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Competition between aggregation and migration processes of a multi-species system*

Ke Jian-Hong(柯见洪)†, Zhuang You-Yi(庄友谊),
Lin Zhen-Quan(林振权), and Ye Peng(叶 鹏)

School of Physics and Electronic Information, Wenzhou University, Wenzhou 325027, China

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We propose a solvable multi-species aggregation–migration model, in which irreversible aggregations occur between any two aggregates of the same species and reversible migrations occur between any two different species. The kinetic behaviour of an aggregation–migration system is then studied by means of the mean-field rate equation. The results show that the kinetics of the system depends crucially on the details of reaction events such as initial concentration distributions and ratios of aggregation rates to migration rate. In general, the aggregate mass distribution of each species always obeys a conventional or a generalized scaling law, and for most cases at least one species is scaled according to a conventional form with universal constants. Moreover, there is at least one species that can survive finally.

Keywords: kinetic behaviour, aggregation, migration, scaling law
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1. Introduction

Aggregation is an important kinetic phenomenon that underlies a wide variety of nonequilibrium processes in nature, such as aerosol physics,[1] gelation,[2] and crystal growth.[3] The aggregates in these processes evolve typically according to an irreversible reaction scheme $A_i + A_j \to A_{i+j}$. That is, the aggregates A_i and A_j, which respectively consist of i and j monomers, can bond spontaneously and result in a larger aggregate A_{i+j}. In the last few decades, much work has been devoted to understanding the kinetics of these irreversible aggregation processes and considerable developments have been achieved.[4−9] Recently, some studies turned to focusing on another growth mechanism which also arises frequently in physics, especially in social science.[10−15] This is an evaporation/condensation mechanism. Ispolatov *et al*[13] and Ben-Naim *et al*[14] introduced several kinetic exchange models which may be used to describe the wealth distribution of individuals in economical activities, and Leyvraz *et al*[15] proposed a migration-driven aggregation model with a preferential migration kernel that can provide a description for the evolution of city populations. We investigated the kinetics of general reversible migration-driven aggregation models.[16−18] In these models, the aggregates evolve according to a migration scheme $A_k + A_l \to A_{k-1} + A_{l+1}$, i.e. a monomer transfers from an aggregate of mass k to that of mass l. The results exhibited that the kinetics of such a migration-driven aggregation process falls into a scaling regime different from that for conventional aggregation processes. In order to understand thoroughly the similarities and differences between the aggregation and migration processes, we also studied the kinetics of a two-species aggregation–migration process and found that the evolution behaviour of the system depended strongly on the ratio of the aggregation rate to migration rate coefficient.[19] It is thus believed that the multi-species aggregation–migration processes may give rise to richer evolution kinetics.

In this work, we investigate the competition between aggregation and migration processes of n types of species A^l ($l = 1, 2, \ldots, n$, $n \geq 3$). We assume that irreversible aggregations occur between any two aggregates of the same type, $A_i^l + A_j^l \xrightarrow{I_l(i;j)} A_{i+j}^l$, and re-

*Project supported by the National Natural Science Foundation of China (Grant Nos 10305009 and 10275048) and the Zhejiang Provincial Natural Science Foundation of China (Grant No 102067).
†Corresponding author. E-mail address: kejianhong@yahoo.com.cn

versible migrations occur simultaneously between any two aggregates of different species, $A_i^l + A_j^m \xrightarrow{K_{lm}(i;j)} A_{i-1}^l + A_{j+1}^m$ $(l, m = 1, 2, \ldots, n,\ l \neq m)$. The above migration scheme indicates that when any two aggregates of different types meet, a monomer of one aggregate will change its type and then join in another aggregate. The rate kernel of the aggregation reaction between the A_i^l and A_j^l aggregates is $I_l(i;j)$ and that of the migration between the A_i^l and A_j^m aggregates is $K_{lm}(i;j)$.

The multi-species aggregation–migration system is assumed to be of spatial homogeneity, so that the fluctuations in the densities of the reactants are ignored and the aggregates are considered to be homogeneously distributed in the space throughout the processes. So, the investigations of our model can be performed in the mean-field framework. We believe that the aggregation–migration model may mimic a wide variety of physical, biological, and social phenomena. For example, two separate population districts in one country can merge cooperatively to form a larger district, and the population migrations occur simultaneously between any two different countries. Thus, the global population distribution may be driven by mergence of population districts as well as by population migrations. It is also believed that the multi-species aggregation–migration processes are of interest on their own rights such as the rich scaling properties of their evolution behaviours. According to our work, it is found that the evolution kinetics of the system depends strongly on the ratios of the aggregation rates to the migration rate coefficient. Meanwhile, the initial concentrations of the species also play important roles in some cases.

