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Laser pulse modulation instabilities in partially stripped plasma*

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The laser pulse modulation instabilities in partially stripped plasma were discussed based on the phase and group velocities of the laser pulse and the two processes that modulation instabilities excited. The excitation condition and growth rate of the modulation instability were obtained. It was found that the positive chirp and competition between normal and abnormal dispersions play important roles in the modulation instability. In the partially stripped plasma, the increased positive chirp enhances the modulation instability, and the dispersion competition reduces it.

Keywords: partially stripped plasma, modulation instability, photon acceleration
PACC: 5235P, 5240D, 5260

1. Introduction

The study of intense laser pulse interacting with partially stripped plasma (PSP) is getting more and more important in view of the fact that in many experiments of laser target (especially high Z target) interaction, the atoms cannot be fully stripped. For example, in the gold target plasma, the ionization degree is 51 and there are 28 electrons remaining in every atom.[1] In the laser electric field, the partially stripped atoms are polarized, which induce the polarized current; therefore, many characteristics of the laser pulse propagating in PSP experienced obvious change.[2−8]

The modulation instability is one of the most important phenomena in the process of intense laser pulse interacting with plasma; it can distort the shape of the laser pulse and affect the propagation of laser energy in the plasma. The transfer of laser energy to fusion core is a very important problem in inertial confinement fusion (ICF). In PSP, the atomic modulation instability (AMI) resulting from bound electrons can induce great change in phase modulation instability. Sprangle *et al* have reported that the AMI could dominate the conventional relativistic modulation instability (RMI) and forward Raman scattering (FRS) instability.[2,3] The mechanism of modulation instability can be divided into two steps:[9] first, the local refraction index gradient induces a change of local laser phase velocity, which modulates the local laser frequency, i.e. photon acceleration; second, the local group velocity varies according to the modulated frequency, causing energy to bunch longitudinally, also referred to as group velocity dispersion (GVD).[2,3,10] Since it is a two-step process, the modulation instability in fully stripped plasma (FSP) has a lower growth rate than FRS. At present, there are also some problems about modulation instability that are not clear such as the excitation condition of the instability, the relationship among the maximum growth rate, the laser frequency and radiation intensity.

In this paper, we assume that within a local volume the photon number is conserved. The laser pulse modulation instabilities in PSP are discussed based on the phase and group velocities of laser pulse and the two processes that the modulation instabilities are excited. The excitation condition and the growth rate of the modulation instability will be obtained. We will show that the positive chirp and competition between normal and abnormal dispersions play important roles in the modulation instability. In PSP, the increased positive chirp enhances the modulation instability and the dispersion competition reduces it.

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2. Phase and group velocities

When a laser pulse propagates in PSP, the local phase velocity v_ϕ and the local group velocity v_g are varied, which can induce many nonlinear phenomena such as longitudinal bunching, transverse focusing, and photon acceleration. In PSP, free electrons and bound electrons exist simultaneously, so the phase and group velocities of laser pulse are different in FSP or in other nonionized medium. The phase and group velocities can be derived from the propagation equation. In our discussion, we will only consider the weak relativistic effect and neglect the harmonic.

In the PSP, the propagation equation can be written as

$$\left(c^2\nabla^2 - \eta_0^2\frac{\partial^2}{\partial t^2}\right)\hat{a} = \frac{\omega_p^2}{\gamma}\hat{a} + \alpha\frac{\partial^2\hat{a}}{\partial t^2}, \tag{1}$$

where c is the light velocity in vacuum, $\eta_0^2 = 1 + 4\pi\chi^{(1)}$, $\alpha = 4\pi\chi^{(3)}\left(\frac{m_e c\omega}{e}\right)^2\langle a^2\rangle$, $\chi^{(1)}$ and $\chi^{(3)}$ are the first and third susceptibility associated with the bound electrons, and the order of magnitude are $\chi^{(1)} = 4\times10^{-24}n_a[\mathrm{cm}^{-3}]$ and $\chi^{(3)}[\mathrm{esu}] = 10^{-38}n_a[\mathrm{cm}^{-3}]$,[2,3,7] n_a is the density of partially stripped atoms, ω is the laser frequency, $\hat{a}$ is the normalized vector potential, $\gamma = \left(1 - \frac{v^2}{c^2}\right)^{-1/2}$, $\boldsymbol{v}$ is the velocity of a free electron, $\omega_p = \left(\frac{4\pi e^2 n_e}{m_e}\right)^{1/2}$ is the plasma frequency, n_e is the plasma density, which is a function of time and space. Assume that $\hat{a} = \frac{1}{2}\boldsymbol{a}e^{i(kc-\omega t)} + \text{c.c.}$, where $\boldsymbol{a}$ is supposed to be a slowly varying function both in time on the laser radiation period $(2\pi/\omega)$ and in space on its wavelength $(2\pi/k)$, and k is the wave number. From Eq.(1), the dispersion relation for linearly polarized pump wave can be derived as

