Vol 14 No 2, February 2005
1009-1963/2005/14(02)/0223-08

Chinese Physics

© 2005 Chin. Phys. Soc.
and IOP Publishing Ltd

Nonlocality and purity in atom–field coupling system*

Cai Xin(蔡 欣), Huang Guang-Ming(黄光明)†, and Li Gao-Xiang(李高翔)

Department of Physics, Huazhong Normal University, Wuhan 430079, *China*

(Received 3 March 2004; revised manuscript received 4 October 2004)

The effects of initial field state and thermal environment on quantum nonlocality and linear entropy in an atom–field coupling system are investigated. We found that if the cavity is lossless and the reservoir is in vacuum, the atom–field state can exhibit quantum nonlocality periodically and the linear entropies of the atom and the field also oscillate periodically with a period the same as that of quantum nonlocality. And if the cavity dissipation is very weak and the average photon number of the reservoir is very small, the quantum nonlocality will be lost and the linear entropies of the atom and the field oscillate with a decreasing amplitude. The rapidity of the loss of the quantum nonlocality depends on the amplitude of the initial squeezed coherent state, the cavity damping constant κ and the average photon number N of the thermal reservoir. The stronger the field and the larger the constant κ and the average photon number N could be, the more rapidly the nonlocality decreases.

Keywords: quantum nonlocality, linear entropy, thermal environment, squeezed coherent state
PACC: 0155, 0365

1. Introduction

The paradox suggested by Einstein, Podolsky, and Rosen[1] aroused controversy about nonlocality of quantum states. Bell[2] proposed a remarkable inequality imposed by a local hidden variable theory, which enables a quantitative test on quantum nonlocality. Numerous theoretical studies and experimental demonstrations have been performed to understand nonlocal properties of quantum states.

Gisin and Peres[3] found pairs of observables whose correlations violate Bell's inequality for a discrete N-dimensional entangled state. Banaszek and Wodkiewicz[4] studied Bell's inequality for continuous-variable states, in terms of Wigner representation in phase space based upon parity measurement and displacement operation.Following the parity measurement approach to test quantum nonlocality for a field, Kim and Lee[5] proposed a scheme to test quantum nonlocality for the field prepared in two spatially separated cavities, by sending two independent two-level atoms which interact odd-resonantly with cavity fields separately, or sending single atoms sequentially to interact with the cavities. Majumdar and Nayak[6] investigated a Bell-type inequality probabilities of detected atoms formulated using atom-photon interactions in a cavity. They considered decoherence brought about by both atomic decay and cavity photon loss, and studied its action quantitatively in diminishing the atom–field and resultant atom–atom secondary correlations. Jeong *et al*[7] analysed various observables to give the maximum violation of Bell's inequality for continuous-variable states and discussed an observable which gives the violation of Bell's inequality for any entangled pure continuous-variable states. They found that it does not have to be a maximally entangled state to give the maximal violation of Bell's inequality. Wu *et al*[8] solved a class of coupled-channel cavity quantum electronic dynamics (QED) models describing two-photon resonant Raman process in multiple-Λ configuration analytically by using an algebraic method that introduces an angular momentum algebra for this multiwave mixing system. We have once investigated the property of quantum nonlocality in the atom–field coupling system via degenerate Raman processes with cavity damping.[9]

The aim of this paper is to analyse the dynamical behaviour of the quantum nonlocality and purity, which is quantified by the linear entropy, in the atom–field coupling system in a cavity. In this system, the Λ-type atom is initially in one of its two lower states and the cavity in a squeezed coherent state.

This paper is organized as follows. In Section

*Project supported by the National Natural Science Foundation of China (Grant No 10204009).
†E-mail: gmhuang@phy.ccnu.edu.cn

2 we discuss the master equation describing the interaction of the system "atom + field mode" with a thermal reservoir. In Section 3 we use the Wigner function to calculate the Bell function, with which we study the dynamical behaviour of quantum nonlocality of the system. In Section 4 we introduce the linear entropies of the atomic and field subsystems, which can serve as measures of quantum purity and entanglement, putting emphasis on the usefulness of the Wigner function in the calculation of these entropies. Section 5 gives the conclusion.

2. Model system

The system under consideration is a degenerate Λ-type three-level atom placed in a single-mode cavity which is coupled to a reservoir in the usual way.[10] The upper atomic level with energy w_i and magnetic quantum number $m_j = 0$ is denoted by $|i\rangle$ and the two lower levels with magnetic quantum number $m_j = \pm 1$ are denoted by $|e\rangle$ and $|g\rangle$ with energy w_e and w_g, respectively. In order to simplify the calculation further, we assume that the two levels are degenerate, i.e. $w_e = w_g$.[11] If the cavity field is of x-polarization, the dipole-allowed transitions $|e\rangle \leftrightarrow |g\rangle$ and $|g\rangle \leftrightarrow |e\rangle$ are mediated by photons of the single-mode cavity field, which is characterized by the photon annihilation and creation operators a and a^+ with frequency ω_a. Following Refs.[11, 12], one can adiabatically remove the upper level $|i\rangle$ when the one-photon detuning $|\delta|$ (where $\delta = \omega_i - \omega_a - \omega_e = \omega_i - \omega_a - \omega_g$) is much larger than the Rabi frequency of the oscillation between $|i\rangle$ and $|e\rangle$ ($|g\rangle$), so that the effective Hamiltonian of the atom–field coupling system including the ac Stark shifts is written in the interaction picture as[13]

