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Influence of Doppler-broadening on absorption-dispersion properties in a resonant coherent medium*

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We investigate the influence of Doppler broadening on absorption-dispersion properties in a four-level atomic system that can evolve from a normal dispersion to an anomalous dispersion. Our results show that the absorption-dispersion properties become strongly dependent on the propagation directions of the applied fields if Doppler broadening is taken into account. Especially, the switchover in the sign of the dispersion is still achievable even in the presence of Doppler broadening if properly arranging the propagation directions of the applied fields, which is in contrast with the otherwise behaviours in some other configurations.

Keywords: Doppler broadening, absorption, dispersion, subluminal, superluminal
PACC: 4250, 4210J, 4210K, 4210M

1. Introduction

The dispersion and absorption are a pair of fundamental properties of an optical medium, which can be dramatically modified by employing coherent fields to drive the medium. The quantum coherence and interference created by the coherent fields have led to many interesting optical phenomena such as electromagnetically induced transparency (EIT),[1,2] lasing without inversion (LWI),[3,4] enhancement of the index of refraction,[5] efficient hyper-Raman scattering[6] and enhancement of four-wave mixing,[7] Associated with the technique of EIT, recently, the study of subluminal and superluminal light propagations has aroused a great deal of interest.[8−19] Subluminal light propagation has been demonstrated in ultracold[8] and hot[9] atomic gases, and in solids;[10] Superluminal light propagation also has been experimentally observed in Cs gas[11] and Rb gas.[12] More recently, Bigelow *et al* have observed both superluminal and ultraslow light propagations in the same solid-state material by using spectral hole burning.[13] The slowing down of the group velocity of the light is usually a consequence of the normal (positive) dispersion that occurs between two absorption lines in an EIT system,[8] whereas "fast" light is obtained as a result of the anomalous (negative) dispersion between two closely spaced gain lines.[11] It has been predicted in several different schemes that the dispersion of the probe field can be changed from positive to negative by controlling the intensities or phases of the applied fields.[14−16]

In Ref.[16], a four-level atomic system that can evolve from a normal dispersion and amplification without population inversion into an anomalous dispersion and amplification with population inversion is presented. The superluminal propagation in a similar scheme is also noted in Ref.[17]. The scheme analysed in this paper is the same as that in Ref.[16]. We note that the authors of Ref.[16] performed their calculations under the situation without the Doppler broadening, and their experiment on light amplification was carried out in cold atoms, which means that the Doppler broadening can be ignored in their experiment. Under such a condition, amplification was the dominant feature at the line center for the probe field.

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However, in a realistic experiment at normal temperature, one has to consider the effect of Doppler broadening, which tends to mask the interference effect.[20] In the present paper, we extend the study of Ref.[16] to the situation including the Doppler broadening. We find that in the presence of the Doppler broadening, the absorption-dispersion properties of the probe field are strongly dependent on the propagation directions of the applied fields. In different propagation schemes, the absorption-dispersion properties for the probe field may be quite different. The amplification at the line center, as well as the switchover from a positive to a negative dispersion demonstrated in Ref.[16] may never be obtained in some cases if Doppler broadening is included. In order to observe the phenomena demonstrated in Ref.[16], we need to properly arrange the propagation directions of the applied fields. We note that Goren *et al* have found that the tunability from subluminal to superluminal is lost completely due to the Doppler broadening effect in the configuration they considered.[15] In the present configuration, however, we show that the switchover in the sign of the dispersion is still achievable even in the presence of Doppler broadening if we properly arrange the propagation directions of the applied fields.

2. Model and equations

A four-level atomic system considered is shown in Fig.1. Levels $|2\rangle$ and $|1\rangle$ are two lower levels, a coupling field E_c with a Rabi frequency $\Omega_c = E_c \cdot d_{23}/2\hbar$ drives a transition $|2\rangle \leftrightarrow |3\rangle$; a weak probe field E_p with a Rabi frequency $\Omega_p = E_p \cdot d_{13}/2\hbar$ drives a transition $|1\rangle \leftrightarrow |3\rangle$; a pump field E_d with a Rabi frequency $\Omega_d = E_d \cdot d_{14}/2\hbar$ is applied to the optically allowed transition $|1\rangle \leftrightarrow |4\rangle$. $\Delta_c = \omega_{32} - \omega_c$, $\Delta_p = \omega_{31} - \omega_p$ and $\Delta_d = \omega_{41} - \omega_d$ are the detunings of the coupling field, probe field and the pump field, respectively. $\gamma_3 = \gamma_{32} + \gamma_{31}$ and $\gamma_4 = \gamma_{41} + \gamma_{42}$ denote the spontaneous decay rates from levels $|3\rangle$ and $|4\rangle$, respectively, and the collisional relaxation rate between the states $|2\rangle$ and $|1\rangle$ is γ_{21}. Such a four-level configuration can be found in many real atomic systems, such as alkali-metal atoms.