The paper is organized as follows. In Sec.2, we propose a general n-species aggregation–migration model, in which monomer migrations occur between any two different species, and then derive the asymptotic solutions of the aggregate mass distributions in several different cases. A brief summary is given in Sec.3.

2. The model of general n-species aggregation–migration processes

In our investigation, the theoretical approach to the aggregation–migration processes is based on the mean-field rate equation, which assumes that the reaction proceeds at a rate proportional to the reactant concentrations. We propose a general n-species aggregation–migration model in which monomer migrations occur between any two different species. The concentration of A^l aggregates of k-mers is denoted as a_{lk}.

Consider a simple model in which the aggregation rate kernel $I_l(i;j)$ of A^l species are different constants I_l $(l = 1, 2, \ldots, n)$. Meanwhile, we assume that the migration rate of an aggregate A_k^l gaining one monomer is proportional only to its mass k and independent of the mass of its opposite aggregate which loses a monomer, namely, $K_{lm}(i;j) = Jj$, where J is a constant coefficient and $l \neq m$. The governing mean-field rate equation for this general aggregation–migration system then reads

$$\begin{aligned}\frac{\mathrm{d}a_{lk}}{\mathrm{d}t} =& \frac{I_l}{2}\left(\sum_{i+j=k} a_{li}a_{lj} - 2a_{lk}\sum_{j=1}^{\infty} a_{lj}\right) \\ &+ J(a_{l,k+1} - a_{lk}) \sum_{1\le m\le n, m\neq l} \sum_{j=1}^{\infty} j a_{mj} \\ &+ J[(k-1)a_{l,k-1} - k a_{lk}] \sum_{1\le m\le n, m\neq l} \sum_{j=1}^{\infty} a_{mj}, \\ & l = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

where we impose the boundary condition $a_{l0}(t) \equiv 0$. When $n = 2$, Eq.(1) will reduce to the governing rate equation for the two-species aggregation–migration system in Ref.[19].

As we aim to find out the analytical solutions of the aggregate mass distributions and investigate their long-time scaling properties, we consider a simplest but important case in which there exist only monomer aggregates of each species at $t = 0$ and the concentration of A^l monomers is equal to A_{l0}. Then this monodisperse initial condition is

$$a_{lk}(0) = A_{l0}\delta_{k1}, \quad l = 1, 2, \ldots, n. \tag{2}$$

According to the algebra technique used in Ref.[19], we believe that under the above initial condition, the governing rate equation (1) can also be solved with the help of ansatz (see, e.g., Refs.[19, 20])

$$a_{lk}(t) = A_l(t)[a_l(t)]^{k-1}. \tag{3}$$

Substituting Eq.(3) into Eq.(1), we can transform the rate equation (1) into the following differential equa-

tions:

$$\frac{\mathrm{d}a_l}{\mathrm{d}t} = \frac{I_l A_l}{2} + J \sum_{1\le m\le n, m\ne l} \frac{A_m(1-a_l)}{1-a_m},$$

$$\frac{\mathrm{d}A_l}{\mathrm{d}t} = -\frac{I_l A_l^2}{1-a_l} - JA_l \times \sum_{1\le m\le n, m\ne l} \left[\frac{A_m(1-a_l)}{(1-a_m)^2} + \frac{A_m}{1-a_m}\right]. \quad (4)$$

The corresponding initial condition of Eqs.(4) is

$$a_l(0) = 0, \quad A_l(0) = A_{l0}, \quad l = 1, 2, \ldots, n. \quad (5)$$

From Eqs.(4) we can obtain

$$\frac{\mathrm{d}\ln[(1-a_m)/(1-a_l)]}{\mathrm{d}t} = \frac{I_l - 2J}{2}\frac{A_l}{1-a_l} - \frac{I_m - 2J}{2}\frac{A_m}{1-a_m}, \quad (6)$$

$$\frac{\mathrm{d}[A_l/(1-a_l)^2]}{\mathrm{d}t} = \frac{JA_l}{(1-a_l)^2} \sum_{1\le m\le n, m\ne l} \left[A_m \frac{a_l - a_m}{(1-a_m)^2}\right]. \quad (7)$$

Equation (7) directly yields

$$\sum_{l=1}^{n}\sum_{k=1}^{\infty} k a_{lk} = \sum_{l=1}^{n} \frac{A_l}{(1-a_l)^2} \equiv \sum_{l=1}^{n} A_{l0}, \quad (8)$$

which indicates that the total mass of all species is conserved by the kinetics of the aggregation–migration processes. It is natural for the system without any annihilation reactions. Equation (8) also implies that the total mass of each species is bounded.