$$\begin{aligned} H =& \frac{g_2^2}{\delta} a^+ a |e\rangle\langle e| + \frac{g_1^2}{\delta} a^+ a |g\rangle\langle g| \\ &+ \frac{g_1 g_2}{\delta} a^+ a (|e\rangle\langle g| + |g\rangle\langle e|), \end{aligned} \tag{1}$$

where $g_1(g_2)$ represents the coupling strength for the dipole-allowed transition $|i\rangle \leftrightarrow |g\rangle(|i\rangle \leftrightarrow |e\rangle)$. The first two terms in Eq.(1) describe the ac Stark shifts of the atomic levels $|e\rangle$ and $|g\rangle$ respectively, the last two characterize the degenerate Raman transition processes between $|e\rangle$ and $|g\rangle$. Taking the influence of cavity dissipation into account, we can write the master equation which governs the time evolution of the density operator for the atom–field coupling system

$$\frac{\mathrm{d}}{\mathrm{d}t}\rho(t) = -\mathrm{i}[H, \rho(t)] + L_c\rho(t), \tag{2}$$

with

$$\begin{aligned} L_c\rho(t) =& \kappa(N+1)[2a\rho(t)a^+ - a^+a\rho(t) - \rho(t)a^+a] \\ &+ \kappa N[2a^+\rho(t)a - aa^+\rho(t) - \rho(t)aa^+], \end{aligned} \tag{3}$$

where κ is the damping constant for the cavity mode and N is the average photon number of the thermal reservoir.

In order to solve Eq.(2) analytically, we describe two new atomic states as

$$\begin{aligned} |\psi_1\rangle &= \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_1|e\rangle - g_2|g\rangle), \\ |\psi_2\rangle &= \frac{1}{\sqrt{g_1^2 + g_2^2}}(g_2|e\rangle + g_1|g\rangle), \end{aligned} \tag{4}$$

which form a new basis for the atomic Hilbert space. Transforming Eq.(2) into the new basis defined by Eq.(4), the density operator for the atom–field coupling system satisfies

$$\frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}(t) = -\mathrm{i}\left(\frac{g_1^2}{\delta} + \frac{g_2^2}{\delta}\right)[a^+a|\psi_2\rangle\langle\psi_2|, \tilde{\rho}(t)] + L_c\tilde{\rho}(t). \tag{5}$$

By expressing $\tilde{\rho}(t)$ as

$$\begin{aligned} \tilde{\rho}(t) =& \tilde{\rho}_{22}(t)|\psi_2\rangle\langle\psi_2| + \tilde{\rho}_{11}(t)|\psi_1\rangle\langle\psi_1| \\ &+ \tilde{\rho}_{21}(t)|\psi_2\rangle\langle\psi_1| + \tilde{\rho}_{12}(t)|\psi_1\rangle\langle\psi_2|, \end{aligned} \tag{6}$$

where $\tilde{\rho}_{jk}(t) = \langle\psi_j|\tilde{\rho}(t)|\psi_k\rangle$, $(j,k = 1,2)$, the operators $\tilde{\rho}_{jk}(t)$ satisfy the following equations:

$$\begin{aligned} \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}_{11}(t) &= L_c\tilde{\rho}_{11}(t), \\ \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}_{22}(t) &= -2\mathrm{i}\lambda[a^+a, \tilde{\rho}_{22}(t)] + L_c\tilde{\rho}_{22}(t), \\ \frac{\mathrm{d}}{\mathrm{d}t}\tilde{\rho}_{21}(t) &= -2\mathrm{i}\lambda a^+a\tilde{\rho}_{21}(t) + L_c\tilde{\rho}_{21}(t), \end{aligned} \tag{7}$$

where we have assumed that $g_1 = g_2 = g$ and $\lambda = g^2/\delta$.

Let us assume that the field is in the squeezed coherent state $|\alpha, r\rangle$ and the atom initially in the state $|e\rangle$. Although the two atomic lower levels $|e\rangle$ and $|g\rangle$ are degenerate, they have different magnetic quantum number $m_j = \pm 1$, so the initial atomic state $|e\rangle$ can be realized by the technique of population transfer by using a sequence of two counter-intuitively ordered delayed laser pulses.[14] And $|\alpha, r\rangle$ is given by

$$|\alpha, r\rangle = \hat{D}(\alpha)\hat{S}(\varepsilon)|0\rangle, \tag{8}$$

with $\hat{D}(\alpha)$ being the displacement operator, $\hat{D}(\alpha) = \exp(\alpha\hat{a}^* - \alpha^*\hat{a})$, and $\hat{S}(\varepsilon)$ the squeeze operator, $\hat{S}(\varepsilon) =$

$\exp\left[\frac{1}{2}\varepsilon^*\hat{a}^2 - \frac{1}{2}\varepsilon(\hat{a}^+)^2\right]$. Here, $\varepsilon = re^{i\varphi}$ is any complex number with modulus r and argument φ, which determine the squeezing intensity and the direction of squeezing respectively.

