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New exact solutions of nonlinear Gross–Pitaevskii equation with weak bias magnetic and time-dependent laser fields*

Li Hua-Mei(李画眉)

Department of Physics, Zhejiang Normal University, Jinhua 321004, China

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By using the mapping method and an appropriate transformation, we find new exact solutions of nonlinear Gross–Pitaevskii equation with weak bias magnetic and time-dependent laser fields. The solutions obtained in this paper include Jacobian elliptic function solutions, combined Jacobian elliptic function solutions , triangular function solutions, bright and dark solitons, and soliton-like solutions.

Keywords: Gross–Pitaevskii equation, exact solution, mapping method
PACC: 0340K, 0290, 0230

1. Introduction

The study on nonlinear property of matter wave has received much attention due to the experimental realization of Bose–Einstein condensation (BEC) in weakly interacting atomic gases.[1,2] The most spectacular experimental progress achieved is the demonstration of atomic four-wave mixing,[3,4] the observation of solitons[5−9] and matter–wave amplification[10] in BEC. Conceptually, a kind of macroscopic quasiparticle soliton provides a link of BEC physics with fluid mechanics, nonlinear optics, and fundamental particle physics. Bright and dark solitons in BEC correspond to the attractive and repulsive interactions (s-wave scattering length $a < 0$ or $a > 0$). Recently, theoretical and experimental studies have mainly concentrated on the dynamics and stability of dark solitons of two-component BEC.

The nonlinear excitations of BEC in magnetic trap can be described by the Gross–Pitaevskii equation

$$\begin{aligned} \mathrm{i}\hbar\psi_\tau = &-\frac{\hbar^2}{2M}\psi_{yy} + \frac{4\pi\hbar^2}{M}a|\psi|^2\psi \\ &+ y[n_1 + n_2\sin(\nu\tau)]\psi, \end{aligned} \tag{1}$$

where τ is the time, y the coordinate, M the atomic mass, and a the scattering length between atoms; n_1 and n_2 are bias magnetic and time-dependent laser-field parameters, respectively. By the following dimensionless transformation:

$$\begin{aligned} u(x,t) &= \psi(y,\tau), \\ \tau &= \frac{m\varepsilon t}{4\pi\hbar^2 a}, \\ y &= \sqrt{\frac{\varepsilon}{4\pi a}}x, \\ b &= \sqrt{\frac{\varepsilon}{\pi a}}\frac{m\varepsilon n_1}{8\pi a\hbar^2}, \\ \omega &= \frac{\nu m\varepsilon}{4\pi\hbar a}, \\ c &= \sqrt{\frac{\varepsilon}{\pi a}}\frac{m\varepsilon n_2}{8\pi a\hbar^2}, \end{aligned}$$

we can rewrite Eq.(1) as

$$\mathrm{i}u_t = -\frac{1}{2}u_{xx} + \varepsilon|u|^2u + x[b + c\sin(\omega t)]u, \tag{2}$$

where x is measured in units of $\varsigma = 1\mu m$, a characteristic length unit in this type of experiment, t in units of $m\varsigma^2/\hbar$, u in units of the square root of n_0a^3, the maximum density in the initial distribution of the condensate, and the interaction constant is defined as $\varepsilon = 4\pi n_0 a\varsigma^2$, with a being the interatomic scattering length. It is very difficult to solve a variable coefficient nonlinear equation. The bright soliton solutions of Eq.(2) have been obtained when c=0.[11−13]

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Very recently, Liu *et al* obtain the exact bright and dark soliton solutions of Eq.(2) by applying the inverse scattering method and selecting the appropriate transformation.[14]