$$\eta_0^2\omega^2 - k^2c^2 = \frac{\omega_p^2}{\gamma} - \alpha\omega^2. \tag{2}$$

So the phase velocity of the laser pulse is

$$v_\phi = \frac{\omega}{k} = \frac{c}{\sqrt{\eta_0^2 + \alpha - \omega_p^2/(\gamma\omega^2)}}. \tag{3}$$

Decker and Mori[11,12] have proved that under the relativistic condition there is some ambiguity in Rayleigh's group velocity definition $v_g = \frac{\partial\omega}{\partial k}$, and redefined the group velocity as the local energy transport velocity. According to this definition, the group velocity of the laser in PSP is given by

$$v_g = \frac{\left\langle -\frac{c^2}{4\pi v_\phi}\left(\frac{m_e c\omega}{e}\right)^2 a^2\hat{k} + n_e m_e c^2(\gamma-1)\boldsymbol{v}\right\rangle}{\left\langle\left(\varepsilon + \frac{c^2}{v_\phi^2}\right)\left(\frac{m_e c\omega}{e}\right)^2\frac{a^2}{8\pi} + n_e m_e c^2(\gamma-1)\right\rangle}, \tag{4}$$

where $\langle\ \rangle$ represents averaging over the fast oscillations, $\hat{k}$ is the unit vector of wave number, $\varepsilon = \eta_0^2 + \alpha$, and the longitudinal electric field is neglected. Using $\gamma \approx \gamma_\perp$ yields

$$v_g \approx \frac{c}{\eta_0}\left(1 - \frac{\alpha}{2\eta_0^2} - \frac{\omega_p^2}{\eta_0^2(\gamma_\perp+1)\omega^2}\right) \tag{5}$$

using

$$\gamma_\perp = (1 + \langle a^2\rangle)^{1/2}$$

and

$$R(\chi^{(3)},\omega) = \frac{P_p}{P_a} = 8\pi\chi^{(3)}\left(\frac{m_e c}{e}\right)^2\frac{\omega^4}{\omega_p^2},$$

where $R(\chi^{(3)},\omega)$ is a function of the laser and plasma frequencies and related to the third susceptibility; P_p and P_a are the critical powers for the relativistic focusing in a plasma and the nonlinear focusing in a gas respectively, thus the phase and group velocities can be derived as

$$v_\phi = \frac{c}{\eta_0}\left[1 + \frac{\omega_p^2}{2\eta_0^2\omega^2}\left(\frac{1}{\gamma} - \frac{R(\chi^{(3)},\omega)}{2}\langle a^2\rangle\right)\right],$$

$$v_g = \frac{c}{\eta_0}\left[1 - \frac{\omega_p^2}{2\eta_0^2\omega^2}\left(\frac{2}{(\gamma_\perp+1)} + \frac{R(\chi^{(3)},\omega)}{2}\langle a^2\rangle\right)\right].$$

Let $\omega = \omega_0 + \delta\omega$, $n_e = n_0 + \delta n_e$, $\langle a^2\rangle = \langle a^2\rangle + \delta\langle a^2\rangle$. Linearly expanding the phase and group velocities, and neglecting the products of the perturbed quantities, we obtain

$$v_\phi \approx \frac{c}{\eta_0}\left\{1 + \frac{\omega_{p0}^2}{2\eta_0^2\omega_0^2}\left[\left(1 + \frac{\delta n_e}{n_0} - \frac{\langle a^2\rangle}{2} - 2\frac{\delta\omega}{\omega_0}\right) - \frac{1}{2}R_0\langle a^2\rangle\left(1 + 2\frac{\delta\omega}{\omega_0} + \frac{\delta\langle a^2\rangle}{\langle a^2\rangle}\right)\right]\right\}, \tag{6}$$