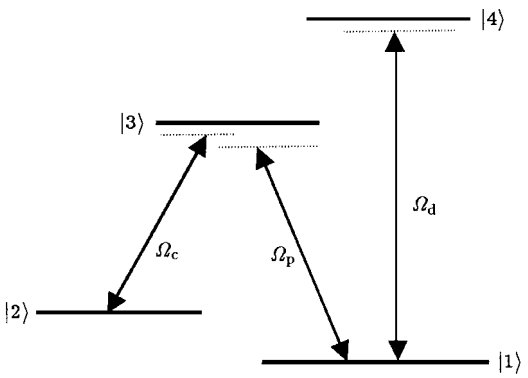


Fig.1. Schematic diagram of a four-level atomic system driven by three coherent fields.

In the interaction picture, the semiclassical interaction Hamiltonian of this system in a rotating-wave frame can be written as

$$H_I = -\Delta_p |3\rangle\langle 3| - (\Delta_p - \Delta_c)|2\rangle\langle 2| - \Delta_d |4\rangle\langle 4| - [\Omega_p |3\rangle\langle 1| + \Omega_c |3\rangle\langle 2| + \Omega_d |4\rangle\langle 1| + \text{H.c.}], \quad (1)$$

where $|i\rangle\langle j|$ is the population operator for $i = j$ or the polarization operator when $i \neq j$. The density-matrix equations of motion in the electric-dipole approximation and the rotating-wave approximation can be derived as follows:

$$\begin{aligned}
\dot{\rho}_{44} &= -\gamma_4\rho_{44} + i\Omega_d(\rho_{14} - \rho_{41}),\\
\dot{\rho}_{33} &= -\gamma_3\rho_{33} + i\Omega_p(\rho_{13} - \rho_{31}) + i\Omega_c(\rho_{23} - \rho_{32}),\\
\dot{\rho}_{22} &= \gamma_{21}(\rho_{11} - \rho_{22}) + \gamma_{32}\rho_{33} + \gamma_{42}\rho_{44} + i\Omega_c(\rho_{32} - \rho_{23}),\\
\dot{\rho}_{12} &= [-\gamma_{12} - i(\Delta_p - \Delta_c)]\rho_{12} - i\Omega_c\rho_{13} + i\Omega_p\rho_{32} + i\Omega_d\rho_{42},\\
\dot{\rho}_{13} &= [-(\gamma_3 + \gamma_{21})/2 - i\Delta_p]\rho_{13} + i\Omega_p(\rho_{33} - \rho_{11}) - i\Omega_c\rho_{12} + i\Omega_d\rho_{43},\\
\dot{\rho}_{14} &= [-(\gamma_4 + \gamma_{21})/2 - i\Delta_d]\rho_{14} + i\Omega_d(\rho_{44} - \rho_{11}) + i\Omega_p\rho_{34},\\
\dot{\rho}_{23} &= [-(\gamma_3 + \gamma_{21})/2 - i\Delta_c]\rho_{23} + i\Omega_c(\rho_{33} - \rho_{22}) - i\Omega_p\rho_{21},\\
\dot{\rho}_{24} &= [-(\gamma_4 + \gamma_{21})/2 - i(\Delta_d + \Delta_c - \Delta_p)]\rho_{24} + i\Omega_c\rho_{34} - i\Omega_d\rho_{21},\\
\dot{\rho}_{34} &= [-(\gamma_3 + \gamma_4)/2 - i(\Delta_d - \Delta_p)]\rho_{34} - i\Omega_d\rho_{31} + i\Omega_p\rho_{14} + i\Omega_c\rho_{24}. \quad (2)
\end{aligned}$$

The above equations are constrained by $\rho_{ij} = \rho_{ji}^*$ and $\rho_{11}+\rho_{22}+\rho_{33}+\rho_{44} = 1$, where ρ_{ii} is the population of the level i, $\rho_{ij}(i \neq j)$ represents the atomic coherence between levels i and j.