The construction of the exact solutions of nonlinear partial differential equations (NLPDEs) is one of the most important and essential tasks in nonlinear science. With the help of exact solutions, when they exist, the phenomena modelled by these NLPDEs can be better understood. They can also help to analyse the stability of the solutions and to check numerical analysis for these NLPDEs. In recent decades, many powerful methods to construct exact analytical solutions have been proposed, such as the inverse scattering method,[15] Bäcklund transformation and Darboux transformation,[16] Painlevé truncation expansion,[17−19] the homogeneous balance method,[20] sine–cosine function method,[21] tanh-function method,[22,23] and Jacobian elliptic function method.[24−27] Very recently, a unified method called the mapping method has been developed to obtain the Jacobian elliptic function, solitons and periodic functions to some NLPDEs.[28−33] Compared with the previous methods mentioned above, the mapping method gives more general solutions without much extra effort and without repetitive calculations. What is remarkable about the mapping method is that it allows one to find both the Jacobian elliptic function, triangular functions and solitons using the same unique procedure. More importantly, the method permits the classification of the solutions depending on three parameters.

The rest of this paper is organized as follows. In Section 2, we simply introduce the mapping method.[28] In Section 3, applying the method, with the aid of Maple, we obtain abundant explicit and exact solutions of Eq.(2), which include Jacobian elliptic function solutions, combined Jacobian elliptic function solutions, triangular function solutions, bright and dark solitons, and soliton-like solutions. The final section gives a short summary and discussion.

2. The mapping method

The key idea in this generalized indirect method is to take full advantage of the solutions of an auxiliary ordinary differential equation (the first kind of Jacobian equation containing three parameters) to construct abundant families of Jacobian elliptic solutions for the above-mentioned equation. The auxiliary equation reads

$$\phi'^2 = C + B\phi^2 + \frac{A}{2}\phi^4, \tag{3}$$

where $\phi' := \mathrm{d}\phi/\mathrm{d}\xi, \xi = \xi(x,t)$ and A, B, C are constants. Equation (3) has the following solutions:

$\phi(\xi)$	A	B	C,
$\mathrm{sn}\xi; \mathrm{cd}\xi = \mathrm{cn}\xi/\mathrm{dn}\xi$	$2m^2$	$-(m^2+1)$	1,
$\mathrm{cn}\xi$	$-2m^2$	$2m^2-1$	$1-m^2$,
$\mathrm{dn}\xi$	-2	$2-m^2$	m^2-1,
$\mathrm{ns}\xi = \dfrac{1}{\mathrm{sn}\xi};\quad \mathrm{dc} = \dfrac{\mathrm{dn}\xi}{\mathrm{cn}\xi}$	2	$-(m^2+1)$	m^2,
$\mathrm{nc}\xi = 1/\mathrm{cn}\xi$	$2(1-m^2)$	$2m^2-1$	$-m^2$,
$\mathrm{nd}\xi = 1/\mathrm{dn}\xi$	$2(m^2-1)$	$2-m^2$	-1,
$\mathrm{cs}\xi = \mathrm{cn}\xi/\mathrm{sn}\xi$	2	$2-m^2$	$1-m^2$,
$\mathrm{sc}\xi = \mathrm{sn}\xi/\mathrm{cn}\xi$	$2(1-m^2)$	$2-m^2$	1,
$\mathrm{sd}\xi = \mathrm{sn}\xi/\mathrm{dn}\xi$	$2m^2(m^2-1)$	$2m^2-1$	1,
$\mathrm{ds}\xi = \mathrm{dn}\xi/\mathrm{sn}\xi$	2	$2m^2-1$	$m^2(m^2-1)$,
$m\mathrm{cn}\xi \pm \mathrm{dn}\xi$	$-1/2$	$(1+m^2)/2$	$-(1-m^2)^2/4$,
$\mathrm{ns}\xi \pm \mathrm{cs}\xi$	$1/2$	$(1-2m^2)/2$	$1/4$,
$\mathrm{nc}\xi \pm \mathrm{sc}\xi$	$(1-m^2)/2$	$(1+m^2)/2$	$(1-m^2)/4$,
$\mathrm{ns}\xi \pm \mathrm{ds}\xi$	$1/2$	$(m^2-2)/2$	$m^4/4$,
$\mathrm{sn}\xi \pm \mathrm{icn}\xi;\quad \dfrac{\mathrm{dn}\xi}{\sqrt{1-m^2}\mathrm{sn}\xi \pm \mathrm{cn}\xi}$	$m^2/2$	$(m^2-2)/2$	$m^2/4$,
$m\mathrm{sn}\xi \pm \mathrm{idn}\xi;\quad \dfrac{\mathrm{sn}\xi}{1 \pm \mathrm{cn}\xi}$	$1/2$	$(1-2m^2)/2$	$1/4$,