$$v_g \approx \frac{c}{\eta_0}\left\{1 - \frac{\omega_{p0}^2}{2\eta_0^2\omega_0^2}\left[\left(1 + \frac{\delta n_e}{n_0} - \frac{\langle a^2\rangle}{4} - 2\frac{\delta\omega}{\omega_0}\right) + \frac{1}{2}R_0\langle a^2\rangle\left(1 + 2\frac{\delta\omega}{\omega_0} + \frac{\delta\langle a^2\rangle}{\langle a^2\rangle}\right)\right]\right\}, \tag{7}$$

where

$$\omega_{\rm p0}=\left(\frac{4\pi e^2 n_0}{m_{\rm e}}\right)^{1/2}$$

and

$$R_0=8\pi\chi^{(3)}\left(\frac{m_{\rm e}c}{e}\right)^2\frac{\omega_0^4}{\omega_{\rm p0}^2}.$$

In deriving Eqs.(6) and (7), $\chi^{(1)}$ and $\chi^{(3)}$ are assumed to be constants and independent of laser frequency.

3. Modulation instability

The present theory is based on the assumption that the plasma is produced but no further ionization and combination occur. So within a local volume we can assume that the photon number, i.e. the classical action, is conserved. The conservation of action can be stated as[9]

$$\left\langle a^2\right\rangle \omega r^2 L=\text{const.}, \tag{8}$$

where L is some initial longitudinal extent and r is the spot size. So the laser's vector potential can be modulated in three ways: modulate L, then induce the longitudinal bunching; modulate r, then induce the transverse focusing; modulate ω, then induce the photon acceleration. Here, we focus our attention on longitudinal bunching and photon acceleration, which are closely related with the modulation instability. Use the speed of light frame variables $\xi=t-x/c$ and $\tau=t$, the longitudinal bunching and photon acceleration can be stated as follows:[9]

$$\frac{1}{L}\frac{\partial L}{\partial\tau}=-\frac{1}{c}\frac{\partial v_{\rm g}}{\partial\xi}, \tag{9}$$

$$\frac{1}{\omega}\frac{\partial\omega}{\partial\tau}=\frac{1}{c}\frac{\partial v_\phi}{\partial\xi}=-\frac{1}{c}\eta^{-2}\frac{\partial\eta}{\partial\xi}, \tag{10}$$

where η is the local refraction index.

Assume the change of $\left\langle a^2\right\rangle$ is only caused by the change in L due to different local group velocity, then the differential of Eq.(8) gives

$$\Delta\left\langle a^2\right\rangle=-\frac{\Delta L}{L}\left\langle a^2\right\rangle. \tag{11}$$

Therefore, $\Delta\left\langle a^2\right\rangle$evolves in time τ as

$$\frac{\partial}{\partial\tau}\Delta\left\langle a^2\right\rangle=-\frac{1}{L}\frac{\partial L}{\partial\tau}\left\langle a^2\right\rangle. \tag{12}$$

Substituting Eq.(9) into Eq.(12) yields

$$\frac{\partial}{\partial\tau}\Delta\left\langle a^2\right\rangle=\frac{1}{c}\frac{\partial v_{\rm g}}{\partial\xi}\left\langle a^2\right\rangle. \tag{13}$$

The change in $v_{\rm g}$ is mainly due to the change in ω, so substituting Eq.(7) into Eq.(13) results in

$$\frac{\partial}{\partial\tau}\Delta\left\langle a^2\right\rangle=\frac{\omega_{\rm p0}^2}{\eta_0^3\omega_0^2}\left[1-\frac{R_0}{4}a_0^2\right]\frac{a_0^2}{2}\frac{1}{\omega_0}\frac{\partial\omega}{\partial\xi}, \tag{14}$$

where $\left\langle a^2\right\rangle=\dfrac{a_0^2}{2}$ is used, and a_0 is the peak normalized radiation intensity in the pulse. Equation (14) shows that the partially stripped atoms can reduce the longitudinal bunching of laser energy, and later we will show it actually is the competition of normal and abnormal dispersion. The time derivative of Eq.(14) is

$$\frac{\partial^2}{\partial\tau^2}\Delta\left\langle a^2\right\rangle=\frac{\omega_{\rm p0}^2a_0^2}{2\eta_0^3\omega_0^2}\left(1-\frac{R_0}{4}a_0^2\right)\frac{\partial}{\partial\xi}\frac{1}{\omega_0}\frac{\partial\omega}{\partial\tau}. \tag{15}$$