3. Wigner function and quantum nonlocality of the system

Wodkiewicz[15] proposed a scheme to test whether or not the atom–field entangled state displays the nonlocality, in which the state of the atom is detected by standard spin-1/2 projections $\hat{\sigma}(\theta)$, but the state of the field is probed by displaced parity operator $\hat{\Pi}(\beta)$. In the following, we adopt Wodkiewicz's scheme to investigate the influence of the cavity dissipation on the quantum nonlocality.

The state of the atom is detected by measuring the projection $\hat{\sigma}(\theta)$ of the atomic Bloch vector on to a direction θ, represented by $|\theta\rangle = \cos(\theta/2)|e\rangle + \sin(\theta/2)|g\rangle$. Therefore, the joint measurement on the system is described by the following dichotomous observable for the field and the atom:

$$\hat{\Pi}(\beta,\theta) = \hat{\Pi}(\beta)\otimes\hat{\sigma}(\theta). \tag{9}$$

It is easily derived that the local hidden variable theory results in the following Bell inequality:[10]

$$\begin{aligned} -2 \le B =& \Pi(\beta',\theta') + \Pi(\beta',\theta) \\ &+ \Pi(\beta,\theta') - \Pi(\beta,\theta) \le 2 \end{aligned} \tag{10}$$

for any choice of the parameters β and β' and the angles θ and θ', where we call B the Bell function, and $\Pi(\beta,\theta)$ is the quantum expectation value of operator $\hat{\Pi}(\beta,\theta)$. If the Bell inequality (10) is violated, we can obtain the result that the atom–field coupling state exhibits quantum nonlocality.

On the other hand, the mean parity of the field is proportional to the Wigner function:[15]

$$\Pi(\beta) = (\pi/2)W(\beta). \tag{11}$$

So, we can obtain the quantum expectation value of the displaced parity operator by calculating the Wigner function which is defined as

$$\begin{aligned} W(\beta,\beta^*) =& \int \mathrm{Tr}[\exp(\xi\hat{a}^+ - \xi^*\hat{a})\rho] \\ &\times \exp(\beta\xi^* - \beta^*\xi)/\pi^2 \mathrm{d}^2\xi. \end{aligned} \tag{12}$$

It is easy to calculate the Wigner functions corresponding to $\rho_{11}^{\sim}$, $\rho_{22}^{\sim}$ and $\rho_{21}^{\sim}$ respectively; here we have assumed that the direction of squeezing $\varphi = 0$:

$$\begin{aligned} W_{11}(\beta,\beta^*) =& \frac{1}{\pi}\sqrt{\frac{1}{t(r)t(-r)}}\exp\left[\frac{(\beta-\beta^*)^2}{2t(r)}\right] \\ &\times \exp\left[-\frac{(2\alpha e^{-\kappa t} - \beta - \beta^*)^2}{2t(-r)}\right], \\ W_{22}(\beta,\beta^*) =& \frac{1}{2\pi}\sqrt{\frac{1}{C_1^2 - 4C_2^2}} \\ &\times \exp\left[\frac{(\beta e^{2i\lambda t} - \beta^* e^{-2i\lambda t})^2}{4(2C_2 + C_1)}\right] \\ &\times \exp\left[\frac{(2\alpha e^{-\kappa t} - \beta e^{2i\lambda t} - \beta^* e^{-2i\lambda t})^2}{4(2C_2 - C_1)}\right], \\ W_{21}(\beta,\beta^*) =& \frac{1}{\pi}\frac{(\Omega+\kappa)e^{3\Omega t} + (\Omega-\kappa)e^{\Omega t}}{\Omega\sqrt{f_1(r)f_1(-r)}} \\ &\times \exp\left[(\kappa + i\lambda)t - \frac{2i\lambda\alpha^2(e^{2\Omega t}-1)}{\Omega g(r)}\right] \\ &\times \exp\left[-\frac{(\beta e^{i\lambda t} - \beta^* e^{-i\lambda t})^2 f_2(r)}{4f_1(r)}\right. \\ &\left.- \frac{\left(\frac{4\alpha e^{\Omega t}}{g(r)} - \beta e^{i\lambda t} - \beta^* e^{-i\lambda t}\right)^2 f_2(-r)}{4f_1(-r)}\right], \end{aligned} \tag{13}$$

with

$$\begin{aligned} t(r) =& (2N+1)(1 - e^{-2\kappa t}) + e^{-2\kappa t + 2r}, \\ C_1 =& -\frac{2N+1}{2}(e^{-2\kappa t} - 1) + \frac{\mathrm{ch}2r}{2}e^{-2\kappa t}, \\ C_2 =& \frac{\mathrm{sh}r\mathrm{ch}r}{2}e^{-2\kappa t}, \\ \Omega =& \sqrt{\kappa^2 + 2i(2N+1)\kappa\lambda - \lambda^2}, \end{aligned}$$