$$\begin{array}{lccc}
\dfrac{\mathrm{sn}\xi}{1\pm\mathrm{dn}\xi} & m^2/2 & (m^2-2)/2 & 1/4, \\
\dfrac{\mathrm{dn}\xi}{1\pm m\mathrm{sn}\xi} & (m^2-1)/2 & (m^2+1)/2 & (m^2-1)/4, \\
\dfrac{\mathrm{cn}\xi}{1\pm\mathrm{sn}\xi} & (1-m^2)/2 & (m^2+1)/2 & (1-m^2)/4, \\
\dfrac{\mathrm{sn}\xi}{\mathrm{dn}\xi\pm\mathrm{cn}\xi} & (1-m^2)^2/2 & (m^2+1)/2 & 1/4, \\
\dfrac{\mathrm{cn}\xi}{\sqrt{1-m^2}\pm\mathrm{dn}\xi} & m^4/2 & (m^2-2)/2 & 1/4,
\end{array} \tag{4}$$

where $\mathrm{i}^2 = -1$. The Jacobian elliptic functions $\mathrm{sn}\xi = \mathrm{sn}(\xi, m)$, $\mathrm{cn}\xi = \mathrm{cn}(\xi, m)$ and $\mathrm{dn}\xi = \mathrm{dn}(\xi, m)$, where $m(0 < m < 1)$ is the modulus of the elliptic function, are double periodic and possess properties of triangular functions, namely

$$\begin{aligned}
&\mathrm{sn}^2\xi + \mathrm{cn}^2\xi = 1, \\
&\mathrm{dn}^2\xi + m^2\mathrm{sn}^2\xi = 1, \\
&\frac{\mathrm{d}}{\mathrm{d}\xi}\mathrm{sn}\xi = \mathrm{cn}\xi\mathrm{dn}\xi, \\
&\frac{\mathrm{d}}{\mathrm{d}\xi}\mathrm{cn}\xi = -\mathrm{sn}\xi\mathrm{dn}\xi, \\
&\frac{\mathrm{d}}{\mathrm{d}\xi}\mathrm{dn}\xi = -m^2\mathrm{cn}\xi\mathrm{sn}\xi.
\end{aligned} \tag{5}$$

In addition, we see that other solutions are obtained from solutions (4) in the case of degeneracies as follows.

When $m \to 0$, the Jacobian elliptic functions degenerate to the trigonometric functions solutions of the given NLPDE i.e.

$$\mathrm{sn}\xi \to \sin\xi, \quad \mathrm{cn}\xi \to \cos\xi, \quad \mathrm{dn}\xi \to 1. \tag{6}$$

When $m \to 1$, the Jacobian elliptic functions degenerate to the solitonic solutions of the given NLPDE i.e.

$$\mathrm{sn}\xi \to \tanh\xi, \quad \mathrm{cn}\xi \to \mathrm{sech}\xi, \quad \mathrm{dn}\xi \to \mathrm{sech}\xi. \tag{7}$$

Given a nonlinear NLPDE with, say, two variables: x, t,

$$P(u_x, u_t, u_{xx}, u_{tt}, u_{xt}, \cdots) = 0. \tag{8}$$

Step 1 We assume that Eq.(8) has the following solutions:

$$u(x,t) = \sum_{i=0}^{n} u_i \phi^n(\xi), \tag{9}$$

where $u_i = u_i(x,t), (i = 0, 1, \cdots, n), \xi = \xi(x,t)$ are all unknown functions of x and t, $\phi(\xi)$ satisfying Eq.(3).

The parameter n can be found by balancing the highest order derivative term and the nonlinear term in Eq.(8) (n is a positive integer).