In the limit of a weak probe, the indices of refraction and gain (absorption) for the probe field are given by

$$R(\Delta_{\rm p}) = k{\rm Re}(\rho_{13}/\Omega_{\rm p}),$$
$$A(\Delta_{\rm p}) = k{\rm Im}(\rho_{13}/\Omega_{\rm p}), \qquad (3)$$

where $k = \dfrac{3N\lambda_{31}^2\gamma_{31}}{8\pi}$, N is the density of atoms, λ_{31} is the wavelength of the probe laser, γ_{31} is the decay along the probe transition. In our notation, $A(\Delta_{\rm p}) > 0$ means that the probe field is amplified, and $A(\Delta_{\rm p}) < 0$ means that the probe field is absorbed.

To consider the effect of Doppler broadening, we need to substitute $\Delta_{\rm p}$, $\Delta_{\rm c}$, and $\Delta_{\rm d}$ with velocity-dependent detunings, which are given by

$$\Delta_{\rm p}(v) = \Delta_{\rm p} + \omega_{\rm p}\frac{v}{c},$$
$$\Delta_{\rm c}(v) = \Delta_{\rm c} + \omega_{\rm c}\eta_{\rm c}\frac{v}{c},$$
$$\Delta_{\rm d}(v) = \Delta_{\rm d} + \omega_{\rm d}\eta_{\rm d}\frac{v}{c}, \qquad (4)$$

where $\Delta_{\rm p}$, $\Delta_{\rm c}$ and $\Delta_{\rm d}$ are nominal detunings, $\omega_{\rm p}$, $\omega_{\rm c}$ and $\omega_{\rm d}$ the corresponding field frequencies, c the velocity of light and v the component of the atom's velocity in the propagation directions of the applied fields. η_i is a parameter that depends on the propagation direction of the applied fields: $\eta_i = 1$ ($\eta_i = -1$) means that the coupling or the pump field is co-propagation (counter-propagating) with the probe laser. According to Eqs. (2) the response of the atoms to the fields is now velocity-dependent, the following integrations over the velocity distribution $N(v)$ lead to the contribution of all the atoms to the total absorption and dispersion of the probe

$$A(\Delta_{\rm p}) = \int_{-\infty}^{\infty} k(v){\rm Im}\rho_{13}(v)/\Omega_{\rm p}{\rm d}v,$$
$$R(\Delta_{\rm p}) = \int_{-\infty}^{\infty} k(v){\rm Re}\rho_{13}(v)/\Omega_{\rm p}{\rm d}v, \qquad (5)$$

where $k(v) = \dfrac{3N(v)\lambda_{31}^2\gamma_{31}}{8\pi}$, and $N(v){\rm d}v$ is the number of atoms per unit volume with velocity v. Assuming that the atomic velocities obey the Maxwell-Boltzmann distribution, one has the probability distribution function for v as follows:

$$N(v) = \frac{N_0}{u\sqrt{\pi}}\exp(-v^2/u^2), \qquad (6)$$

where N_0 is the total number of atoms per unit volume, and u represents the most probable velocity. For a purely Doppler-broadened medium, the full width at half maximum of the absorption profile is given by

$$\Delta\omega_{\rm D} = 2\sqrt{\ln 2}\frac{u}{c}\omega_{\rm p}. \qquad (7)$$

3. Results and discussion

To obtain the numerical results we have integrated Eqs.(5) and Eqs.(2) by using a Runge–Kutta–Felhberg routine. We have split the velocity distribution into discrete groups with all atoms belonging to a given group, which are taken to be moving at the same velocity. We have considered a large enough number of velocity groups to ensure the convergence of the results.