and

$$\begin{aligned} g(r) =& \left(1 + \frac{i\lambda e^{-2r}}{\Omega} + \frac{\kappa}{\Omega}\right)e^{2\Omega t} \\ &+ \left(1 - \frac{i\lambda e^{-2r}}{\Omega} - \frac{\kappa}{\Omega}\right), \\ f_1(r) =& \left(\frac{(2N+1)\kappa}{\Omega} + \frac{i\lambda}{2\Omega}\right)\left(\frac{\kappa}{\Omega} + \frac{i\lambda e^{2r}}{\Omega} + 1\right)e^{4\Omega t} \\ &+ \left[2e^{2r} - 2\left(\frac{(2N+1)\kappa}{\Omega} + \frac{i\lambda}{2\Omega}\right)\right. \\ &\left.\times\left(\frac{\kappa}{\Omega} + \frac{i\lambda e^{2r}}{\Omega}\right)\right]e^{2\Omega t} \\ &+ \left(\frac{(2N+1)\kappa}{\Omega} + \frac{i\lambda}{2\Omega}\right)\left(\frac{\kappa}{\Omega} + \frac{i\lambda e^{2r}}{\Omega} - 1\right), \\ f_2(r) =& \left(1 + \frac{2\kappa}{\Omega} + \frac{i\lambda e^{2r}}{\Omega} + \frac{\kappa^2}{\Omega^2} + \frac{i\kappa\lambda e^{2r}}{\Omega^2}\right)e^{4\Omega t} \\ &+ 2\left(1 - \frac{i\kappa\lambda e^{2r}}{\Omega^2} - \frac{\kappa^2}{\Omega^2}\right)e^{2\Omega t} \\ &+ \left(1 - \frac{2\kappa}{\Omega} - \frac{i\lambda e^{2r}}{\Omega} + \frac{i\kappa\lambda e^{2r}}{\Omega^2} + \frac{\kappa^2}{\Omega^2}\right). \end{aligned} \tag{14}$$

Substituting Eqs.(9),(11) and (13) into Eq.(10) and taking $\beta' = 0$, $\beta = x$, $\alpha = D$, where x and D are the real parameters for simplicity, we obtain the Bell function at time t

$$\begin{aligned} B(x,\theta,t) =& (1+\cos\theta)\mathrm{Re}\left(\frac{(\Omega+\kappa)\mathrm{e}^{3\Omega t}+(\Omega-\kappa)\mathrm{e}^{\Omega t}}{\Omega\sqrt{f_1(r)f_1(-r)}}\right. \\ & \left.\times\exp\left[(\kappa+\mathrm{i}\lambda)t-\frac{4D^2\mathrm{e}^{2\Omega t}f_2(-r)}{g^2(r)f_1(-r)}\right]\right) \\ & +\frac{\sin\theta}{2}\left[\frac{1}{2}\sqrt{\frac{1}{C_1^2-4C_2^2}}\exp\left(\frac{D^2\mathrm{e}^{-2\kappa t}}{2C_2-C_1}\right)\right. \\ & \left.-\sqrt{\frac{1}{t(r)t(-r)}}\exp\left(-\frac{2D^2\mathrm{e}^{-2\kappa t}}{t(-r)}\right)\right] \\ & +\pi(1-\cos\theta)\mathrm{Re}(W_{21}(x)) \\ & -\frac{\pi\sin\theta}{2}[\mathrm{Re}(W_{22}(x)-W_{11}(x))]. \end{aligned} \tag{15}$$

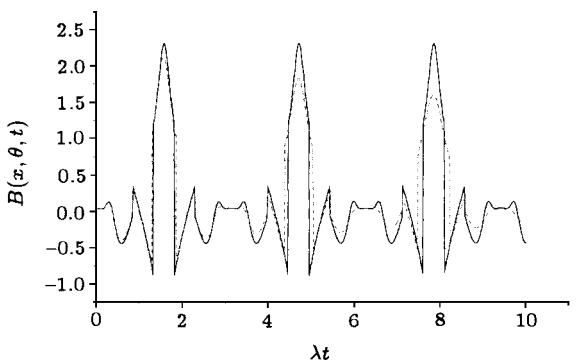


Fig.1. Time evolution of Bell function $B(x,\theta,t)$ for D=1, r=1, x=0.9995, $\cos\theta$=0.9636, $\sin\theta$=0.2675. The damping constant κ and the average photon number of the reservoir N is chosen to be κ/λ=0, N=0 (solid) κ/λ=0.01, N=0.1 (dashed).

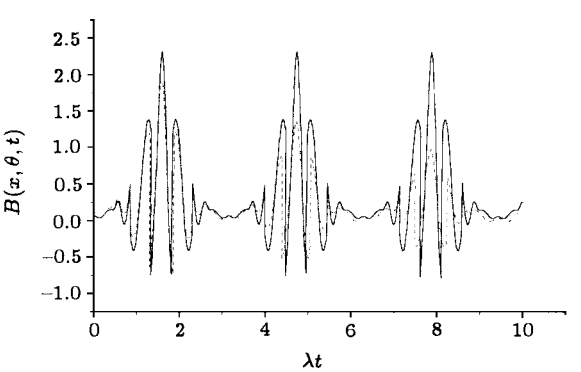


Fig.2. The same as Fig.1, but D=2, x=1.9998, $\cos\theta$=0.9425, and $\sin\theta$=0.3342.

