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Detecting dynamical complexity changes in time series using the base-scale entropy

Li Jin(李 锦)[a)b)], Ning Xin-Bao(宁新宝)[a)†], Wu Wei(吴 巍)[a)], and Ma Xiao-Fei(马小飞)[a)]

[a)] *State Key Laboratory of Modern Acoustics, Institute for Biomedical Electronical Engineering, Nanjing University, Nanjing 210093, China*

[b)] *College of Physics and Information Technology, Shaanxi Normal University, Xi'an 710062, China*

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Timely detection of dynamical complexity changes in natural and man-made systems has deep scientific and practical meanings. We introduce a complexity measure for time series: the base-scale entropy. The definition directly applies to arbitrary real-word data. We illustrate our method on a practical speech signal and in a theoretical chaotic system. The results show that the simple and easily calculated measure of base-scale entropy can be effectively used to detect qualitative and quantitative dynamical changes.

Keywords: dynamical complexity, the base-scale entropy, the zero-crossing rate, Lyapunov exponent
PACC: 0545, 0547, 4360

1. Introduction

Detection of dynamical changes and complexity for outputting time series of a complex system is one of the most important problems. During the last two decades, a number of interesting methods have been proposed to detect dynamical complexity changes. They include recurrence plots[1] and recurrence quantification analysis,[2] spacetime separation plots[3] and their associated probability distributions,[4] statistical tests using discretized invariant distribution in the reconstructed phase space,[5,6] cross-correlation sum analysis,[7] and nonlinear cross prediction analysis.[8] Most of these methods are based on quantifying certain aspects of the nearest neighbours in phase space, and, as a result, are expensive computationally.

We introduce a base-scale entropy method, which estimates the wave information contained in m consecutive values of the time series. The advantages of our method are its simplicity and extremely fast in calculation. At the same time a small noise does not essentially change the complexity of a chaotic signal. We apply this method to a theoretical chaotic system and the complex speech signals whose equations are unknown and whose dynamics are characteristically non-linear and non-stationary. The results show that the base-scale entropy can sensitively capture dynamical changes of time series.

2. The base-scale entropy method

Considering that a time series u of N points is as follows: $\{u(i) : 1 \leq i \leq N\}$. First we embed the time series to an m-dimensional space.[9,10] For each point $u(i)$, we pick m data points from the series to make an m-dimensional vector:

$$X(i) = [u(i), u(i+L), \cdots, u(i+(m-1)L)], \quad (1)$$

where m is called the embedding dimension and L the delay time. Choosing $L = 1$, $N-m+1$ m-dimensional vectors are used in the paper. For each m-dimensional vector, the base scale BS is calculated. Define the base scale as the root-mean-square of the differences between every two contiguous data points in an m-dimensional vectors.

$$BS(i) = \sqrt{\sum_{j=1}^{m-1} ((u(i+j) - u(i+j-1))^2/(m-1)}. \quad (2)$$

Based on the base scale, the partition standard can be selected as $a \times BS$, we transform each m-dimensional vector $X(i)$ into symbolic sequences $S_i(X(i)) = \{s(i), s(i+1), \cdots, s(i+m-1)\}$, $s \in A$ on the basis of the alphabet A (A=0, 1, 2, 3). The

†E-mail: xbning@nju.edu.cn

transformation into symbols refers to three given levels:

$$S_i(X(i)) = \begin{cases} 0, & \bar{u} < u_{i+k} \le \bar{u} + \alpha \times BS, \\ 1, & u_{i+k} > \bar{u} + \alpha \times BS, \\ 2, & \bar{u} - \alpha \times BS < u_{i+k} \le \bar{u}, \\ 3, & u_{i+k} \le \bar{u} - \alpha \times BS, \end{cases} \quad (3)$$

where $i = 1, 2, 3, \cdots, N - m + 1, k = 0, 1, 2, \cdots, m - 1$, $\bar{u}$ denotes the mean of the ith m-dimensional vector $X(i)$, BS denotes the base scale of the ith m-dimensional vector $X(i)$. The symbols 0, 1, 2, and 3 are used only as a mark for each region, whose values have no practical meaning. α is a special parameter. If α is too large during transforming original time series to symbolic time series, the detail information will be lost and the dynamic information can not be caught. Too small an α value will result in salient influence from noise. In this paper we obtain α value by use of testing method.[11] We have tested many kinds of sequences and believe that the α value should be chosen from 0.1 to 2. In time series analysis, the choice of α value is closely related with the original data. For example, the logistic chaotic series generated directly by iterative equation have no noise influence, so the range of α value can be bigger. For the real-word data with the influence of noise, the range of α value can be smaller.