The following numerical results are derived by using the parameters in ^{87}Rb atoms (The coupling and probe fields drive transitions $5{\rm S}_{1/2}F = 2 \leftrightarrow 5{\rm P}_{1/2}F' = 2$ and $5{\rm S}_{1/2}F = 1 \leftrightarrow 5{\rm P}_{1/2}F' = 2$, respectively. The pump field drives the transition $5{\rm S}_{1/2}F = 1 \leftrightarrow 5{\rm P}_{3/2}F = 2$): $\gamma_3/2\pi = 5.3$MHz, $\gamma_4/2\pi = 5.9$MHz, $u = 239$m/s, $\Delta\omega_{\rm D} = 500$MHz (room temperature), $N_0 = 2\times10^{11}/{\rm cm}^3$. Allowing for the practical situation, we suppose a collisional relaxation rate $\gamma_{21} = 0.001\gamma_3$ between the states $|1\rangle$ and $|2\rangle$.

For the three applied lasers, there are four possible propagation schemes:

(i) Both the coupling and the pump lasers co-propagate with the probe laser ($\eta_{\rm c} = \eta_{\rm d} = 1$);

(ii) The coupling laser co-propagates with the probe field, the pump laser counter-propagates with the probe laser ($\eta_{\rm c} = 1$, $\eta_{\rm d} = -1$);

(iii) Both the coupling and the pump lasers counter-propagation with the probe laser ($\eta_{\rm c} = \eta_{\rm d} = -1$);

(iv) The coupling laser counter-propagates with the probe field, the pump laser co-propagates with the probe laser ($\eta_{\rm c} = -1$, $\eta_{\rm d} = 1$).

For Scheme (i), in Fig.2 we plot absorption and refraction versus the probe detuning for two different intensities of the pump field: $\Omega_{\rm d} = 0.5\gamma_3$ in Figs.(a) and (a') and $\Omega_{\rm d} = \gamma_3$ in Figs.(b) and (b'), from which we can see that, in the presence of the Doppler broadening, and with the small pump, there appears an extremely narrow gain peak at the line center, accompanied with a very steep positive dispersion. When

the intensity of the pump field increases, the narrow gain peak splits into a doublet, and the corresponding dispersive response of the probe laser becomes anomalous and exhibits a negative slope at resonance (the inset shows a magnified part near the line center). Under such a propagation scheme, the extremely narrow feature caused by Doppler broadening is a notable phenomenon. As is known, the narrow resonance is usually induced by quantum interference,[21,22] it has been reported that the Doppler broadening may suppress and broaden the narrow resonance.[20] Whereas in the present configuration, Doppler broadening plays an opposite role, the ultra-narrow feature takes place just in the presence of the Doppler broadening. To our knowledge, there has been no report on such an unexpected phenomenon of ultra-narrow feature associated with Doppler broadening.