Considering the phase velocity variation mainly caused by the variation of radiation intensity, from Eqs.(6) and (10), we obtain

$$\frac{1}{\omega}\frac{\partial\Delta\omega}{\partial\tau}=-\frac{\omega_{\rm p0}^2}{4\eta_0^3\omega_0^2}\left(1+R_0\right)\frac{\partial\Delta\left\langle a^2\right\rangle}{\partial\xi}. \tag{16}$$

Equation (16) shows that in the front edge of the perturbation, $\dfrac{1}{\omega}\dfrac{\partial\Delta\omega}{\partial\tau}<0$, the frequency is decreasing, while in the back edge of the perturbation, $\dfrac{1}{\omega}\dfrac{\partial\Delta\omega}{\partial\tau}>0$, the frequency is increasing, i.e. the perturbation is positively chirped; and since $R_0>>0$,[2,3] the partially stripped atoms enhance the chirp greatly.

Substituting Eq.(16) into Eq.(15) yields

$$\begin{aligned}&\frac{\partial^2}{\partial\tau^2}\Delta\left\langle a^2\right\rangle\\&=-\frac{a_0^2\omega_{\rm p0}^4}{8\eta_0^6\omega_0^4}\left(1-\frac{R_0}{4}a_0^2\right)\left(1+R_0\right)\frac{\partial^2\Delta\left\langle a^2\right\rangle}{\partial\xi^2}.\end{aligned} \tag{17}$$

If $\Delta\left\langle a^2\right\rangle=\dfrac{\left\langle a^2\right\rangle_1}{2}\mathrm{e}^{\mathrm{i}(k'x-\omega' t)}+\text{c.c.}=\dfrac{\left\langle a^2\right\rangle_1}{2}\mathrm{e}^{-\mathrm{i}k'c\xi}+\text{c.c.}$, then

$$\frac{\partial^2}{\partial\tau^2}\left\langle a^2\right\rangle_1=\frac{k'^2c^2a_0^2\omega_{\rm p0}^4}{8\eta_0^6\omega_0^4}\left(1-\frac{R_0}{4}a_0^2\right)\left(1+R_0\right)\left\langle a^2\right\rangle_1. \tag{18}$$

According to Ref.[9], we can add

$$-\frac{1}{4}\frac{k'^4}{\omega_0^6}\omega_{\rm p0}^4c^4\left\langle a^2\right\rangle_1$$

to the right-hand side of Eq.(18) and obtain the final equation for phase modulation

$$\begin{aligned}\frac{\partial^2}{\partial\tau^2}\left\langle a^2\right\rangle_1=&\left[\frac{k'^2c^2a_0^2\omega_{\rm p0}^4}{8\eta_0^6\omega_0^4}\left(1-\frac{R_0}{4}a_0^2\right)\left(1+R_0\right)\right.\\&\left.-\frac{1}{4}\frac{k'^4}{\omega_0^6}\omega_{\rm p0}^4c^4\right]\left\langle a^2\right\rangle_1.\end{aligned} \tag{19}$$

Letting $\tilde{\omega} = \dfrac{\omega_0}{\omega_{p0}}$ and $\tilde{k} = \dfrac{k'}{k_0}$, k_0 being the wave number of the pump, we have

$$\frac{\partial^2}{\partial\tau^2}\left\langle a^2\right\rangle_1 = \frac{\tilde{k}^2 k_0^2 c^2}{4\tilde{\omega}^4}\left[\frac{a_0^2}{2\eta_0^6}\left(1-\frac{R_0}{4}a_0^2\right)\times(1+R_0)-\eta_0^2\tilde{k}^2\right]\left\langle a^2\right\rangle_1, \quad (20)$$

where $\omega_0 = \dfrac{k_0 c}{\eta_0}$.

4. Numerical solution

The excitation condition of the modulation instability derived from Eq.(20) is

$$\frac{a_0^2}{2\eta_0^6}\left(1-\frac{R_0}{4}a_0^2\right)(1+R_0)-\eta_0^2\tilde{k}^2>0. \quad (21)$$

Figure 1(a) shows the stability boundary described by Eq.(21) for different wavelengths, and the instability-excited region is on the left side of the curves. It also shows that the instability region decreases as the laser frequency increases. Figure 1(b) shows the stability boundary for different radiation intensities and the instability-excited region is on the left side of the curves. It shows that the instability region decreases as the radiation intensity increases.