We study the distributing probability about the symbolic sequences S_i of m-dimensional vector $X(i)$. The symbolic sequences S_i are made up of four symbols that are 0, 1, 2, and 3, which at most have 4^m different types of states π. For each π we determine the relative frequency.

$$p(\pi) = \#\{t|(u_t, ..., u_{t+m-1}) \text{has type } \pi\}/(N - m + 1) \quad (4)$$

where $1 \le t \le N - m + 1$, the symbol "#" means number.

The base-scale entropy with m dimensions is defined as

$$H(m) = -\sum P(\pi) \log_2 P(\pi), \quad (5)$$

where the value of $m = 3, 4, 5, 6, 7$ seems to be suitable for the convenience of calculation and it is also reasonable that N is bigger than 4^m, where the sum runs over all types of states π. This is the wave information contained in m consecutive values of the time series. It is clear that $0 \le H(m) \le \log_2 4^m$, where the lower bound is attained for a series that presents only one type among 4^m possible types, and the upper bound for a completely random series where all 4^m possible types appear in the same probability. The time series presents some sort of dynamics when $H(m) < \log_2 4^m$. The bigger entropy denotes the more complicated wave mode of m-dimensional vector and higher complexity of time series.

3. Application of base-scale entropy to a speech signal

To demonstrate how the base-scale entropy applies to real data, we consider in Fig.1(a) the utterance "Attention can be paid to some interesting speech of human" (in Chinese) spoken by a male speaker in a modal register. The signal was sampled at 8kHz, using a PC sound card without special filtering. The signal was analysed using a sliding window of $T_{win} = 500$ samples. The 34300 windows of the 4.3s signal were processed on a PC in less than several seconds, which means that the base-scale entropy can be used in real-time applications. We choose $m = 4$, and $\alpha = 0.2$ to calculate the base-scale entropy of this speech signal. The results are shown in Fig.1(b). (We test several α values from 0.1 to 0.4, and the results have no significant difference.) The signal starts and ends with noise, and the corresponding normalized entropies are close to 1, as well as for unvoiced sounds, while all voiced sounds can be recognized by decreasing the entropy. The base-scale entropy can capture the dynamical changes very well.

A well-known complexity parameter for short-time speech analysis is the zero-crossing rate (ZCR):[12]

$$\text{ZCR} = \#\{t\,|u(t)u(t+1) < 0\}/(T_{win} - 1). \quad (6)$$

Assuming that the mean of the signal is zero, ZCR is expected to be close to 0.5 for noisy data like pure fricatives (unvoiced sounds) or noisy breaks, and close to zeros for voiced speech (Fig.1(c)). Therefore, it is recommended for detection of voiced segments.

In Figs.1(b) and 1(c), at first glance, ZCR and base-scale entropy behave similarly. However, at some places the base-scale entropy seems to be a better indicator, e.g. in Fig.1(c), at points 1.6×10^4 and 2.9×10^4, ZCR misleadingly indicates low complexity.

Various experiments have shown that the base-scale entropy, in contrast to ZCR, is robust with respect to the choice of window length (even 200 samples are sufficient), to the sample frequency (5–12kHz gives very similar results). The $m = 3, 4, 5$ also have little influence on the results.