In Figs.1 and 2 we show the time evolution of the Bell function. In these figures, the parameters x and θ have been chosen such that the Bell function $B(x,\theta,t=\pi/2\lambda)$ reaches its maximal value when $N=0$. For $\kappa=0$ and $N=0$, which means that the cavity dissipation and the average photon number of the reservoir can be ignored, the value of B can be larger than 2 around $t_s=(k+1/2)\pi/\lambda(k=0,1,2,\cdots)$, which means that the atom–field coupling state can exhibit the quantum nonlocality. The Bell function exhibits an exact periodic collapses and revivals with the period $T=\pi/\lambda$. From the two figures, we find that the larger the amplitude of the initial squeezed coherent state $|D,r\rangle$, the more rapidly the quantum nonlocality is lost, and the maximal value of B decreases with increasing D. Thus, it is noted that the weaker the intensity of the initial squeezed coherent state, the more evident the quantum nonlocality, which is helpful to observe the quantum nonlocality. If the influence of the cavity and the average photon number of the reservoir is very small but cannot be ignored, for example, $\kappa/\lambda=0.01, N=0.1$, B also behaves as a periodic collapse-and-revival phenomenon, the periodic period is the same as that of $\kappa=0, N=0$, but the quantum nonlocality only exists in a short period around $t=\pi/2\lambda$, and the maximal value of B decreases around t_s, which means that the quantum nonlocality becomes weaker. This is because the system evolves into a mixed state, in which the quantum coherence loses irreversibly due to the cavity dissipation. In addition, by comparing with our previous results[9] we found that the time evolution of the Bell function in coherent state $|D\rangle$ has the same period as in the squeezed coherent state $|D,r\rangle$ and the two systems exhibit quantum nonlocality at about the same time, and the maximal value of B is larger in the squeezed coherent state than in the coherent state. However, the quantum nonlocality is lost more rapidly in the squeezed coherent state than in the coherent state (see Fig.3). In other words, the weaker the intensity of the state of field, the more evident the quantum nonlocality. The quantum nonlocality for the atom–field coupling state originates from the quantum interference between the coherent component states for the joint measurement with the atom and the field. The Raman-type interaction between the atom and the field leads the atom–field coupling system to evolve into a Schrödinger cat state, $\frac{1}{\sqrt{2}}(|\psi_1\rangle|D,r\rangle+|\psi_2\rangle|-D,-r\rangle)$, periodically when $\kappa=0$, $N=0$ with the period $T=\pi/\lambda$, so the quantum interference will recover periodically (see Eq.(13)). However, the quantum interference is destroyed by the cavity dissipation. The speed of de-

struction depends on the damping constant κ. Thus, the strong cavity dissipation leads to the phenomenon that the quantum nonlocality of the system becomes weaker and disappears finally with the time evolution.

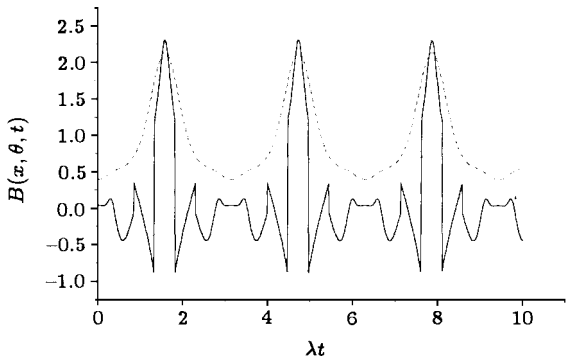


Fig.3. Time evolution of the Bell function $B(x,\theta,t)$ for squeezed coherent state (solid), and coherent state (dashed).

4. Linear entropies of atom, field and system

Entropy is a key concept in quantum information theory. On the one hand, it can measure how much uncertainty there is in the state of a physical system.[16,17] Usually, the von Neumann entropy, defined as

$$S = -\mathrm{Tr}(\rho \ln \rho), \tag{16}$$

is used to measure the uncertainty of a mixed state. For a pure state, $S = 0$, while for a mixed state, $S > 0$. And for a bipartite system in a pure state, the von Neumman entropy of a subsystem can be used to measure the entanglement between the two subsystems. The larger the von Neumann entropy of a subsystem, the greater the entanglement. On the other hand, information entropy can overcome the disadvantage of the definition based on the Heisenberg uncertainty relation (HUR), for example, when applying to the squeezing of the two-level atom; and Fang *et al*[18] defined the squeezing of the atom using the entropy uncertainty relation (EUR), named as entropy squeezing, which can be used to describe the nonclassical characteristics of a two-level atom. From Eq.(16) we find that the von Neumann entropy is difficult to be used in calculation since it is necessary to diagonalize the operator ρ. Therefore, we study the evolution of linear entropy defined as

$$S_{\mathrm{linear}} = 1 - \mathrm{Tr}\rho^2. \tag{17}$$

An advantage of using the linear entropy instead of the von Neumann entropy lies in the great simplification of calculations, without changing the qualitative physical conclusions, especially when discussing the purity of the atom–field coupling system. For a pure state, $S_{\mathrm{linear}} = 0$, while for a mixed state, $S_{\mathrm{linear}} > 0$. In the following, we will use Wigner function to calculate linear entropies and discuss their evolutions with time.