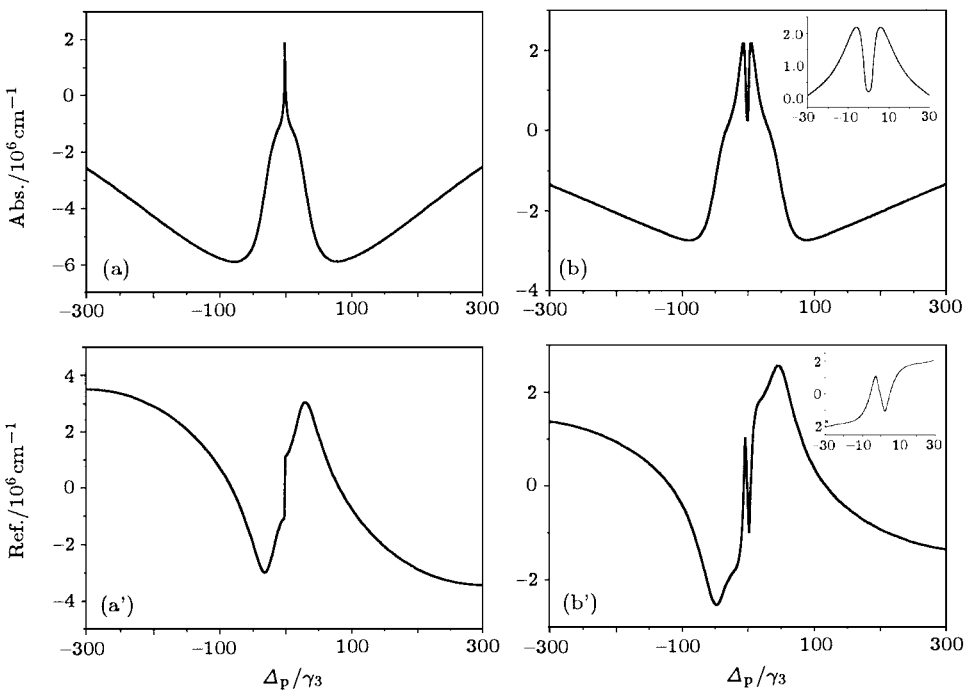


Fig.2. Absorption $A(\Delta_p)$ and Refraction $R(\Delta_p)$ as a function of the probe detuning Δ_p in the presence of the Doppler broadening, and with both the coupling and the pump lasers co-propagating with the probe laser ($\eta_c = \eta_d = 1$). The parameters are $\gamma_3/2\pi = 5.3$MHz, $\gamma_4/2\pi = 5.9$MHz, $u = 239$m/s, $\Delta\omega_D = 500$MHz (room temperature), $N_0 = 2\times10^{11}/\text{cm}^3$, $\gamma_{21} = 0.001\gamma_3$, $\Omega_c = 0.5\gamma_3$, $\Omega_d = 0.5\gamma_3$ in Figs.(a, a') and $\Omega_d = \gamma_3$ in Figs.(b, b'). The inset shows a magnified part near the line center correspondingly.

For Scheme (ii), the situation is very similar to that of Scheme (i) (as shown in Fig.3, where the parameters are the same as those in Fig.2, the inset shows a magnified part near the line center), and the negative dispersion also can be obtained when the intensity of the pump laser increases. It should be noted that for the same parameters there are some differences between the spectra for the two schemes: first, the extremely narrow feature disappears in Scheme (ii); second, in Scheme (ii), between the gain doublets, the negative dispersion is accompanied by the absorption at the line center, while in Scheme (i), the negative dispersion is accompanied by the gain at the line center.

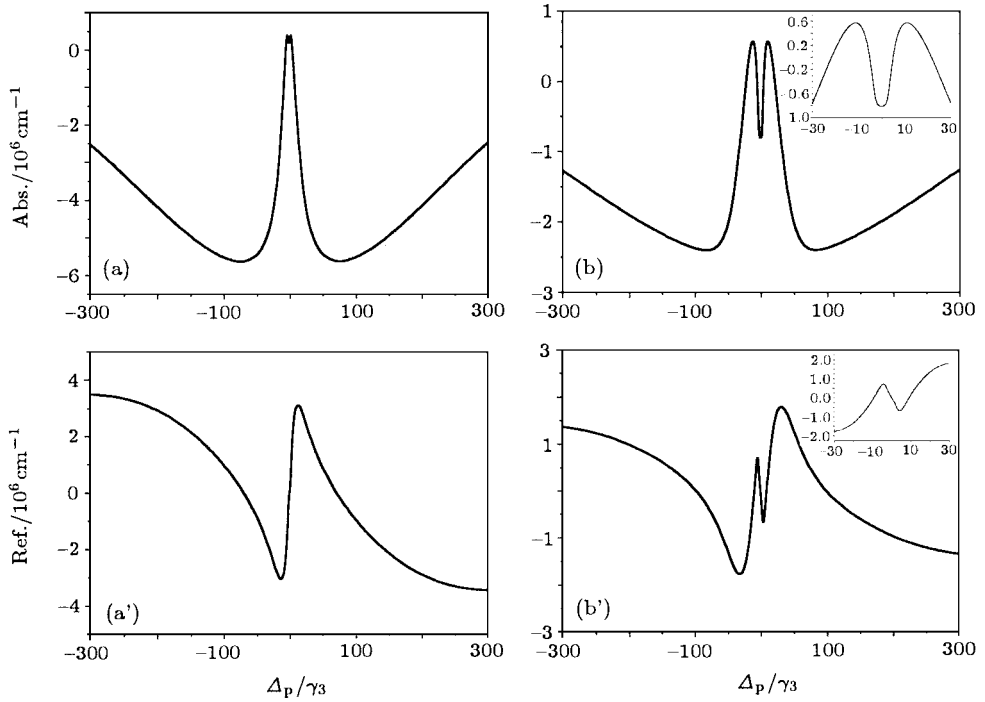


Fig.3. Absorption $A(\Delta_p)$ and Refraction $R(\Delta_p)$ as a function of the probe detuning Δ_p in the presence of the Doppler broadening, and with the coupling laser co-propagating with the probe field, and the pump laser counter-propagating with the probe laser ($\eta_c = 1$, $\eta_d = -1$). The parameters in Figs.(a), (a'), (b) and (b') are the same as those in Figs. 2(a) (a'), (b) and (b'), respectively. The inset shows a magnified part near the line center correspondingly.