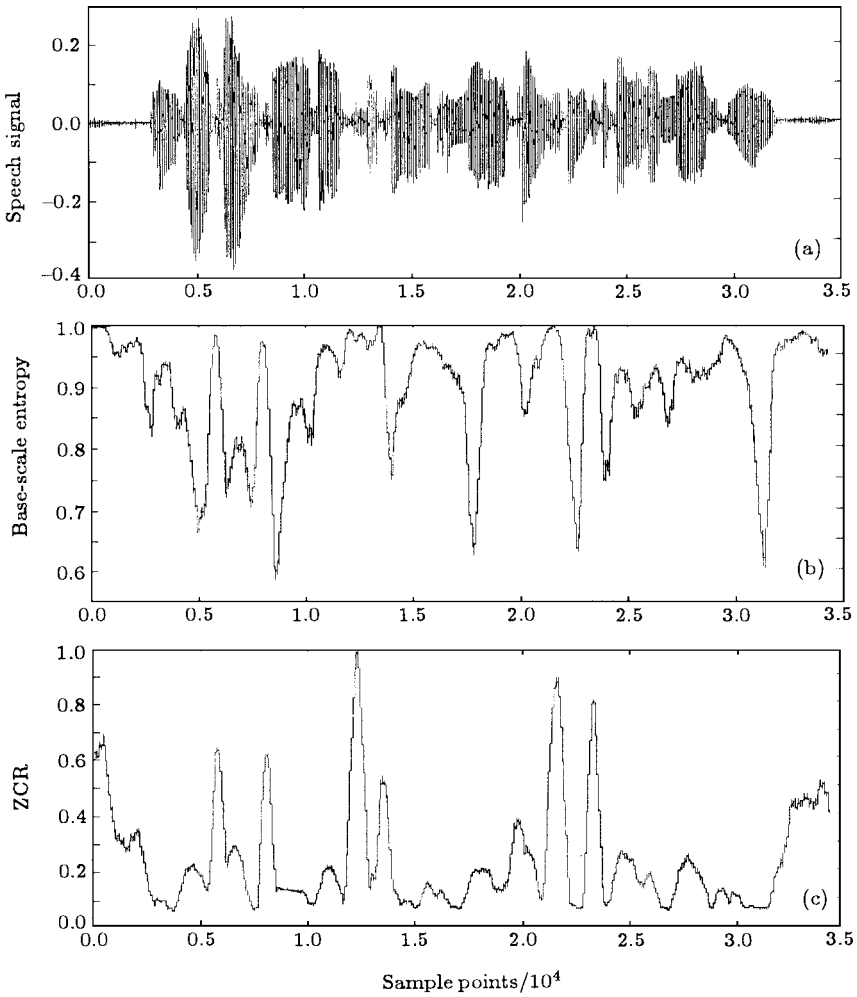


Fig.1. Sliding window complexity analysis of the utterance. (a) Speech signals. (b) The normalized base-scale entropy. (c) Zero-crossing rate.

4. Application of base-scale entropy to chaotic time series

The logistic map is given by

$$x_{i+1} = rx_i(1 - x_i), \tag{7}$$

where r denotes the control parameter ranging from 0 to 4.[13]

We generate the time series of logistic map $x(r)$, by starting from $x_0 = 0.65$ and $r_0 = 3.4$ and consistently increasing r in steps of 10^{-4} at each iteration. Figure 2(a) shows the resulting time series. The change region of parameter r is $(3.4, 4.0)$. From Fig.2(a) we can see that the logistic system presents a 2^n period series in a region of parameter r from 3.4 to near 3.57. For each r, the iterative x_i converges into an iterant series whose period is 2^n. n is increased by 1 through each bifurcate point. The dashed vertical lines in the region (3.4, 3.57) indicate the bifurcate points (Limited by the accuracy in calculation, Figure 2(a) only shows the period 2, 4, 8, 16, and do not show the period 32, 64,..., 2^∞. For the convenience of depiction, we use the approximate rational numbers to denote the parameter $r$of bifurcate points and period windows). For example, near $r = 3.565$ the logistic map is translated from period 8 to period 16, and near $r = 3.57$ the logistic map is translated from period 16 to chaotic series. The logistic system presents a chaotic series and frequently intermits by periodic windows in the region $(3.57, 4.0)$ of parameter r. Near $r = 3.61, 3.625, 3.69, 3.73, 3.83, 3.91, 3.96$, the dashed vertical lines indicate the periodic windows.