The linear entropies of the atom, field and total system are defined respectively as

$$s_A = 1 - \mathrm{Tr}\rho_A^2,\ s_C = 1 - \mathrm{Tr}\rho_C^2,\ s_{AC} = 1 - \mathrm{Tr}\rho^2. \tag{18}$$

where the reduced density operator of each subsystem is obtained by taking a partial trace of the total statistical operator ρ over the complementary subsystem: $\rho_A = \mathrm{Tr}_C\rho$, $\rho_C = \mathrm{Tr}_A\rho$. We use the subscripts A, C and AC to denote the atom, the field and the total system of them respectively. If the total system is in a pure quantum state (that happens in the absence of dissipation, i.e. for $\kappa = 0$), each of the partial linear entropies can be used as a measurement of entanglement between the atomic and field subsystems.[19,20]

In the generic case of the statistical operator as described in Eq.(6), we have

$$\begin{aligned}\rho_C =&\tilde{\rho}_{11} + \tilde{\rho}_{22},\\ \rho_A =&\mathrm{Tr}\tilde{\rho}_{11}|\psi_1\rangle\langle\psi_1| + \mathrm{Tr}\tilde{\rho}_{22}|\psi_2\rangle\langle\psi_2|\\ &+ \mathrm{Tr}\tilde{\rho}_{12}|\psi_1\rangle\langle\psi_2| + \mathrm{Tr}\tilde{\rho}_{21}|\psi_2\rangle\langle\psi_1|.\end{aligned} \tag{19}$$

Calculating all necessary traces, we obtain the following expressions for the linear entropies of the atom, the field and the total system:[21]

$$\begin{aligned}s_{AC} =&1 - \mathrm{Tr}(\tilde{\rho}_{22}^2) - \mathrm{Tr}(\tilde{\rho}_{11}^2) - 2\mathrm{Tr}(\tilde{\rho}_{12}\tilde{\rho}_{21}),\\ s_C =&1 - \mathrm{Tr}(\tilde{\rho}_{22}^2) - \mathrm{Tr}(\tilde{\rho}_{11}^2) - 2\mathrm{Tr}(\tilde{\rho}_{11}\tilde{\rho}_{22}),\\ s_A =&2\mathrm{Tr}(\tilde{\rho}_{11})\mathrm{Tr}(\tilde{\rho}_{22}) - 2|\mathrm{Tr}\tilde{\rho}_{21}|^2.\end{aligned} \tag{20}$$

One of the simplest ways to calculate the traces of partial statistical operators and their products is to use the Wigner function which is associated with the operator ρ:[22]

$$\begin{aligned}\mathrm{Tr}\rho =&\int W(\beta,\beta^*)\mathrm{d}^2\beta,\\ \mathrm{Tr}(\rho_a\rho_b) =&\pi\int W_a(\beta,\beta^*)W_b(\beta,\beta^*)\mathrm{d}^2\beta.\end{aligned} \tag{21}$$

Substituting Eqs.(13), (21) into Eq.(20), the linear entropies of the atom, the field and the total system can be given as follows:

$$
\begin{aligned}
s_{AC}(t) =& 1 - \frac{1}{4\sqrt{t(r)t(-r)}} - \frac{1}{8\sqrt{C_1^2 - 4C_2^2}} - 2\frac{1}{\sqrt{\frac{f_2(r)}{f_1(r)} + \frac{f_2^*(r)}{f_1^*(r)}}\sqrt{\frac{f_2(-r)}{f_1(-r)} + \frac{f_2^*(-r)}{f_1^*(-r)}}} \\
& \times \left| \frac{(\Omega+\kappa)\mathrm{e}^{3\Omega t} + (\Omega-\kappa)\mathrm{e}^{\Omega t}}{\Omega\sqrt{f_1(r)f_2(r)}} \right|^2 \\
& \times \left| \exp\left[(\kappa + \mathrm{i}\lambda)t - \frac{2\mathrm{i}\lambda D^2(\mathrm{e}^{2\Omega t} - 1)}{\Omega g(r)} - \frac{4f_2(-r)D^2\mathrm{e}^{2\Omega t}}{f_1(-r)g^2(-r)} \right] \right|^2 \\
& \times \exp\left[\frac{4\left(\frac{f_2(-r)D\mathrm{e}^{\Omega t}}{f_1(-r)g(r)} + \frac{(D\mathrm{e}^{\Omega t} f_2(-r))^*}{(f_1(-r)g(-r))^*} \right)^2}{\frac{f_2(-r)}{f_1(-r)} + \frac{f_2^*(-r)}{f_1^*(-r)}} \right], \\
s_A(t) =& \frac{1}{2} - 2\left| \frac{(\Omega+\kappa)\mathrm{e}^{3\Omega t} + (\Omega-\kappa)\mathrm{e}^{\Omega t}}{\Omega\sqrt{f_2(r)f_2(-r)}} \exp\left[(\kappa + \mathrm{i}\lambda)t - \frac{2\mathrm{i}\lambda D^2(\mathrm{e}^{2\Omega t} - 1)}{\Omega g(r)} \right] \right|^2, \\
s_C(t) =& 1 - \frac{1}{4\sqrt{t(r)t(-r)}} - \frac{1}{8\sqrt{C_1^2 - 4C_2^2}} \\
& - \sqrt{\frac{1}{t(r)t(-r)}}\sqrt{\frac{1}{C_1^2 - 4C_2^2}}\sqrt{\frac{1}{M^2 - 4L^2}} \exp\left[- \frac{2D^2\mathrm{e}^{-2\kappa t}}{t(-r)} + \frac{D^2\mathrm{e}^{-2\kappa t}}{2C_2 - C_1} \right] \\
& \times \exp\left[\frac{2D^2\mathrm{e}^{-2\Omega t}(\cos\phi + 1)}{t^2(-r)(2L + M)} \right] \\
& \times \exp\left[\frac{D^2\mathrm{e}^{-2\Omega t}(\cos\phi\cos 4\lambda t + \sin\phi\sin 4\lambda t + 1)}{2(2C_2 - C_1)^2(2L + M)} \right] \\
& \times \exp\left[- \frac{2D^2\mathrm{e}^{-2\Omega t}(\cos\phi\cos 2\lambda t + \sin\phi\sin 2\lambda t + \cos 2\lambda t)}{t(-r)(2C_2 - C_1)(2L + M)} \right] \\
& \times \exp\left[\frac{2D^2\mathrm{e}^{-2\Omega t}(1 - \cos\phi)}{t^2(-r)(M - 2L)} \right] \\
& \times \exp\left[\frac{D^2\mathrm{e}^{-2\Omega t}(1 - \cos\phi\cos 4\lambda t - \sin\phi\sin 4\lambda t)}{2(2C_2 - C_1)^2(M - 2L)} \right] \\
& \times \exp\left[\frac{2D^2\mathrm{e}^{-2\Omega t}(\cos\phi\cos 2\lambda t + \sin\phi\sin 2\lambda t - \cos 2\lambda t)}{t(-r)(2C_2 - C_1)(M - 2L)} \right]. \qquad (22)
\end{aligned}
$$

