alpha = \sqrt{3}$, the solutions are known,[4] and in the case $\alpha = 1$, an exact rotating dilatonic black hole solution is derived by Sen[5] in the model (2). When $\alpha = 0$, the rotating charged solution is the familiar Kerr–Newman solution. The coupling plays some important roles in black hole physics. Koga and Maeda[1] have studied the evaporation of the spherically symmetric and rotating black holes with dilaton coupling via Hawking's quantum radiation. They found that the dilaton coupling constant α drastically affects the emission rate. In addition to quantum thermal effect depending on the temperature, there is also an important quantum nonthermal effect in the spacetime of the dilatonic black holes. This is a spontaneous radiation independent of temperature. In this paper, we will study the nonthermal radiation from the dilatonic black holes. We expect that the coupling constant dependence is noticeable.

2. Nonthermal radiation from spherically symmetric dilatonic black holes

The spherically symmetric exact solution with arbitrary coupling constant α is given by[2,3]

$$\mathrm{d}s^2 = -\frac{\Delta(r)}{R^2(r)}\mathrm{d}t^2 + \frac{R^2(r)}{\Delta(r)}\mathrm{d}r^2 + R^2(r)\left(\mathrm{d}\theta^2 + \sin^2\theta\mathrm{d}\phi^2\right), \quad (3)$$

$$A_t = -\frac{Q}{r}, \qquad \Phi = \frac{\alpha}{1+\alpha^2}\ln\left(1 - \frac{r_-}{r}\right), \quad (4)$$

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where

$$\begin{aligned}\Delta(r) =&(r-r_+)(r-r_-),\\ R(r) =&r\left(1-\frac{r_-}{r}\right)^{\alpha^2/(1+\alpha^2)},\end{aligned} \tag{5}$$

and A_t is the t component of the four-potential A_μ. The outer and "inner" horizons r_+ and r_- are related to the mass M, the electric charge Q of the black hole, and α as

$$r_\pm = \frac{(1+\alpha^2)\left[M \pm \sqrt{M^2-(1-\alpha^2)Q^2}\right]}{(1\pm\alpha^2)}, \tag{6}$$

when $|Q| = Q_{\max} = \sqrt{1+\alpha^2}M$, r_+ and r_- coincide. However, when $r_+ = r_- = (1+\alpha^2)M$, the area of the black hole vanishes for $\alpha \neq 0$, therefore it is not a black hole solution.[2]

Motions of particles with charge e and mass m in curved spacetimes are described by the Hamilton–Jacobi equation

$$g^{\mu\nu}\left(\frac{\partial S}{\partial x^\mu} - eA_\mu\right)\left(\frac{\partial S}{\partial x^\nu} - eA_\nu\right) + m^2 = 0, \tag{7}$$

where $S = \int L\mathrm{d}\tau$ is the Hamilton–Jacobi principal function, and A_μ is the electromagnetic field four-potential in the spacetime. Inserting Eqs.(3) and (4) into the Hamilton–Jacobi equation (7), one obtains

$$\begin{aligned}&-\frac{R^2}{\Delta}\left(\frac{\partial S}{\partial t}+e\frac{Q}{r}\right)^2 + \frac{\Delta}{R^2}\left(\frac{\partial S}{\partial r}\right)^2 + \frac{1}{R^2}\left(\frac{\partial S}{\partial\theta}\right)^2\\ &+\frac{1}{R^2\sin^2\theta}\left(\frac{\partial S}{\partial\phi}\right)^2 + m^2 = 0.\end{aligned} \tag{8}$$