Since it is difficult to obtain the exact solutions of Eqs.(4), we turn to discussing their asymptotic solutions in several different cases.

2.1. The case of all aggregation rates greater than $2J$

In this case, from Eqs.(6) and (7) we find that the solution to the governing rate equation is dependent on the values of $A_{l0}(I_l - 2J)$. Without any loss of generality, one can assume that $A_{10}(I_1 - 2J) \ge A_{20}(I_2 - 2J) \ge \cdots \ge A_{n0}(I_n - 2J)$.

When $A_{10}(I_1 - 2J) > A_{l'0}(I_{l'} - 2J)$ $(l' = 2, 3, \ldots, n)$, it can be concluded from Eq.(6) that $(1-a_{l'})/(1-a_l) \to \infty$ for all $l' > l$ $(l = 1, 2, \ldots, n)$ in the $t \to \infty$ limit. We then examine the final total mass of each species. Set $A_l/(1-a_l)^2 = M_l(\infty)$ at $t \to \infty$. If at least one of $M_{l'}(\infty)$ is greater than zero, we then assume that among the species whose final total mass is non-zero, A^q $(q \ge 2)$ is the one with the smallest value of $A_{q0}(I_q - 2J)$. Thus we know that in the $t \to \infty$ limit, $M_l(\infty) \to 0$ for $n \ge l > q$ and $(1-a_l)/(1-a_q) \to 0$ for $1 \le l < q$. From Eqs.(4) we deduce

$$\frac{\mathrm{d}(1-a_q)^{-1}}{\mathrm{d}t} = \frac{I_q A_q}{2(1-a_q)^2} + J \sum_{1\le m\le n, m\ne q} \left[\frac{A_m}{(1-a_m)^2}\frac{1-a_m}{1-a_q}\right]. \quad (9)$$

In the long-time limit, Eq.(9) reduces to

$$\frac{\mathrm{d}(1-a_q)^{-1}}{\mathrm{d}t} \simeq \frac{I_q A_q}{2(1-a_q)^2} \to \frac{I_q M_q(\infty)}{2}, \quad (10)$$

which indicates that $(1-a_q) \sim t^{-1}$ at $t \gg 1$. So we have $\int_0^\infty \mathrm{d}t[a_1(t) - a_q(t)] = \int_0^\infty \mathrm{d}t\{[1-a_q(t)] - [1-a_1(t)]\} \sim \ln t \to \infty$ at $t \to \infty$. Moreover, $a_1 > a_{l'}$ for all $l' \ge 2$ at long times. From Eq.(7) we can then deduce that $A_1/(1-a_1)^2 \to \infty$ at $t \to \infty$, which is in discrepancy with the above result that the total mass of each species is bounded. One can conclude that $M_{l'}(\infty) = 0$ for all $l' \ge 2$. Hence, $A_1/(1-a_1)^2 \to \sum_{l=1}^{n} A_{l0}$ and $A_{l'}/(1-a_{l'})^2 \to 0$ at $t \to \infty$. From Eqs. (4) one can then derive the asymptotic solutions of $a_l(t)$ and $A_l(t)$ in the long-time limit,

$$\begin{aligned} a_1(t) &\simeq 1 - C_{1,1}t^{-1}, \\ A_1(t) &\simeq \frac{2C_{1,1}}{I_1}t^{-2}, \\ a_{l'}(t) &\simeq 1 - C_{1,l'}t^{-2J/I_1}, \\ A_{l'}(t) &\simeq C_{2,l'}\exp\left[-\frac{2JC_{1,l'}}{(I_1-2J)C_{1,1}}t^{1-2J/I_1}\right], \end{aligned} \quad (11)$$

where $C_{1,1} = (I_1\sum_{l=1}^{n} A_{l0}/2)^{-1}$, $C_{1,l'}$ and $C_{2,l'}$ are integration constants. We then obtain the asymptotic solution of $a_{lk}(t)$ at long times,

$$\begin{aligned} a_{1k}(t) &\simeq \frac{2C_{1,1}}{I_1}t^{-2}(1 - C_{1,1}t^{-1})^{k-1}, \\ a_{l'k}(t) &\simeq C_{2,l'}\exp\left[-\frac{2JC_{1,l'}}{(I_1-2J)C_{1,1}}t^{1-2J/I_1}\right] \times (1 - C_{1,l'}t^{-2J/I_1})^{k-1}. \end{aligned} \quad (12)$$