Step 2 Substituting Eq.(9) along with Eq.(3) into Eq.(8), extracting the numerator of the resulting system, we can obtain a set of algebraic polynomials for $\phi^i(\xi)(i = 0, 1, \cdots)$. Setting the coefficients of these terms $\phi^i(\xi)$ to zero, we have a system of overdetermined PDEs with respect to unknown functions $\{u_i(i = 0, 1, \cdots), \xi\}$.

Step 3 Solving the above system by use of the symbolic computation system Maple, we would end up with the explicit expressions for $u_i, (i = 0, 1, \cdots, n)$ and ξ or the constraints among them. Thus, according to Eqs.(4), (6), (7), (9) and the conclusions in Step 3, we can obtain many families of exact solutions for Eq.(8).

3. Exact solutions of Eq.(2)

In order to obtain some exact solutions of Eq.(2), firstly we make the transformation

$$u(x,t) = V(x,t)\exp(\mathrm{i}\Theta(x,t)). \tag{10}$$

Then substituting Eq.(10) into Eq.(2) and setting the real and imaginary parts of the resulting equation equal to zero, we obtain the following sets of partial differential equations:

$$\begin{aligned}
&-V\Theta_t + \frac{1}{2}(V_{xx} - V\Theta_x^2) - \varepsilon V^3 \\
&- x(b + c\sin(\omega t))V = 0,
\end{aligned} \tag{11}$$

$$V_t + \frac{1}{2}V\Theta_{xx} + V_x\Theta_x = 0. \tag{12}$$

Balancing the highest order derivative term V_{xx} with the nonlinear term V^3 in Eq.(11) yields $n = 1$. Therefore, by the use of the mapping method, we assume the solutions of Eqs.(11) and (12) in the following special forms:

$$V(x,t) = f(t) + h(t)\phi(\xi), \quad \xi = p(t)x + q(t), \tag{13}$$

$$\Theta(x,t) = \Gamma(t)x + \Omega(t), \tag{14}$$

where $f(t)$, $h(t)$, $p(t)$, $q(t)$, $\Gamma(t)$ and $\Omega(t)$ are functions of t to be determined, with $\phi(\xi)$ satisfying Eq.(3).

Substituting Eqs.(13), (14) and (3) into Eqs.(11), (12) and collecting coefficients of power of $x^k\phi^l$, and $\sqrt{C+B\phi^2+\frac{A}{2}\phi^4}$ $(k=0,1;l=0,1,2,3)$ with the aid of Maple, then setting each of the coefficients to zero, we can obtain the following set of over-determined partial differential equations with respect to unknown derivative functions $\{f(t), h(t), p(t), q(t), \Gamma(t), \Theta(t)\}$:

$$f_t = 0, \tag{15}$$
$$h_t = 0, \tag{16}$$
$$p_t = 0, \tag{17}$$
$$q_t + p\Gamma = 0, \tag{18}$$
$$-\frac{1}{2}Ahp^2 + \varepsilon h^3 = 0, \tag{19}$$
$$3h^2 f = 0, \tag{20}$$
$$h(\Gamma_t + b + c\sin(\omega t)) = 0, \tag{21}$$
$$h(\Omega_t - \frac{1}{2}(Bp^2 - \Gamma^2) + 3\varepsilon f^2) = 0. \tag{22}$$

Solving Eqs.(15)–(22) by means of Maple, we obtain the following results:

$$\begin{aligned}
f =& 0, \quad p = C_1, \quad h = \pm\sqrt{\frac{A}{2\varepsilon}}C_1,\\
\Gamma =& -bt + \frac{c}{\omega}\cos(\omega t) + C_2,\\
q =& C_1\left(\frac{b}{2}t^2 - \frac{c}{\omega^2}\sin(\omega t) - C_2 t + C_3\right),\\
\Omega =& -\frac{b^2}{6}t^3 + \frac{1}{2}bC_2t^2 + \frac{1}{2}(BC_1^2 - \frac{c^2}{2\omega^2} - C_2^2)t\\
&+ \frac{bc}{\omega^2}t\sin(\omega t) - \frac{c^2}{8\omega^3}\sin(2\omega t)\\
&+ \frac{bc}{\omega^3}\cos(\omega t) - \frac{cC_2}{\omega^2}\sin(\omega t) + C_4,
\end{aligned} \tag{23}$$