The Hamilton–Jacobi equation (8) in this spacetime can be made separable, by setting

$$S = -\omega t + F(r) + H(\theta) + l\phi, \tag{9}$$

where $\omega = -\partial S/\partial t$ is the energy of the particle, and $l = \partial S/\partial\phi$ is the azimuthal component of the angular momentum. Then, the radial equation of Eq.(8) is

$$\left(\frac{\mathrm{d}F}{\mathrm{d}r}\right)^2 = \frac{R^4\left(\omega - e\frac{Q}{r}\right)^2}{\Delta^2} - \frac{(m^2R^2+\lambda)}{\Delta}, \tag{10}$$

where λ is a constant. Because $(\mathrm{d}F/\mathrm{d}r)^2 \geq 0$, we have

$$R^4\left(\omega - e\frac{Q}{r}\right)^2 - \Delta(m^2R^2+\lambda) \geq 0. \tag{11}$$

This is the condition that the energy of particles have to satisfy in this spacetime. Adopting the equality in Eq.(11), we have

$$\omega_0^\pm = e\frac{Q}{r} \pm \frac{1}{R^2}\sqrt{\Delta\,(\lambda + m^2R^2)}. \tag{12}$$

The distribution of the energy levels of the Dirac vacuum is given by

$$\omega \geq \omega_0^+ \quad \text{and} \quad \omega \leq \omega_0^-. \tag{13}$$

The forbidden region of the energy is

$$\omega_0^- < \omega < \omega_0^+. \tag{14}$$

The width of the forbidden region is

$$\Delta\omega = \frac{2}{R^2}\sqrt{\Delta\,(m^2R^2+\lambda)}. \tag{15}$$

(i) When $r \to \infty$, $\omega_0^\pm \to \pm m$, and $\Delta\omega = 2m$. The distribution of the Dirac energy levels becomes that in the Minkowski spacetime. (ii) When $r \to r_{\mathrm{h}} = M + \sqrt{M^2-(1-\alpha^2)Q^2}$ (the event horizon), $\Delta = 0$, and we have

$$\omega_0 \equiv \omega_0^+ = \omega_0^- = \frac{eQ}{M+\sqrt{M^2-(1-\alpha^2)Q^2}}. \tag{16}$$

The width of the forbidden region vanishes at the event horizon,

$$\Delta\omega = 0. \tag{17}$$

Therefore, there exists a crossing of the positive and negative energy levels near the event horizon. When $\omega_0 > +m$, the particle can escape to infinity from the event horizon. This means that there is radiation from the region near the event horizon. This quantum effect is independent of the temperature of the black hole. This is the Starobinsky–Unruh process of spontaneous radiation.

From Eq.(16), we see that the energy of particles is dependent on the coupling parameter α. When $\alpha = 0$, it reduces to the results of the Reissner–Nordström solution. To see how the energy of particles ω_0 at the event horizon varies with the coupling parameter α, we plot the specific energy ω_0/e of the particles in Fig.1. We assume the mass of the black hole is conserved. Figure 1 shows ω_0/e as a function of α for four values of Q/M: 0.6, 0.8, 0.9, 1.0 and for the extreme case ($Q = Q_{\max} = \sqrt{1+\alpha^2}M$). We notice that as the electric charge of the black hole Q increases, the energy of particles increases; and as the coupling parameter α increases, the energy decreases. The increase of ω_0/e with Q/M is larger at low values of α than at high values of α, and ω_0/e for each value of Q/M intends to coincide at the large values of α. This means that the stronger the coupling, the

smaller the effect of the electric charge of the black hole on the energy of particles.