The results show that the aggregate mass distributions approach the scaling forms

$$\begin{aligned} a_{1k}(t) &\sim t^{-2}\exp(-x), \quad x \propto kt^{-1}, \\ a_{l'k}(t) &\sim \exp\left[-\frac{2JC_{1,l'}}{(I_1-2J)C_{1,1}}t^{1-2J/I_1}\right]\exp(-y), \\ & \quad y \propto kt^{-2J/I_1}. \end{aligned} \quad (13)$$

Equations (13) indicate that the evolution of the aggregate mass distribution of the A^1 species obeys the conventional scaling law (see, e.g., Ref.[20])

$$c_k(t) \sim t^{-w}\Phi[k/S(t)], \quad S(t) \propto t^z, \tag{14}$$

where $c_k(t)$ is the concentration of the aggregates of mass k, and $S(t)$ is the characteristic mass which plays a role analogous to that of the correlation length in ordinary critical phenomena. For the A^1 species, the scaling function is exponential, $\Phi(x) = \exp(-x)$, and the governing exponents are $w = 2$ and $z = 1$. The results indicate that the A^1 species evolves as in the irreversible single-species aggregation system. On the other hand, the $A^{l'}$ species are not scaled according to the conventional definition (14), but the aggregate mass distribution of the $A^{l'}$ species takes the generalized scaling form (see, e.g., Refs.[19, 20])

$$a_k(t) \sim F(t)\Phi[k/S(t)], \quad S(t) \propto t^z, \tag{15}$$

where $F(t)$ is an unusual time-dependent decreasing function. In this case, the typical mass of each $A^{l'}$ species is the same, $S(t) \sim t^{2J/I_1}$, which is forced mainly by migrations between the $A^{l'}$ and A^1 species as well as self-coagulations of the A^1 aggregates. Then we discuss the evolution behaviours of the total density and the total mass. By summing Eqs.(12) we find that in the long-time limit,

$$\begin{aligned} N_1(t) &= \sum_{k=1}^{\infty} a_{1k}(t) \sim t^{-1}, \\ N_{l'}(t) &= \sum_{k=1}^{\infty} a_{l'k}(t) \sim \exp\left[-\frac{2JC_{1,l'}t^{1-2J/I_1}}{(I_1-2J)C_{1,1}}\right] t^{2J/I_1}. \end{aligned} \tag{16}$$

These indicate that the total density of the A^1 species decays as t^{-1} while that of the $A^{l'}$ species decreases rapidly as $\exp[-2JC_{1,l'}t^{1-2J/I_1}/(I_1-2J)C_{1,1}]$. Moreover, when $t \to \infty$,

$$\begin{aligned} M_1(t) &= \sum_{k=1}^{\infty} k a_{1k}(t) \to \sum_{l=1}^{n} A_{l0}, \\ M_{l'}(t) &= \sum_{k=1}^{\infty} k a_{l'k}(t) \to \\ &\quad \exp\left[-\frac{2JC_{1,l'}t^{1-2J/I_1}}{(I_1-2J)C_{1,1}}\right] t^{4J/I_1}. \end{aligned} \tag{17}$$

These indicate that the large A^1 aggregates dominate over the corresponding $A^{l'}$ aggregates in the long-time limit and all masses of the $A^{l'}$ species will transfer to the A^1 species through the monomer migration processes at the end. Thus, we can conclude that for this case only the A^1 species can survive finally. Moreover, this consequence is independent of the values of initial concentrations.