where C_1, C_2, C_3 and C_4 are arbitrary constants. Hence,the solution of Eq.(2) reads

$$u = \pm\sqrt{\frac{A}{2\varepsilon}}C_1\phi(\xi)\exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{BC_1^2}{2}\right)\right), \tag{24}$$

where

$$\begin{aligned}
\xi =& px + q = C_1\left(x + \frac{b}{2}t^2 - \frac{c}{\omega^2}\sin(\omega t) - C_2 t + C_3\right),\\
\Gamma =& -bt + \frac{c}{\omega}\cos(\omega t) + C_2,\\
\theta =& -\frac{b^2}{6}t^3 + \frac{1}{2}bC_2t^2 - \frac{1}{2}\left(\frac{c^2}{2\omega^2} + C_2^2\right)t\\
&+ \frac{bc}{\omega^2}t\sin(\omega t) - \frac{c^2}{8\omega^3}\sin(2\omega t)\\
&+ \frac{bc}{\omega^3}\cos(\omega t) - \frac{cC_2}{\omega^2}\sin(\omega t) + C_4,
\end{aligned}$$

with ϕ being the solution of Eq.(3).

3.1. Jacobian elliptic function solution

Using the solutions of Eq.(3), we obtain the following Jacobian elliptic function solutions of Eq.(2):

$$u_1 = \pm\frac{mC_1}{\sqrt{\varepsilon}}\mathrm{sn}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{m^2+1}{2}C_1^2 t\right)\right), \tag{25}$$

$$u_2 = \pm\frac{mC_1}{\sqrt{\varepsilon}}\mathrm{cd}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{m^2+1}{2}C_1^2 t\right)\right), \tag{26}$$

$$u_3 = \pm\frac{mC_1}{\sqrt{-\varepsilon}}\mathrm{cn}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2m^2-1}{2}C_1^2 t\right)\right), \tag{27}$$

$$u_4 = \pm\frac{C_1}{\sqrt{-\varepsilon}}\mathrm{dn}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2-m^2}{2}C_1^2 t\right)\right), \tag{28}$$

$$u_5 = \pm\frac{C_1}{\sqrt{\varepsilon}}\mathrm{ns}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{m^2+1}{2}C_1^2 t\right)\right), \tag{29}$$

$$u_6 = \pm\frac{C_1}{\sqrt{\varepsilon}}\mathrm{dc}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{m^2+1}{2}C_1^2 t\right)\right), \tag{30}$$

$$u_7 = \pm\frac{\sqrt{1-m^2}C_1}{\sqrt{\varepsilon}}\mathrm{nc}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2m^2-1}{2}C_1^2 t\right)\right), \tag{31}$$

$$u_8 = \pm\frac{\sqrt{m^2-1}C_1}{\sqrt{\varepsilon}}nd(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2-m^2}{2}C_1^2 t\right)\right), \tag{32}$$

$$u_9 = \pm\frac{C_1}{\sqrt{\varepsilon}}\mathrm{cs}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2-m^2}{2}C_1^2 t\right)\right), \tag{33}$$

$$u_{10} = \pm\frac{\sqrt{1-m^2}C_1}{\sqrt{\varepsilon}}\mathrm{sc}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2-m^2}{2}C_1^2 t\right)\right), \tag{34}$$

$$u_{11} = \pm\frac{m\sqrt{m^2-1}C_1}{\sqrt{\varepsilon}}\mathrm{sd}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2m^2-1}{2}C_1^2 t\right)\right), \tag{35}$$

$$u_{12} = \pm \frac{C_1}{\sqrt{\varepsilon}}\mathrm{ds}(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{2m^2-1}{2}C_1^2 t\right)\right), \quad (36)$$