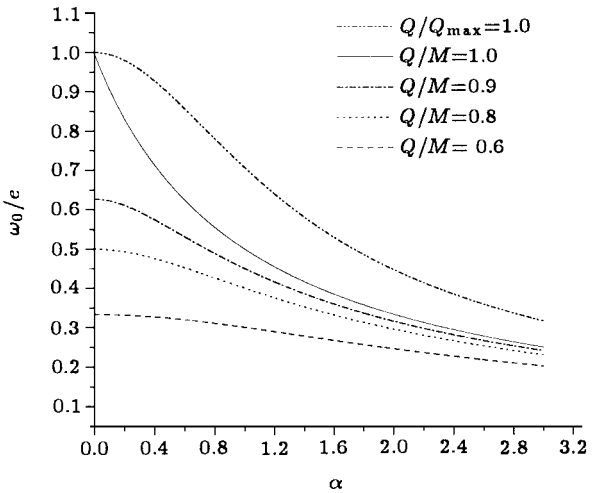


Fig.1. ω_0/e versus the coupling parameter α for several values of Q/M. $Q/M = 0.6$ (the dashed line), $Q/M = 0.8$ (the dotted line), $Q/M = 0.9$ (the dot-dashed line), $Q/M = 1.0$ (the solid line), and $Q/Q_{\max} = 1.0$ (the dot-dot-dashed line).

3. Nonthermal radiation from rotating dilatonic black holes

Next, we consider the nonthermal radiation from rotating dilatonic black holes. Because we do not yet have explicit rotating black hole solutions for all values of α, we will study the Sen black hole ($\alpha = 1$). We first summarize the Sen black hole. The rotating dilatonic black hole with coupling constant $\alpha = 1$ is given by[5]

$$\begin{aligned} \mathrm{d}s^2 =& -\frac{\Sigma - a^2\sin^2\theta}{\Delta}\,\mathrm{d}t^2 \\ & - \frac{2a\sin^2\theta}{\Delta}\left[\left(r^2+a^2-2\beta r\right)-\Sigma\right]\mathrm{d}t\mathrm{d}\phi \\ & + \frac{\Delta}{\Sigma}\,\mathrm{d}r^2 + \Delta\mathrm{d}\theta^2 \\ & + \frac{\sin^2\theta}{\Delta}\left[\left(r^2+a^2-2\beta r\right)^2 - \Sigma a^2\sin^2\theta\right]\mathrm{d}\phi^2, \end{aligned} \tag{18}$$

where $\Delta = r^2 - 2\beta r + a^2\cos^2\theta$, and $\Sigma = r^2 - 2\mu r + a^2$, $\beta = -\mu\sinh^2(\sigma/2)$. This metric describes a black hole solution with mass M, charge Q, angular momentum J and magnetic dipole moment P, which are related to parameters μ, σ and a as

$$\begin{aligned} M &= \frac{\mu}{2}(1+\cosh\sigma), \quad Q = \frac{\mu}{\sqrt{2}}\sinh\sigma, \\ J &= \frac{\mu a}{2}(1+\cosh\sigma), \quad P = \frac{1}{\sqrt{2}}\mu a\sinh\sigma. \end{aligned} \tag{19}$$

It will be more convenient to express μ, σ and a in terms of the independent physical parameters M, J and Q. From Eq.(19), we obtain

$$\mu = M - \frac{Q^2}{2M}\,, \quad \sinh\sigma = \frac{2\sqrt{2}QM}{2M^2-Q^2}\,, \quad a = \frac{J}{M}. \tag{20}$$

The outer and "inner" horizons r_{h}^+ and r_{h}^- are given by the mass M, electric charge Q and angular momentum J of the black hole as

$$r_{\mathrm{h}}^{\pm} = M - \frac{Q^2}{2M} \pm \sqrt{\left(M-\frac{Q^2}{2M}\right)^2 - \frac{J^2}{M^2}}, \tag{21}$$

and then the regular horizons exist if

$$|J| \le M^2 - \frac{Q^2}{2}. \tag{22}$$

Thus the extreme limit of the black hole corresponds to $|J| = M^2 - Q^2/2$. For the extreme Sen black holes, r_{h}^+ and r_{h}^- coincide. The angular velocity Ω_{h} of the Sen black hole at the event horizon is[5]

$$\Omega_{\mathrm{h}} = \frac{J}{M\left[2M^2 - Q^2 + \sqrt{(2M^2-Q^2)^2 - 4J^2}\,\right]}. \tag{23}$$

The location of the infinite red-shift surfaces of the black hole is determined by

$$r_{\mathrm{s}}^{\pm} = M - \frac{Q^2}{2M} \pm \sqrt{\left(M-\frac{Q^2}{2M}\right)^2 - \frac{J^2}{M^2}\cos^2\theta}. \tag{24}$$