Now let us turn to the general case in which there are n_1 kinds of species ($1 < n_1 < n$) with the same largest values of $A_{m0}(I_m - 2J)$ ($1 \le m \le n_1$). Assume that $A_{m0}(I_m - 2J) = A_{10}(I_1 - 2J)$ for $1 \le m \le n_1$ and $A_{m'0}(I_{m'} - 2J) < A_{10}(I_1 - 2J)$ for $n_1 < m' \le n$. By analysing the evolution properties of Eqs.(4) and (6) we find that $a_m(t) = a_1(t)$ and $(I_m - 2J)A_m(t) = (I_1 - 2J)A_1(t)$ for $1 \le m \le n_1$. Moreover, by employing a method similar to that used in the above case one can deduce that $A_{m'}(t)/[1 - a_{m'}(t)]^2 \to 0$ ($n_1 < m' \le n$) at $t \to \infty$. The asymptotic solutions of $a_l(t)$ and $A_l(t)$ are then determined as follows:

$$\begin{aligned} &a_m(t) \simeq 1 - B_1 t^{-1}, \\ &A_m(t) \simeq C_{3,m} t^{-2}, \quad m = 1, 2, \ldots, n_1, \\ &a_{m'}(t) \simeq 1 - C_{3,m'} t^{-\gamma}, \\ &A_{m'}(t) \simeq C_{4,m'} \exp\left[-\left(\frac{JC_{3,m'}}{1-\gamma}\sum_{l=1}^{\infty} A_{l0}\right) t^{1-\gamma}\right], \\ &m' = n_1 + 1, n_1 + 2, \ldots, n, \end{aligned} \tag{18}$$

where

$$B_1 = \Big\{\sum\nolimits_{l=1}^{n} A_{l0}[J + 1/\sum\nolimits_{m=1}^{n_1}(I_m/2 - J)^{-1}]\Big\}^{-1},$$

$$C_{3,m} = B_1^2 \sum\nolimits_{l=1}^{n} A_{l0}/\sum\nolimits_{i=1}^{n_1}[(I_m - 2J)(I_i - 2J)^{-1}],$$

$\gamma = J[J + 1/\sum_{m=1}^{n_1}(I_m/2 - J)^{-1}]^{-1}$, $C_{3,m'}$ and $C_{4,m'}$ are integration constants. It is obvious that Eqs.(18) are similar to Eqs.(11). One can thus conclude that all A^m species are scaled according to the conventional definition of Eq.(14) with the exponents $w = 2$ and $z = 1$; while each of the $A^{m'}$ species approaches the scaling equation (15) with the typical mass $S(t) \propto t^{\gamma}$. In this general case, the large A^m aggregates dominate over the corresponding $A^{m'}$ aggregates at long times. The consequence of the migration processes is that in the $t \to \infty$ limit, all A^m species ($1 \le m \le n_1$) with the same largest value of $A_{m0}(I_m - 2J)$ share up the masses of the $A^{m'}$ species ($n_1 < m' \le n$) according to a certain redistribution scheme, $(I_m - 2J)M_m(\infty) = (I_1 - 2J)M_1(\infty)$ and $M_1(\infty) = \sum_{l=1}^{n} A_{l0}/\sum_{i=1}^{n_1}[(I_1 - 2J)(I_i - 2J)^{-1}]$. Moreover, for the $A^{m'}$ species, its total mass decays as a stretched exponential in time. Hence, only the A^m species ($1 \le m \le n_1$) can survive at the end.

Finally, we discuss the species case in which all A^l species have the same value of $A_{l0}(I_l - 2J)$. From Eqs.(4) we derive the exact solutions

$$a_l(t) = 1 - (B_2 t + 1)^{-1},$$
$$A_l(t) = A_{l0}(B_2 t + 1)^{-2}, \quad l = 1, 2, \ldots, n, \quad (19)$$

where $B_2 = (I_1 - 2J)A_{10}/2 + J\sum_{l=1}^{n} A_{l0}$. By substituting Eqs.(19) into Eq.(3) one can readily obtain the exact solutions of the aggregate mass distributions for all species. In the long-time limit, the aggregate mass distribution of each species approaches the same scaling form

$$c_k(t) \sim t^{-2}\exp(-x), \quad x \propto kt^{-1}, \quad (20)$$

where $c_k(t)$ denotes the concentration of the aggregates of any type that consists of k monomers at time t. It is obvious that Eq.(20) is a reduced form of Eq.(14) with the universal exponents $w = 2$ and $z = 1$. Moreover, each species obeys the mass conservation law.

2.2. The case of all aggregation rates equal to $2J$

In this case, Eqs.(6) and (7) reduce to

$$a_l(t) = a_1(t),$$
$$A_l(t) = A_{l0}[1 - a_1(t)]^2, \quad l = 1, 2, \ldots, n. \quad (21)$$

Substituting Eqs.(21) into Eqs.(4), one can also obtain the exact solutions (19). Thus we can conclude that for this case each species is scaled according to Eq.(20). Moreover, each species also obeys the mass conservation law. This case is equivalent to an irreversible multi-species non-interacting aggregation process.