Figure 2(b) shows the base-scale entropy of logistic map. The x-axis denotes the control parameter r and the y-axis denotes the normalized base-scale entropy. We let $m = 6$, $N = 9000$, $a = 2.0BS$ in calculating entropy values. The dash-dot level line serves as a delimitation between periodic signals and chaotic series, its value is the base-scale entropy of period 16 when r falls in the region $(3.565, 3.57)$. From Fig.2(b)

we can see that when r is in the region $(3.4, 3.57)$, the logistic system presents a 2^n period series whose entropy value is lower than that of boundary. The base-scale entropy will transform at each bifurcation point and keep invariability in the same period, which quantifies the complexity of logistic map very well. The base-scale entropy changes with the parameter r in the region $(3.57, 4.0)$. When the parameter r is in periodic windows, the base-scale entropy is lower than that of the boundary. As is illustrated in Fig.2(b), the base-scale entropy can capture the dynamics complexity transformation of logistic map very well, as well as reveal the dynamics characteristic better. Trulla *et al* [14] have calculated the Lyapunov exponent of logistic map; their obtained results and the results of Fig.2(b) have a very similar appearance over the whole chaotic region. Different appearance within the periodic region $r < 3.57$ shows that the base-scale entropy quantifies the periodic series better than the Lyapunov exponent does. We also investigated the Lorenz chaotic system,[15] the obtained results showed that the base-scale entropy could completely detect the dynamical complexity changes of the Lorenz chaotic system.

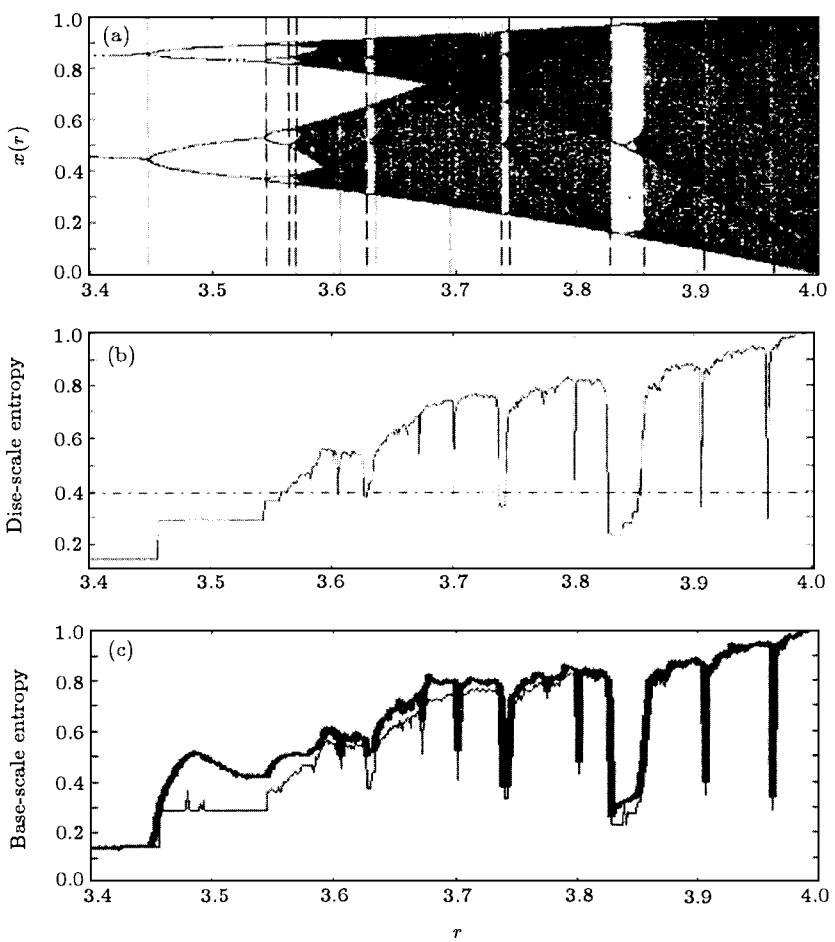


Fig.2. (a) Logistic map for varying parameter r. (b) The normalized base-scale entropy for logistic map. (c) The normalized base-scale entropy with Gaussian noise at standard deviations $s = 0.00025$ (thin line), and $s = 0.1$ (thick line).