Therefore, in the Sen spacetime there exist ergospheres, i.e.

$$\begin{aligned} &\text{outer ergosphere} \quad r_{\mathrm{h}}^+ < r < r_{\mathrm{s}}^+, \\ &\text{inner ergosphere} \quad r_{\mathrm{s}}^- < r < r_{\mathrm{h}}^-. \end{aligned} \tag{25}$$

In the Sen spacetime the four potentials are

$$A_0 = -\frac{Qr}{\Delta},\ A_1 = A_2 = 0,\ A_3 = \frac{Qra}{\Delta}\sin^2\theta. \tag{26}$$

Inserting Eqs.(26) and (18) into Eq.(7), one obtains the Hamilton–Jacobin equation of particles[6–10] in the Sen spacetime

$$\begin{aligned} & \Sigma\left(\frac{\partial S}{\partial r}\right)^2 + \left(\frac{\partial S}{\partial\theta}\right)^2 + \left(a\sin\theta\frac{\partial S}{\partial t} + \frac{1}{\sin\theta}\frac{\partial S}{\partial\phi}\right)^2 \\ & - \frac{1}{\Sigma}\left(B\frac{\partial S}{\partial t} + a\frac{\partial S}{\partial\phi} + Qer\right)^2 + m^2\Delta = 0, \end{aligned} \tag{27}$$

where $B = r^2 + a^2 - 2\beta r$. Separating Eq.(27), we have the radial equation

$$\left(\frac{\mathrm{d}R}{\mathrm{d}r}\right)^2 = \frac{1}{\Sigma^2}\left(-B\omega + al + Qer\right)^2 - \frac{1}{\Sigma}\left[m^2(r^2 - 2\beta r) + \lambda\right], \qquad (28)$$

where λ is a constant, $\omega = -\partial S/\partial t$ is the energy of the particle, and $l = \partial S/\partial\phi$ is the azimuthal component of the angular momentum. The condition $(\mathrm{d}R/\mathrm{d}r)^2 \geq 0$ corresponds to

$$\left(-B\omega + al + Qer\right)^2 - \Sigma\left[m^2(r^2 - 2\beta r) + \lambda\right] \geq 0. \qquad (29)$$

Adopting the equality in Eq.(29), we have

$$\omega_0^{\pm} = \frac{1}{B}\left[al + Qer \pm \sqrt{\Sigma\left(m^2(r^2 - 2\beta r) + \lambda\right)}\right]. \qquad (30)$$

The distribution of the energy levels of the Dirac vacuum is given by

$$\omega \geq \omega_0^+ \quad \text{and} \quad \omega \leq \omega_0^-. \qquad (31)$$

The forbidden region of the energy is

$$\omega_0^- < \omega < \omega_0^+. \qquad (32)$$

The width of the forbidden region is

$$\Delta\omega = \frac{2}{B}\sqrt{\Sigma\left[m^2(r^2 - 2\beta r) + \lambda\right]}. \qquad (33)$$

(i) When $r \to \infty$, electromagnetic effects can be neglected, and we have

$$\omega_0^{\pm} \to \pm m. \qquad (34)$$

The distribution of the Dirac energy levels goes to that in the Minkowski spacetime. The width of the forbidden region is

$$\Delta\omega = 2m. \qquad (35)$$

(ii) When $r \to r_{\mathrm{h}} = M - Q^2/(2M) + \sqrt{[M - Q^2/(2M)]^2 - a^2}$, we get

$$\omega_0 \equiv \omega_0^+ = \omega_0^- = \frac{al + Qer_{\mathrm{h}}}{B}, \qquad (36)$$

which is different from the results of the Kerr–Newman black hole. The width of the forbidden region vanishes at the event horizon, i.e.

$$\Delta\omega = 0. \qquad (37)$$

Therefore, there exists a crossing of the positive and negative energy levels near the event horizon in the Sen black hole. Nonthermal radiation takes place near the event horizon.