2.3. The case of all aggregation rates less than $2J$

In this case, one can readily deduce, by analysing the evolution properties of Eqs.(6) and (7), that $A_l/(1 - a_l)^2 \to M_{l\infty}$ ($M_{l\infty}$ are positive constants, $l = 1, 2, \ldots, n$) at $t \to \infty$. Moreover, we also find that $a_l(t) \simeq a_1(t)$ and $(2J - I_l)A_l \simeq (2J - I_1)A_1$ at $t \gg 1$. In the long-time limit, from Eqs.(4) we then obtain the following asymptotic solutions:

$$a_l(t) \simeq 1 - B_3 t^{-1}, \quad A_l(t) \simeq C_{5,l} t^{-2}, \quad (22)$$

where

$$C_{5,l} = B_3^2 \sum_{m=1}^{n} A_{m0} / \sum_{m=1}^{n} [(2J - I_l)(2J - I_m)^{-1}]$$

and

$$B_3 = \Big\{ \sum_{m=1}^{n} A_{m0}[J - 1/\sum_{m=1}^{n} (J - I_m/2)^{-1}] \Big\}^{-1}.$$

Especially, when all A^l species have the same value of $A_{l0}(2J - I_l)$, Eqs.(22) will reduce to Eqs.(19). From Eqs.(22) one can conclude that for this case the evolution of the aggregate mass distribution of each species also obeys the conventional scaling equation (20). Moreover, the total masses of all species will redistribute among the species according to the scheme $(2J - I_l)M_l(\infty) = (2J - I_m)M_m(\infty)$ $(l, m = 1, 2, \ldots, n)$ and each species can survive finally. It is not difficult to find the final total mass to be $M_l(\infty) = \sum_{m=1}^{n} A_{m0} / \sum_{m=1}^{n} [(I_l - 2J)(I_m - 2J)^{-1}]$, $l = 1, 2, \ldots, n$.

2.4. The case of some aggregation rates greater than $2J$ while others equal to or less than $2J$

Now we investigate the case in which some aggregation rates are greater than $2J$ while the others are equal to or less than $2J$. Without any loss of generality, we assume that $I_i > 2J$ for $i = 1, 2, \ldots, n_2$ $(1 \le n_2 < n)$ and $I_j \le 2J$ for $j = n_2 + 1, n_2 + 2, \ldots, n$. This case is similar to that in Case 2.1. The characteristics of the aggregate mass distributions are concerned with the relative values of $A_{l0}(I_l - 2J)$. For simplicity, we assume that $A_{10}(I_1 - 2J) \ge A_{l'0}(I_{l'} - 2J)$ for $l' = 2, 3, \ldots, n$.

When $n_2 = 1$ or $A_{10}(I_1 - 2J) > A_{l'0}(I_{l'} - 2J)$ for all $l' = 2, 3, \ldots, n$, this case is equivalent to the first subcase of $A_{10}(I_1 - 2J) > A_{l'0}(I_{l'} - 2J)$ in Case 2.1. Only the aggregate mass distribution of the A^1 species obeys the conventional scaling equation (20) while the other species are all scaled according to the generalized scaling equation (15). Additionally, only the A^1 aggregates can be conserved by the dynamics of the aggregation–migration system.

In the general case of $n_2 > 1$ and $A_{10}(I_1 - 2J) \ge A_{l'0}(I_{l'} - 2J)$, we assume that $A_{m0}(I_m - 2J) = A_{10}(I_1 - 2J)$ for $m = 2, 3, \ldots, n_3$ $(1 < n_3 \le n_2)$ and $A_{m'0}(I_{m'} - 2J) < A_{10}(I_1 - 2J)$ for $m' = n_3 + 1, n_3 + 2, \ldots, n$. In this case, it follows from Eqs.(6) and (7) that in the $t \gg 1$ limit, $a_1(t) = a_m(t)$ and $(I_m - 2J)A_m(t) = (I_1 - 2J)A_1(t)$ for $1 < m \le n_3$. Moreover, we have $A_{m'}(t)/[1 - a_{m'}(t)]^2 \to 0$ $(n_3 < m' \le n)$ at $t \to \infty$. The asymptotic solutions of $a_l(t)$