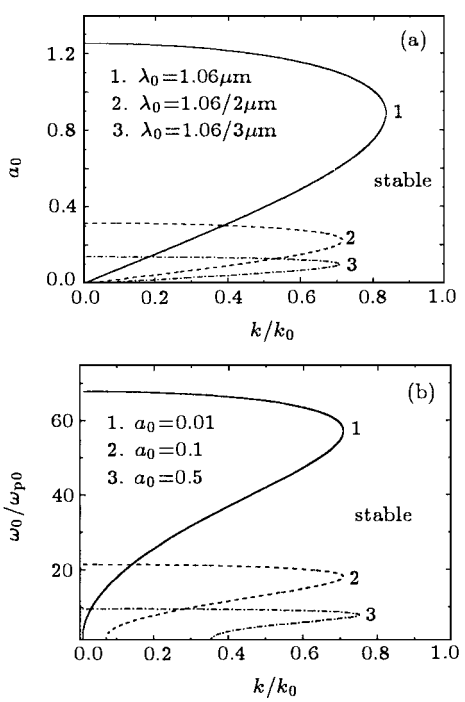


Fig.1. (a) Stability boundary for different wavelengths. The instability-excited region is on the left side of the curve. (b) The stability boundary for different radiation intensities. The instability-excited region is on the left side of the curve.

Sprangle *et al*[10] explained the finite instability-excited region as a result of finite-perturbation-length effect. Here, it is obvious that the basic reason for the existence of the finite region is the competition of normal and abnormal dispersions. Equation (7) shows that for the plasma, the group velocity increases with the increase of frequency, i.e. the plasma acts as an abnormal dispersion medium, but for the partially stripped atoms, the group velocity decreases with the increase of frequency, i.e. it is a normal dispersion medium. When the abnormal dispersion dominates the normal dispersion, i.e. $\dfrac{R_0}{4}a_0^2+\Delta<1$ is satisfied, where $\Delta = 2\eta_0^8\tilde{k}^2/[a_0^2(1+R_0)]$, the positive chirp compresses the perturbation and the perturbation intensity increases, thus the instability can be excited. When the normal dispersion dominates the abnormal dispersion, i.e. $\dfrac{R_0}{4}a_0^2+\Delta>1$ is satisfied, the positive chirp disperses the perturbation and the perturbation intensity decreases, the instability cannot be excited. When the normal dispersion is approximately equal to the abnormal dispersion, i.e. $\dfrac{R_0}{4}a_0^2+\Delta=1$, the perturbation keeps its original state, and a soliton can be present. The laser radiation intensity and its frequency determine the intensity ratio of the normal to the abnormal dispersion, which determines whether the modulation instability can be excited.

If Eq.(21) is satisfied, the growth rate of modulation instability can be derived from Eq.(20):

$$\Gamma_{SM} = \left\{\frac{\tilde{k}^2 k_0^2 c^2}{4\tilde{\omega}^4}\left[\frac{a_0^2}{2\eta_0^6}\left(1-\frac{R_0}{4}a_0^2\right)\times(1+R_0)-\eta_0^2\tilde{k}^2\right]\right\}^{1/2}. \quad (22)$$

Figure 2 shows the variation of growth rate of modulation instability for different wavelengths, where $\Gamma_{FRS} = \dfrac{a_0\omega_{p0}^2}{2\sqrt{2}\omega_0}$ is the maximum growth rate of FRS instability. The solid curves denote the growth rate in PSP and the dot-dashed curve denotes that in FSP. These figures show that the modulation instability growth rate in PSP is much greater than that in FSP, and with the decrease of wavelength, the difference increases. Figures 2(b) and 2(c) show that when the laser frequency is doubled, especially tripled, the modulation instability growth rate in PSP can be greater than the maximum growth rate of FRS instability. The reason is that the partially stripped atoms can enhance the positive chirp greatly.