The base-scale entropy method is effective in the presence of noise. In theory, to avoid a significant influence from noise in base-scale entropy calculation, the partition standard $\alpha \times BS$ should be chosen to be larger than most of the noise.[16] We input stochastic gaussian noises with standard deviations $s = 0.00025$ and $s = 0.1$, then calculate the base-scale entropies of logistic map correspondingly. The results are shown in Fig.2(c), from which we can see that in the period region $(3.4 < r < 3.57)$ the noise causes a small increase in entropy, but the entropy also can capture all bifurcate points. In the chaotic region $(3.57 < r < 4)$ the entropy almost does not change for $s < 0.1$. So a small noise does not change the complexity of a chaotic

signal using this method.

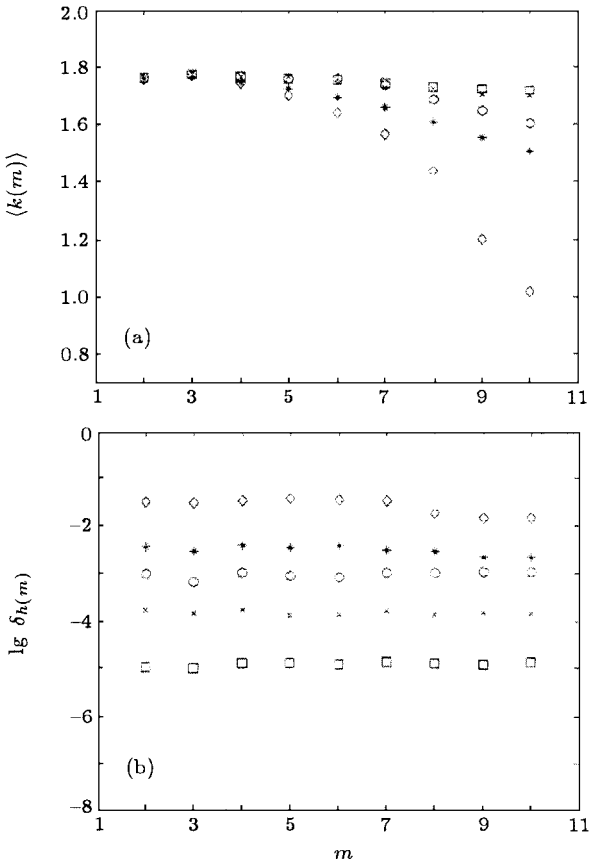


Fig.3. Logistic map (r=4). (a) Mean $\langle h(m)\rangle$ of base-scale entropy $h(m)$ for varying length $N = 10^k$ of time series. (b) The logarithm value $\lg\delta_{h(m)}$ of corresponding standard deviation δ of base-scale entropy $h(m)$, k = 2 (◇), 3 (*), 4 (o), 5 (×), 6 (□).

The case $r = 4$, conjugate to the tent map,[17] is studied for time series of various lengths. We take 540 time series of length $N = 10^k$ with $k = 2, \cdots, 6$. Figure 3 shows the results. In our experiments with chaotic time series, the base-scale entropy $H(m)$ almost increases with m linearly. Thus we divide the base-scale entropy with m dimensions by $m-1$ since comparisons start with the second value, $h(m) = H(m)/m - 1$.In Fig.3(a), we can see that $h(m)$ increases up to $m = 3$, and then decreases. When the series length N is small compared with 4^m, the base-scale entropy decreases clearly. On the contrary, when the series length N is big compared with 4^m, the base-scale entropy $h(m)$ does not change almost. So it is reasonable that N is bigger than 4^m. In Fig.3(b), the variance in estimate of $h(m)$ is rather small. Especially when $k = 5, 6$, the difference among different m is less than 10^{-4}, which indicates that the base-scale entropy $h(m)$ has a strong stability.

5. Conclusion

The base-scale entropy is an appropriate complexity measure for chaotic time series. Since the method is extremely fast and robust, it seems preferable when there are huge data sets and there need be timely capture the dynamical changes. For real-word series, "noise elimination" requires careful preprocessing of the data and fine-tuning of parameters, and the results cannot be reproduced without filtering. For our method a small noise does not essentially change the complexity of chaotic signal. So the base-scale entropy can be directly applied to arbitrary real- word time series. We illustrate our method on a practical speech signal and a theory chaotic system. The results show that the simple and easily calculated measure of base-scale entropy can be effectively used to detect qualitative and quantitative dynamical changes.

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