and $A_l(t)$ are then obtained,

$$\begin{aligned} &a_m(t) \simeq 1 - B_4 t^{-1}, \\ &A_m(t) \simeq C_{6,m} t^{-2}, \quad m = 1, 2, \ldots, n_3, \\ &a_{m'}(t) \simeq 1 - C_{6,m'} t^{-\beta}, \\ &A_{m'}(t) \simeq C_{7,m'} \exp\left[-\frac{J C_{6,m'}}{1-\beta} \sum_{l=1}^{\infty} A_{l0} t^{1-\beta} \right], \\ &\quad m' = n_3 + 1, n_3 + 2, \ldots, n, \end{aligned} \tag{23}$$

where

$$B_4 = \left\{ \sum\nolimits_{l=1}^{n} A_{l0} \left[J + 1 / \sum\nolimits_{m=1}^{n_3} (I_m/2 - J)^{-1} \right] \right\}^{-1},$$

$$C_{6,m} = B_4^2 \sum\nolimits_{l=1}^{n} A_{l0} / \sum\nolimits_{i=1}^{n_3} [(I_m - 2J)(I_i - 2J)^{-1}],$$

$$\beta = J \left[J + 1 / \sum\nolimits_{m=1}^{n_3} (I_m/2 - J)^{-1} \right]^{-1},$$

$C_{6,m'}$ and $C_{7,m'}$ are integration constants. Since Eqs.(23) are similar to Eqs.(18), it can be concluded that all A^m species ($m = 1, 2, \ldots, n_3$) are scaled according to Eq.(20), while each of the $A^{m'}$ species ($m' = n_3 + 1, n_3 + 2, \ldots, n$) approaches Eq.(15) with $S(t) \propto t^{\beta}$. Moreover, all A^m species ($1 \le m \le n_3$) share up the masses of the $A^{m'}$ species ($n_1 < m' \le n$) and their final total masses are

$$M_m(\infty) = \sum\nolimits_{l=1}^{n} A_{l0} / \sum\nolimits_{i=1}^{n_3} [(I_m - 2J)(I_i - 2J)^{-1}].$$

In this general case, only the A^m species with the largest values of $A_{m0}(I_m - 2J)$ ($1 \le m \le n_3$) can survive at the end.

2.5. The case of some aggregation rates equal to $2J$ while the others less than $2J$

For simplicity, we assume that $I_m = 2J$ for $m = 1, 2, \ldots, n_4$ ($1 \le n_4 < n$) and $I_{m'} < 2J$ for $m' = n_4 + 1, n_4 + 2, \ldots, n$. When $n_4 = 1$, from Eqs.(6) and (7) we know $A_1/(1 - a_1)^2 \to \sum_{l=1}^{n} A_{l0}$ and $A_{l'}/(1 - a_{l'})^2 \to 0$ ($l' = 2, 3, \ldots, n$) at $t \to \infty$. In the long-time limit, from Eqs.(4) one can then derive the asymptotic solutions of $a_l(t)$ and $A_l(t)$,

$$\begin{aligned} &a_1(t) \simeq 1 - C_{8,1} t^{-1}, \\ &A_1(t) \simeq J^{-1} C_{8,1} t^{-2}, \\ &a_{l'}(t) \simeq 1 - C_{8,l'} t^{-1}, \\ &A_{l'}(t) \simeq C_{10,l'} t^{-C_{9,l'}}, \quad l' = 2, 3, \ldots, n, \end{aligned} \tag{24}$$

where $C_{8,1} = (J \sum_{l=1}^{n} A_{l0})^{-1}$, $C_{9,l'} = C_{8,l'}/C_{8,1} + 1$, $C_{8,l'}$ and $C_{10,l'}$ are integration constants. Substituting Eqs. (24) into Eq.(6), one can find that $C_{8,l'} > C_{8,1}$ for $l' \ge 2$. Asymptotically, the aggregate mass distributions approach the scaling forms

$$\begin{aligned} &a_{1k}(t) \sim t^{-2} \exp(-x), \\ &a_{l'k}(t) \sim t^{-C_{9,l'}} \exp(-x), \quad x \propto k t^{-1}. \end{aligned} \tag{25}$$

These indicate that the aggregate mass distribution of each species satisfies the conventional scaling equation (14). The governing exponents for the A^1 species are universal, $w = 2$ and $z = 1$; however, the $A^{l'}$ species has the nonuniversal governing exponent $w = J C_{8,l'} \sum_{l=1}^{n} A_{l0} + 1$, which is dependent on the details of the reaction events. A comparison between the total mass of the A^1 species and that of the $A^{l'}$ species, $M_1(t) \simeq \sum_{l=1}^{n} A_{l0}$ and $M_{l'}(t) \simeq t^{-C_{9,l'}+1}$ at $t \gg 1$, shows that only the A^1 species can be conserved at the end. It is not surprising because the aggregation rate of the A^1 species is greater than that of any other species.