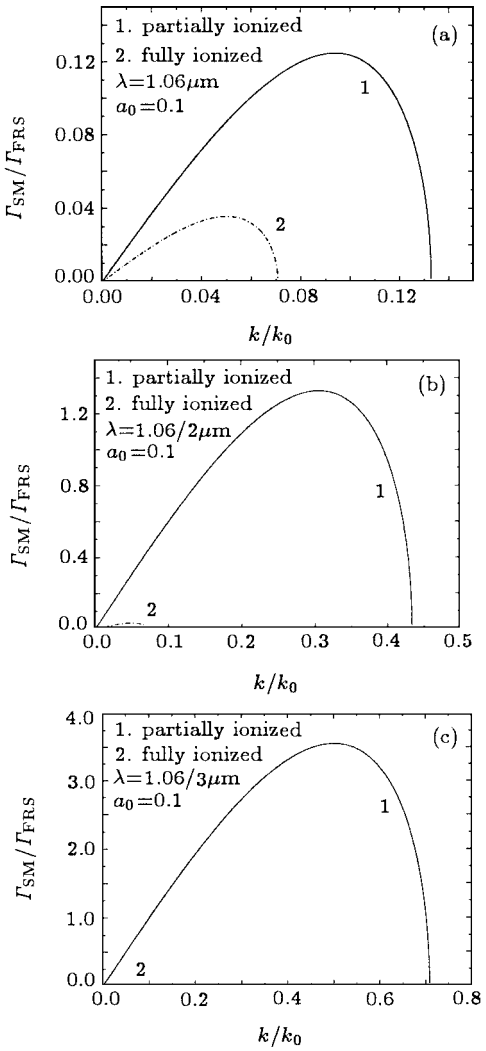


Fig.2. Variation of modulation instability growth rate for different wavelengths. The solid curve denotes the growth rate in the partially stripped plasma, and the dot-dashed curve denotes that in the fully stripped plasma.

Under the condition of $\frac{R_0}{4}a_0^2 + \Delta < 1$, the growth rate shown in Eq.(22) extends from $\tilde{k} = 0$ to $\tilde{k}_{\max} = \left[\frac{a_0^2}{2\eta_0^8}\left(1 - \frac{R_0}{4}a_0^2\right)(1+R_0)\right]^{1/2}$, and peaks at $\tilde{k} = \tilde{k}_{\max}/\sqrt{2}$, so we have the ratio

$$\frac{\Gamma_{\max}}{\Gamma_{FRS}} = \frac{\sqrt{2}a_0}{4\eta_0^6}\left(1 - \frac{R_0}{4}a_0^2\right)(1+R_0). \qquad (23)$$

Figure 3 shows the relationship between the maximum growth rate and the laser radiation intensity. It is shown that when the laser frequency is tripled, i.e. $\lambda_0 = 1.06/3\mu$m, the maximum ratio is approximately equal to 4 at $a_0 \cong 0.08$. It is different from the result $\frac{\Gamma_{\max}}{\Gamma_{FRS}} = \frac{3\sqrt{2}a_0}{8}R$ that Sprangle *et al* have given in Refs.[2,3], where R was taken to be a constant, implying that the ratio varies linearly with the increase of a_0. Here we have shown that the R-ω relationship can induce the competition between normal and abnormal dispersions. In the low-intensity limit $a_0 << 1$, we have $\frac{R_0}{4}a_0^2 << 1$, i.e. the normal dispersion is negligible, and our result is consistent with the result in Refs.[2,3]. But with the increase of a_0, the normal dispersion is enhanced, which competes with the abnormal dispersion and limits the ratio to below 4.

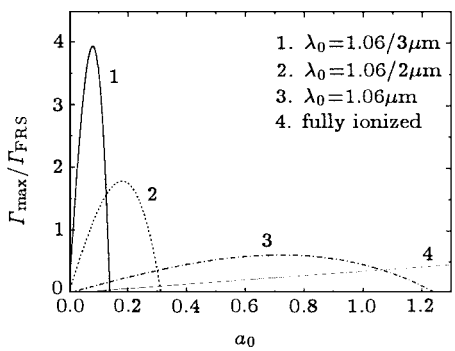


Fig.3. The maximum growth rate versus radiation intensity.

5. Conclusion

In this paper, with the assumption that the photon number is conserved within a local volume, the laser pulse modulation instability has been discussed and the excitation condition and growth rate of the modulation instability were obtained. The analysis gives the following conclusion: first, the excitation of the modulation instability of laser pulses in PSP is closely related with the laser radiation intensity and the laser frequency. Due to the competition between normal and abnormal dispersions, the modulation instabilities could only be excited in a given range of laser radiation intensity and frequency; second, the partially stripped atoms can intensify the positive chirp of the intensity perturbation greatly, so the modulation instability in PSP is much stronger than in FSP; third, the relative growth rate of modulation instability relies strongly on the laser frequency. If the laser frequency increases, under a given radiation intensity, $\Gamma_{\max}/\Gamma_{FRS}$ can be greater than unity, however, due to the competition between normal and abnormal dispersions, $\Gamma_{\max}/\Gamma_{FRS}$ can be only several times greater than unity.

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