Here, $t(r)$, C_1, C_2, in Ω, $g(r)$, $f_1(r)$ and $f_2(r)$ have the same definition as in Eq.(14), and L, M, $\cos\phi$, $\sin\phi$ are defined as

$$
\begin{aligned}
L =& \sqrt{\left(\frac{\mathrm{sh}2r\mathrm{e}^{-2\kappa t}}{t(r)t(-r)} - \frac{C_2\cos 4\lambda t}{4C_2^2 - C_1^2} \right)^2 + \left(\frac{C_2\sin 4\lambda t}{4C_2^2 - C_1^2} \right)^2}, \\
M =& \frac{t(r) + t(-r)}{t(r)t(-r)} - \frac{C_1}{4C_2^2 - C_1^2}, \\
cos\phi =& \frac{\mathrm{e}^{-2\kappa t}\mathrm{sh}2r(4C_1^2 - C_1^2) - C_2 t(r)t(-r)\cos 4\lambda t}{Lt(r)t(-r)(4C_2^2 - C_1^2)}, \\
\sin\phi =& -\frac{C_2\sin 4\lambda t}{L(4C_2^2 - C_1^2)}. \qquad (23)
\end{aligned}
$$

In Figs.4–7 we show the time evolutions of the atom, field and total system linear entropies. In the absence of dissipation and average photon number of the reservoir, i.e. for $\kappa = 0, N = 0$, the linear entropies of the atom and the field vary with time periodically, going to zero at the disentanglement moments $t_d = k\pi/\lambda$, with k being an integer, and it is easy to find that the period is the same as that of the Bell function. And the linear entropy of the total system is zero at any time, that is to say, both the linear entropies of the atom and the field can be used to measure the entanglement between the atom and field subsystems. For $\kappa/\lambda = 0.01$ and $N = 0.1$, the behaviours of $s_A(t)$ and $s_C(t)$ are slightly more complicated, displaying local maxima and minima. The maxima occur when there is maximum dephasing between the two states; the minima occur for minimum dephasing (i.e. during disentanglement). Such oscillations have also been observed in the model with $\kappa/\lambda = 0$ and $N = 0$. However, contrary to what happens here, $s_A(t)$ and $s_C(t)$ vanish at the minima, since these instants correspond to the cases that the atom and the field are in a pure state. In the model with $\kappa/\lambda = 0$ and $N = 0$, the difference between consecutive local

maximum and minimum diminishes with time, since both $s_A(t)$ and $s_C(t)$ tend to asymptotic values.

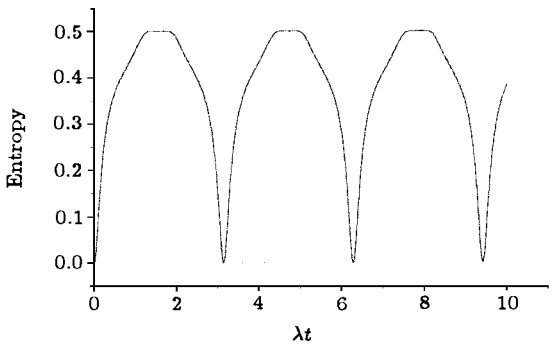


Fig.4. The evolution of the atom (solid), field (dashed) and total system (dotted) linear entropies for $D = 1, N = 0, \kappa/\lambda = 0$, and $r = 1$.

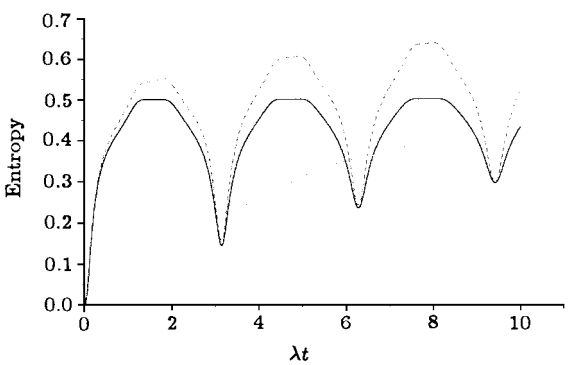


Fig.5. The same as Fig.3, but $N = 0.1$ and $\kappa/\lambda = 0.01$.