$$u_{13} = \pm \frac{C_1}{2\sqrt{-\varepsilon}}(m\mathrm{cn}(\xi) \pm \mathrm{dn}(\xi)) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1+m^2}{4}C_1^2 t\right)\right), \quad (37)$$

$$u_{14} = \pm \frac{C_1}{2\sqrt{\varepsilon}}(\mathrm{ns}(\xi) \pm \mathrm{cs}(\xi)) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1-2m^2}{4}C_1^2 t\right)\right), \quad (38)$$

$$u_{15} = \pm \frac{\sqrt{1-m^2}C_1}{2\sqrt{\varepsilon}}(\mathrm{nc}(\xi) \pm \mathrm{sc}(\xi)) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1+m^2}{4}C_1^2 t\right)\right), \quad (39)$$

$$u_{16} = \pm \frac{C_1}{2\sqrt{\varepsilon}}(\mathrm{ns}(\xi) \pm \mathrm{ds}(\xi)) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{m^2-2}{4}C_1^2 t\right)\right), \quad (40)$$

$$u_{17} = \pm \frac{mC_1}{2\sqrt{\varepsilon}}(\mathrm{sn}(\xi) \pm i\mathrm{cn}(\xi)) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{m^2-2}{4}C_1^2 t\right)\right), \quad (41)$$

$$u_{18} = \pm \frac{mC_1}{2\sqrt{\varepsilon}} \frac{\mathrm{dn}(\xi)}{\sqrt{1-m^2}\mathrm{sn}(\xi) \pm \mathrm{cn}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{m^2-2}{4}C_1^2 t\right)\right), \quad (42)$$

$$u_{19} = \pm \frac{C_1}{2\sqrt{\varepsilon}}(m\mathrm{sn}(\xi) \pm i\mathrm{dn}(\xi)) \times \exp\left(\left(\Gamma x + \theta + \frac{1-2m^2}{4}C_1^2 t\right)\right), \quad (43)$$

$$u_{20} = \pm \frac{C_1}{2\sqrt{\varepsilon}} \frac{\mathrm{sn}(\xi)}{1 \pm \mathrm{cn}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1-2m^2}{4}C_1^2 t\right)\right), \quad (44)$$

$$u_{21} = \pm \frac{mC_1}{2\sqrt{\varepsilon}} \frac{\mathrm{sn}(\xi)}{1 \pm \mathrm{dn}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{m^2-2}{4}C_1^2 t\right)\right), \quad (45)$$

$$u_{22} = \pm \frac{\sqrt{m^2-1}C_1}{2\sqrt{\varepsilon}} \frac{\mathrm{dn}(\xi)}{1 \pm m\mathrm{sn}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{m^2+1}{4}C_1^2 t\right)\right), \quad (46)$$

$$u_{23} = \pm \frac{\sqrt{1-m^2}C_1}{2\sqrt{\varepsilon}} \frac{\mathrm{cn}(\xi)}{1 \pm \mathrm{sn}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1+m^2}{4}C_1^2 t\right)\right), \quad (47)$$

$$u_{24} = \pm \frac{(1-m^2)C_1}{2\sqrt{\varepsilon}} \frac{\mathrm{sn}(\xi)}{\mathrm{dn}(\xi) \pm \mathrm{cn}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1+m^2}{4}C_1^2 t\right)\right), \quad (48)$$

$$u_{25} = \pm \frac{m^2 C_1}{2\sqrt{\varepsilon}} \frac{\mathrm{cn}(\xi)}{\sqrt{1-m^2} \pm \mathrm{dn}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{m^2-2}{4}C_1^2 t\right)\right), \quad (49)$$

3.2. Soliton solution

When $m \to 1$, we can obtain soliton solutions.