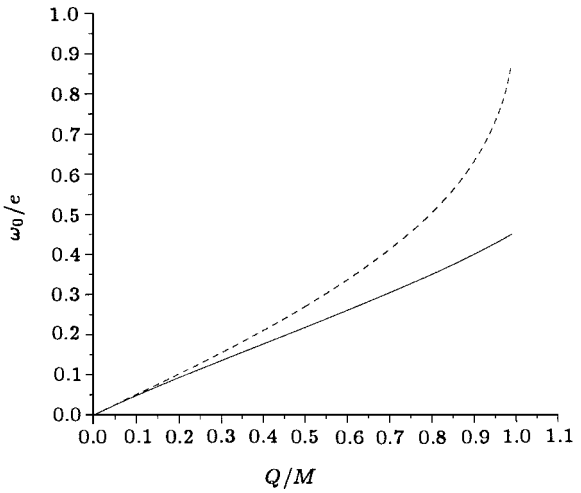


Fig.2. The behaviour of ω_0/e as a function of Q/M for $J = 0.01M^2$ and $l = 0$. The solid line corresponds to the Sen black hole ($\alpha = 1$), and the dashed line corresponds to the Kerr–Newman black hole ($\alpha = 0$).

For the Kerr–Newman black hole, we know

$$\omega_0 = \frac{al + Qer_{\mathrm{h}}}{r_{\mathrm{h}}^2 + a^2}, \qquad (38)$$

where $r_{\mathrm{h}} = M + \sqrt{M^2 - a^2 - Q^2}$. It is obvious that the energy of spontaneously radiant particles is dependent on the coupling parameter α. Figure 2 shows the variation of specific energy ω_0/e with Q/M for the Kerr–Newman black hole ($\alpha = 0$) and the Sen black hole ($\alpha = 1$). In Fig.2, we consider the black holes with a small angular momentum, which is fixed at $J = 0.01M^2$, and the spontaneously radiant particles with the zero azimuthal component of the angular momentum (i.e. $l = 0$). We see that the energy of particles near the Sen black hole is smaller than that of the Kerr-Newman black hole. The behaviour of the energy of particles is very similar to that of nonrotating black holes.

4. Conclusion

In summary, we have studied nonthermal radiation of the spherically symmetric and rotating dilatonic black holes. We give the expressions of the positive and negative energy states of particles in the dilatonic spacetimes. The positive energy state interlaces with the negative state in a region near the event horizon. Therefore, spontaneous nonthermal radiation takes place in the overlap region. We have discussed the effect of the dilaton coupling parameter α on the nonthermal radiation. We find that as the dilaton coupling parameter α increases, the energy of particles decreases, and the more the electric charge of the black holes, the larger the effect of α on the energy of particles. For the spherically symmetric dilatonic black holes, the larger the coupling parameter α, the smaller the effect of electric charge of the black holes Q.

References

[1] Koga J I and Maeda K I 1995 *Phys. Rev.* D **52** 7066
[2] Gibbons G W and Maeda K 1988 *Nucl. Phys.* B **298** 741
[3] Garfinkle D, Horowitz G T and Strominger A 1991 *Phys. Rev.* D **43** 3140
[4] Forlov V, Zelnikov A and Bleyer U 1987 *Ann. Phys.* (Leipzig) **44** 371
[5] Sen A 1992 *Phys. Rev. Lett.* **69** 1006
[6] Yang S and Zhao Z 1996 *Int. J. Theor. Phys.* **35** 2455
[7] Lu J L 1999 *Int. J. Theor. Phys.* **38** 2029
[8] Zhang J H, Meng Q M and Li C A 1996 *Acta Phys. Sin.* **45** 177
[9] Lu J L 2002 *Acta Phys. Sin.* **51** 973
[10] Xie S C, Yang X T, Yang S Z and Lin L B 2001 *Chin. Phys.* **10** 979