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Quantum tunnelling radiation of Einstein–Maxwell–Dilaton–Axion black hole*

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By taking the energy conservation and angular momentum conservation into account, the characteristics of the quantum-tunnelling radiation of Einstein–Maxwell–Dilaton–Axion black hole are studied and the result shows that the tunnelling rate of such a black hole is relevant to Bekenstein–Hawking entropy and that the obtained radiation spectrum is not pure thermal.

Keywords: Einstein–Maxwell–Dilaton–Axion black hole, energy conservation, angular momentum conservation, tunnelling rate

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1. Introduction

Inspired by Bekenstein, Hawking proved that the black hole has thermal radiation, and the spectrum is precisely thermal.[1] The finding plays significantly and actively meaningful roles in further research, and even research on the evolution of fixed stars. On the basis of it, much effort has been devoted to the static, stationary and non-stationary black holes.[2−15] There are usually two methods to investigate the Hawking thermal radiation, namely quantum field theory method and Damour–Ruffini method.[16] The similarity of these two methods is that the background spacetime of the black hole is fixed and the derived thermal radiation spectrum is pure thermal. However, for the sake of authenticity, when a black hole is radiating, the mass of the black hole varies so that the event horizon also varies, and energy conservation as well as angular momentum conservation should be taken into account during the process of radiating particles. Parikh recently put forward the tunnelling model and, in accordance with energy conservation, advocated that the position of event horizon varies after the black hole radiates particles. Moreover, Parikh also did some research on spherically symmetric black holes such as Schwarzschild and Reissner–Nordström black holes.[17−19] And the achievements show that the radiation spectrum of the black hole is not pure thermal, and further, some amendments are made to the thermal radiation of Schwarzschild and Reissner–Nordstrom black holes. As to the stationary axisymmetric black hole, the Einstein–Maxwell–Dilaton–Axion black hole, its characteristic of tunnelling radiation has not been studied previously. Accordingly, the tunnelling radiation characteristics of black hole is researched in this paper; and there are some aspects different from Parikh's results, for the angular speed of black hole $\Omega \neq 0$, and then the analysed results are relatively more meaningful. In the following parts, the spacetime line element of the Einstein–Maxwell–Dilaton–Axion black hole under a dragging coordinate system undergoes a general Painlevé coordinate transformation, then the characteristics of the tunnelling process at the event horizon is analysed, and finally the characteristics of tunnelling at the event horizon of the black hole is discussed as well.

2. General Painlevé coordinate transformation

According to Ref.[20], the spacetime line element

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of Einstein–Maxwell–Dilaton–Axion black hole is

$$\begin{aligned}\mathrm{d}s^2 =& -\frac{\Delta - a^2\sin^2\theta}{\Sigma}\mathrm{d}t_{\mathrm{EMDA}}^2 \\ & -\frac{2a\sin^2\theta[(r^2+a^2-2Dr)-\Delta]}{\Sigma}\mathrm{d}t_{\mathrm{EMDA}}\mathrm{d}\varphi \\ & +\frac{\Sigma}{\Delta}\mathrm{d}r^2+\Sigma\mathrm{d}\theta^2 \\ & +\frac{\left(r^2+a^2-2Dr\right)^2-\Delta a^2\sin^2\theta}{\Sigma}\sin^2\theta\mathrm{d}\varphi^2, \quad (1)\end{aligned}$$

where $\Delta = r^2+a^2-2mr$, $\Sigma = r^2-2Dr+a^2\cos^2\theta$; m, a, and D are the mass of black hole, angular momentum of unit mass and magnetic charge respectively. When t_{EMDA} is the time coordinate of the Einstein–Maxwell–Dilaton–Axion black hole, from Eq.(1) we can obtain the non-zero components of the inverse metric tensor