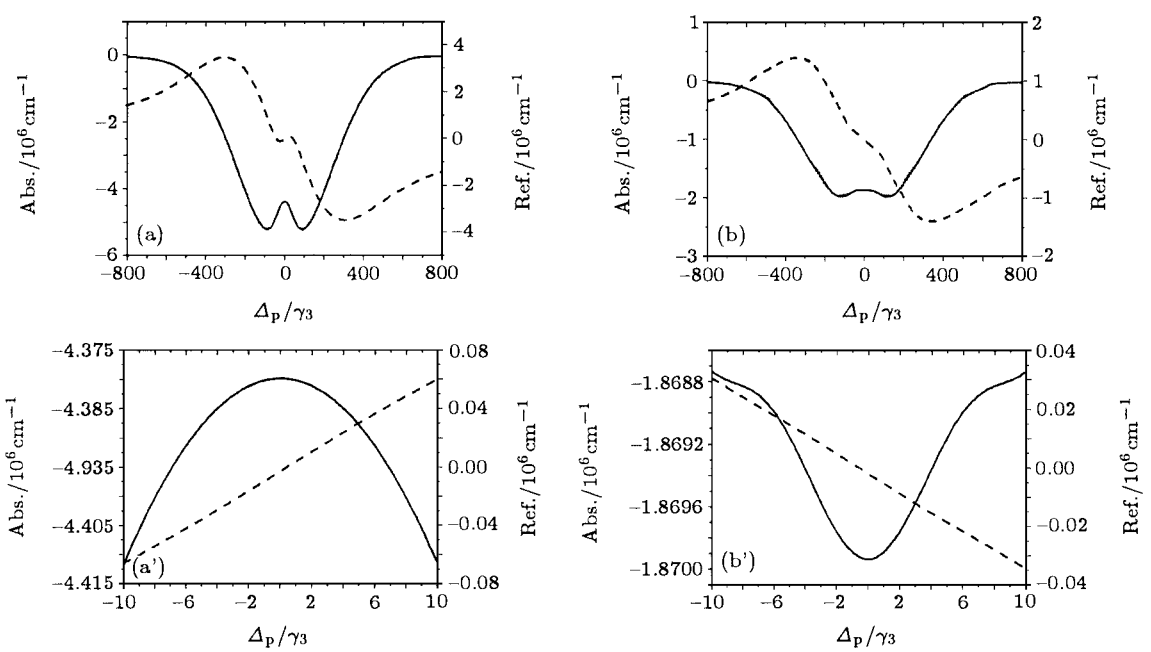


Fig.4. Absorption $A(\Delta_p)$ (solid curves) and Refraction $R(\Delta_p)$ (dashed curves) as a function of the probe detuning Δ_p in the presence of the Doppler broadening and with both the coupling and the pump laser counter-propagating with the probe laser ($\eta_c = \eta_d = -1$). The full spectra are shown in Figs.(a) and (b), and the detailed structure at the line center are shown in Figs.(a') and (b'). The parameters in Figs. (a), (a'), (b) and (b') are the same as that in Fig.2(a), (a'), (b) and (b'), respectively.

Figure 4 depicts the situation for Scheme (iii). It can be seen that when both the coupling and the pump lasers counter-propagate with the probe laser, the whole absorption spectrum is characterized by an absorption profile with a dip in the central region. When the pump is weak, the central absorption is accompanied by a positive dispersion (as shown in Fig.4(a).). As the pump increases, a small peak develops at the center of the dip (Figure 4(b')) shows the detailed structure at the line center.), and the dispersion near the line center changes into negative correspondingly. Please note that in such a propagation scheme, the probe gain can never be obtained.