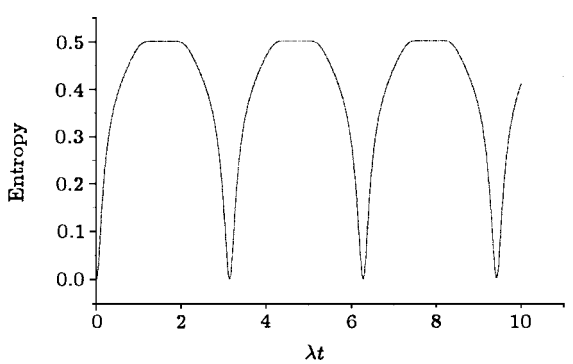


Fig.6. The evolution of the atom (solid), field (dashed) and total system (dotted) linear entropies for $D = 2, N = 0, \kappa/\lambda = 0$, and $r = 1$.

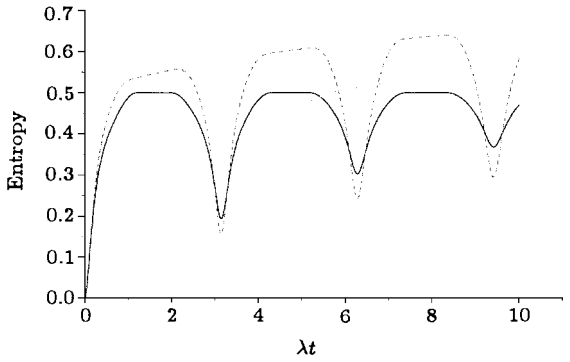


Fig.7. The same as Fig.5, but $N = 0.1$ and $\kappa/\lambda = 0.01$.

5. Conclusion

We have studied the dynamical behaviours of the quantum nonlocality in the atom–field coupling system in a cavity and the linear entropies of the atom, the field and the total system of them. It is found that if the cavity is lossless and the reservoir is in vacuum, the system in which the atom initially in the state $|e\rangle$ and the cavity in a squeezed coherent state $|\alpha, r\rangle$, quantum nonlocality appears periodically with the period $T = \pi/\lambda$, and the linear entropies of the atom and the field also oscillate periodically, with a period the same as that of quantum nonlocality. When the cavity dissipation and the average photon number of the thermal reservoir are taken into account, the quantum nonlocality will be lost and the linear entropies of the atom and the field oscillate with decreasing amplitude, tending to asymptotic values. The rapidity of the loss of the quantum nonlocality depends on the amplitude of the initial squeezed coherent state, the cavity damping constant κ and the average photon number N of the thermal reservoir. The stronger the field and the larger the constant κ and the average photon number N, the more rapidly the nonlocality decreases.

References

[1] Einstein A, Podolsky B and Rosen N 1935 *Phys. Rev.* **47** 777

[2] Bell J S 1964 *Physics, (Long Island City, N. Y.)* **1** 195

[3] Gisin N and Peres A 1992 *Phys. Lett.* A **162** 15

[4] Banaszek K and Wodkiewicz K 1998 *Phys. Rev.* A **58** 4345 Banaszek K *et al* 1999 *Phys. Rev. Lett.* **82** 2009

[5] Kim M S and Lee J 2000 *Phys. Rev.* A **61** 042102

[6] Majumdar A S and Nayak N 2001 *Phys. Rev.* A **64** 013821

[7] Jeong H, Son W, Kim M S, Ahn D and Brukner C 2003 *Phys. Rev.* A **67** 012106

[8] Wu Y and Yang X X 2001 *Phys. Rev.* A **63** 043816

[9] Li G X, Huang G M and Gao Y J 2003 *Chin. Phys. Lett.* **20** 223

[10] Peng J S and Li G X 1998 *Introduction to Modern Quantum Optics* (Singapore: World Scientific)

[11] Schoendroff L and Risken H 1990 *Phys. Rev.* A **41** 5147
Xu L and Zhang Z M 1994 *Z. Phys.* B **95** 507

[12] Li G X and Peng J S 1995 *Phys. Rev.* A **52** 465

[13] Xu X M and Luo D W 1999 *Acta Phys. Sin.* **48** 2154 (in Chinese)

[14] Bergmann K R, Theuer H and Shore B W 1998 *Rev. Mod. Phys.* **70** 1003

[15] Woedkiewicz K 2000 *New. J. Phys.* **21** 21

[16] Fang M F, Zhou Q P and Zhou P 1999 *Chin. Phys.* **8** 401

[17] Shi M J, Du J F and Zhu D P 2000 *Acta Phys. Sin.* **49** 825

[18] Fang M F, Zhou P and Swain S 2000 *J. Mod. Opt.* **47** 1043

[19] Furuya K, Nemes M C and Pellegrino G Q 1998 *Phys. Rev. Lett.* **80** 5524

[20] Gemmer J and Mahler G 2001 *Eur. Phys. J.* D **17** 385

[21] Dodonov V V, Jose W D and Mizrahi S S 2003 *J. Opt.* B **5** s567

[22] Hillery M, O'Connell R F, Scully M O and Wigner E P 1984 *Phys. Rep.* **106** 121