We then turn to the general case of $n_4 > 1$. From Eqs.(6) and (7) we deduce that $a_m(t) = a_1(t)$ and $A_m(t) = A_1(t) A_{l0}/A_{10}$ for $1 < m \le n_4$ and $A_{m'}(t)/[1 - a_{m'}(t)]^2 \to 0$ for $n_4 < m' \le n$ at $t \to \infty$. We then obtain the asymptotic solutions of Eqs.(4),

$$\begin{aligned} &a_m(t) \simeq 1 - C_{8,1} t^{-1}, \\ &A_m(t) \simeq C_{11,m} t^{-2}, \quad m = 1, 2, \ldots, n_4, \\ &a_{m'}(t) \simeq 1 - C_{11,m'} t^{-1}, \\ &A_{m'}(t) \simeq C_{13,m'} t^{-C_{12,m'}}, \\ &m' = n_4 + 1, n_4 + 2, \ldots, n, \end{aligned} \tag{26}$$

where $C_{11,m} = A_{m0} C_{8,1} (J \sum_{i=1}^{n_4} A_{i0})^{-1}$, $C_{12,m'} = C_{11,m'}/C_{8,1} + 1$, $C_{11,m'}$ and $C_{13,m'}$ are integration constants. Substituting Eqs.(26) into Eq.(6), we find $C_{11,m'} > C_{8,1}$ for $m' \ge n_4 + 1$. Equations (26) are similar to Eqs.(24). Hence, the A^m species approaches the scaling equation (14) with the exponents $w = 2$ and $z = 1$; while the other species $A^{m'}$ ($n_4 < m' \le n$) takes the scaling form of Eq.(14) with the exponents $w = J C_{11,m'} \sum_{l=1}^{n} A_{l0} + 1$ and $z = 1$. The results also imply that in the $t \to \infty$ limit, there remain only the aggregates of the A^m species ($m = 1, 2, \ldots, n_4$).

3. Summary

We have studied an n-species ($n > 3$) aggregation–migration model with the aggregate rate kernel $I_l(i; j) = I_l$ and the asymmetrical migration rate kernel $K_{lm}(i; j) = Jj$ ($l, m = 1, 2, \ldots, n$). In this model, irreversible aggregations occur between any two aggregates of the same species and monomer

migrations occur simultaneously between any two distinct species. We have presented the solutions to the governing rate equation and then obtained distinct evolution behaviours of the system in several different cases, which are illustrated in Table 1 (where it is noted that $l, m, m' \in (1, n)$ and $m \neq m'$). The results indicate that the kinetics of the model depends crucially on the ratios of the aggregation rates I_l to the migration rate coefficient J as well as the initial concentrations. The critical ratio for each species, which divides two distinct kinetic evolution regimes, is the same, $I_l/J = 2$. Moreover, all species can be conserved by the dynamics of the system in some cases; while in other cases, at least one species may survive finally.

Table 1. Summary of the results in Sec.2.

Case	Summary of the results
$I_l > 2J$	(i) The A^l species with the largest value of $A_{l0}(I_l - 2J)$ obeys the conventional scaling equation (14) with constant exponents while the others satisfy the generalized scaling equation (15).
	(ii) Only the species with the largest value of $A_{l0}(I_l - 2J)$ can remain finally.
$I_l = 2J$	(i) Each species is scaled according to Eq.(14) with constant exponents.
	(ii) All species obey the mass conservation law.
$I_l < 2J$	(i) Each species obeys the conventional scaling equation (14) with constant exponents.
	(ii) The total masses of all species will redistribute among themselves and each species can survive finally.
$I_m > 2J \geq I_{m'}$	(i) This case is similar to the above $I_l > 2J$ case.
	(ii) The species with the largest value of $A_{l0}(I_l - 2J)$ obeys the conventional scaling equation (14) and can be conserved at the end.
$I_m = 2J > I_{m'}$	(i) The species with the aggregation rate equal to $2J$ obeys the conventional scaling equation (14) with constant exponents while the other species satisfy Eq. (14) with nonuniversal exponents.
	(ii) Only the species with aggregation rate equal to $2J$ can survive finally.

Indeed, the competition between the aggregation and migration processes of the multi-species system gives rise to surprisingly rich kinetic behaviours.

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