Family 1 dark soliton solution ($\varepsilon > 0 (a > 0)$)

$$u_{26} = \pm \frac{C_1}{\sqrt{\varepsilon}} \tanh(\xi) \exp(\mathrm{i}(\Gamma x + \theta - C_1^2 t)\Big). \quad (50)$$

Family 2 bright soliton solution ($\varepsilon < 0 (a < 0)$)

$$u_{27} = \pm \frac{C_1}{\sqrt{-\varepsilon}} \mathrm{sech}(\xi) \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1}{2}C_1^2 t\right)\right). \quad (51)$$

The solutions (50) and (51) are also obtained in Ref.[14] by applying the inverse scattering method.

Family 3 soliton-like solutions ($\varepsilon > 0 (a > 0)$)

$$u_{28} = \pm \frac{C_1}{\sqrt{\varepsilon}} \coth(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - C_1^2 t\right)\right), \quad (52)$$

$$u_{29} = \pm \frac{C_1}{\sqrt{\varepsilon}} \mathrm{sech}(\xi) \coth(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1}{2}C_1^2 t\right)\right), \quad (53)$$

$$u_{30} = \pm \frac{C_1}{2\sqrt{\varepsilon}}(1 + \mathrm{sech}(\xi)) \coth(\xi) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{1}{4}C_1^2 t\right)\right), \quad (54)$$

$$u_{31} = \pm \frac{C_1}{2\sqrt{\varepsilon}}(\tanh(\xi) \pm \mathrm{isech}(\xi)) \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{1}{4}C_1^2 t\right)\right), \quad (55)$$

$$u_{32} = \pm \frac{C_1}{2\sqrt{\varepsilon}} \frac{\tanh(\xi)}{1 \pm \mathrm{sech}(\xi)} \times \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{1}{4}C_1^2 t\right)\right). \quad (56)$$

3.3. Triangular function solution

When $m \to 0$, we can obtain triangular function solutions.

$$u_{33} = \pm \frac{C_1}{\sqrt{\varepsilon}} \csc(\xi) \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{1}{2}C_1^2 t\right)\right), \tag{57}$$

$$u_{34} = \pm \frac{C_1}{\sqrt{\varepsilon}} \sec(\xi) \exp\left(\mathrm{i}\left(\Gamma x + \theta - \frac{1}{2}C_1^2 t\right)\right), \tag{58}$$

$$u_{35} = \pm \frac{C_1}{\sqrt{\varepsilon}} \tan(\xi) \exp\left(\mathrm{i}\left(\Gamma x + \theta + C_1^2 t\right)\right), \tag{59}$$

$$u_{36} = \pm \frac{C_1}{\sqrt{\varepsilon}} \cot(\xi) \exp\left(\mathrm{i}\left(\Gamma x + \theta + C_1^2 t\right)\right), \tag{60}$$

$$u_{37} = \pm \frac{C_1}{2\sqrt{\varepsilon}} \csc(\xi)(1 \pm \cos(\xi))) \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1}{4}C_1^2 t\right)\right), \tag{61}$$

$$u_{38} = \pm \frac{C_1}{2\sqrt{\varepsilon}} \sec(\xi)(1 \pm \sin(\xi)) \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1}{4}C_1^2 t\right)\right), \tag{62}$$

$$u_{39} = \pm \frac{C_1}{2\sqrt{\varepsilon}} \frac{\sin(\xi)}{1 \pm \cos(\xi)} \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1}{4}C_1^2 t\right)\right), \tag{63}$$

$$u_{40} = \pm \frac{C_1}{2\sqrt{\varepsilon}} \frac{\cos(\xi)}{1 \pm \sin(\xi)} \exp\left(\mathrm{i}\left(\Gamma x + \theta + \frac{1}{4}C_1^2 t\right)\right). \tag{64}$$

4. Summary

Based on the mapping method and the use of symbolic computation system Maple, we have constructed explicit, rich and new solutions, such as Jacobian elliptic function solutions, combined Jacobian elliptic function solutions, triangular function solutions, bright and dark solitons, and soliton-like solutions to the nonlinear Gross–Pitaevskii equation with weak bias magnetic and time-dependent laser fields. At the same time, we have successfully recovered the previously known exact solutions in Ref.[14] by using the inverse scattering method.

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