For Scheme (iv), in Fig.5, we plot absorption and refraction versus the probe detuning for various intensities of the pump field: $\Omega_{\rm d} = \gamma_3$, $2\gamma_3$, $5\gamma_3$, and $10\gamma_3$. It can be found that with the increase of the pump field, the absorption at the line center evolves into gain, accompanied by a positive dispersion. Further increasing the pump intensity, a dip appears at the gain center. In order to obtain the switchover from a positive to a negative dispersion, an extremely intense pump is required, which gives an almost zero dispersion in the center region.

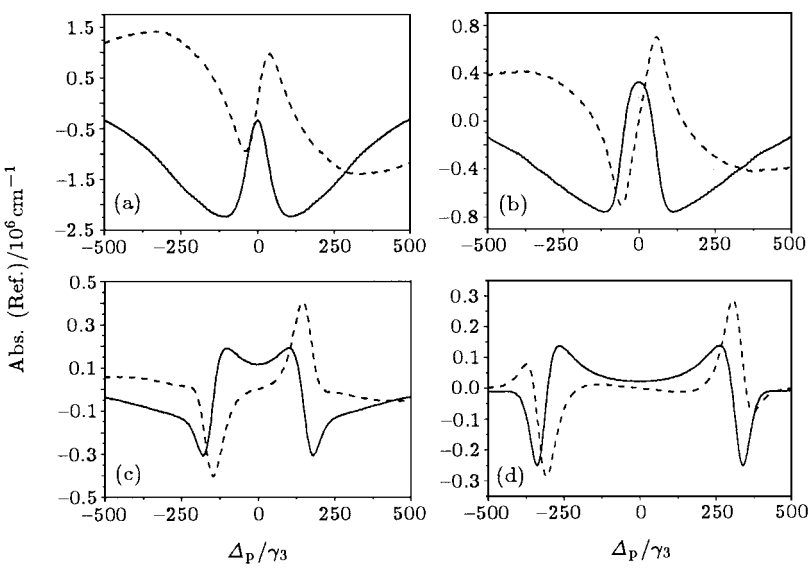


Fig.5. Absorption $A(\Delta_{\rm p})$ (solid curves) and Refraction $R(\Delta_{\rm p})$ (dashed curves) versus the probe detuning $\Delta_{\rm p}$ for various intensities of the pump field in the presence of the Doppler broadening, and with the coupling laser counter-propagating with the probe field and the pump laser co-propagating with the probe laser ($\eta_{\rm c} = -1$, $\eta_{\rm d} = 1$). $\Omega_{\rm d} = \gamma_3$, $2\gamma_3$, $5\gamma_3$, and $10\gamma_3$ in Figs.(a), (b), (c), and (d), respectively. Other parameters are the same as those in Fig.2.

The above calculations show that, when the Doppler broadening is included, the dispersion-absorption properties in this configuration are strongly dependent on the propagation directions of the applied fields. Under different propagation schemes, the absorption-dispersion properties of the probe field may be quite different. In order to obtain probe gain at the line center, we need to properly choose the propagation schemes, and, in order to realize the switchover from subluminal to superluminal, we also need to properly arrange the propagating directions of the applied fields, otherwise the probe amplification at the line center as well as the switchover in the sign of the dispersion demonstrated in Ref.[16] may never be obtained. As long as we properly arrange the propagation directions of the applied fields, the switchover in the sign of the dispersion still can be achieved even in the presence of Doppler broadening. Whereas for the configuration considered in Ref.[15], the switchover in the sign of the dispersion is only achievable in the absence of Doppler broadening. In the present configuration, to obtain superluminality, the first two propagation schemes are preferable (coupling laser co-propagates with the probe laser), carefully choosing the intensities of the pump field, one can obtain an anomalous dispersion with very little gain or absorption at the line center.

4. Conclusion

In summary, we have investigated the influence

of Doppler broadening on the absorption-dispersion properties in a four-level system in ^{87}Rb atom. By taking the effect of Doppler broadening into account, we find that the dispersion-absorption properties in this configuration are strongly dependent on the propagation directions of the applied fields. In the presence of the Doppler broadening, amplification is not the only feature at the line center, under different propagating schemes, the probe field may exhibit extremely narrow resonance, amplification, as well as absorption at the line center. Another meaningful result of our calculation is that, the switchover in the sign of the dispersion is still achievable even in the presence of Doppler broadening if properly arranging the propagating directions of the applied fields, which is in contrast with the otherwise behaviours in some other configurations.

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