$$\begin{aligned}g^{00} =& -\frac{\left(r^2+a^2-2Dr\right)^2-\Delta a^2\sin^2\theta}{\Delta\Sigma}, \\ g^{03} =& -\frac{a\left[\left(r^2+a^2-2Dr\right)-\Delta\right]}{\Delta\Sigma}, \\ g^{11} =& \frac{\Delta}{\Sigma}, \\ g^{22} =& \frac{1}{\Sigma}, \\ g^{30} =& -\frac{a\left[\left(r^2+a^2-2Dr\right)-\Delta\right]}{\Delta\Sigma}, \\ g^{33} =& \frac{\Delta-a^2\sin^2\theta}{\Sigma\Delta\sin^2\theta}. \quad (2)\end{aligned}$$

From the null super-surface equation, we can obtain the event horizon of the black hole

$$r_{\mathrm{H}} = m+\sqrt{m^2-a^2}. \quad (3)$$

From the equation $g_{00}=0$, we can derive the infinite red-shift surface

$$r_+^{\infty} = m+\sqrt{m^2-a^2\cos^2\theta}. \quad (4)$$

By comparison, the event horizon (3) is not coincident with the infinite red-shift surface (4), and then the geometrical optics limit is not valid. To carry out dragging transformation on Eq.(1) we can obtain

$$\begin{aligned}\frac{\mathrm{d}\varphi}{\mathrm{d}t_{\mathrm{EMAD}}} =& -\frac{g_{03}}{g_{33}} \\ =& \frac{a\left[\left(r^2+a^2-2Dr\right)-\Delta\right]}{\left(r^2+a^2-2Dr\right)^2-\Delta a^2\sin^2\theta}, \quad (5)\end{aligned}$$

and the line element (1) is transformed to

$$\mathrm{d}s^2 = \hat{g}_{00}\mathrm{d}t_{\mathrm{EMDA}}^2+\frac{\Sigma}{\Delta}\mathrm{d}r^2+\Sigma\mathrm{d}\theta^2, \quad (6)$$

where

$$\begin{aligned}\hat{g}_{00} =& g_{00}-\frac{g_{03}^2}{g_{33}} \\ =& -\frac{\Delta\Sigma}{\left(r^2+a^2-2Dr\right)^2-\Delta a^2\sin^2\theta}. \quad (7)\end{aligned}$$

In the dragging coordinate system, from Eq.(6) we can derive the event horizon of the black hole which is in coincidence with the infinite red-shift surface

$$r_{\mathrm{H}} = r_+^{\infty} = m+\sqrt{m^2-a^2}. \quad (8)$$

But the line element (6) still has a coordinate singularity. Make the general Painlevé coordinate transformation[21,22]

$$\mathrm{d}t_{\mathrm{EMDA}} = \mathrm{d}t+F(r,\theta)\mathrm{d}r+G(r,\theta)\mathrm{d}\theta, \quad (9)$$

under the integrability condition

$$\frac{\partial F(r,\theta)}{\partial\theta} = \frac{\partial G(r,\theta)}{\partial r}. \quad (10)$$

Substituting Eq.(9) into Eq.(6), we have

$$\begin{aligned}\mathrm{d}s^2 =& \hat{g}_{00}\mathrm{d}t^2+2\hat{g}_{00}F(r,\theta)\mathrm{d}t\mathrm{d}r \\ & +2\hat{g}_{00}G(r,\theta)\mathrm{d}t\mathrm{d}\theta \\ & +\left[\hat{g}_{00}F^2(r,\theta)+\frac{\Sigma}{\Delta}\right]\mathrm{d}r^2 \\ & +2\hat{g}_{00}F(r,\theta)G(r,\theta)\mathrm{d}r\mathrm{d}\theta \\ & +\left[\hat{g}_{00}G^2(r,\theta)+\Sigma\right]\mathrm{d}\theta^2. \quad (11)\end{aligned}$$

Considering the flat Euclidean space in radial terms, we have

$$\hat{g}_{00}F^2(r,\theta)+\frac{\Sigma}{\Delta} = 1. \quad (12)$$

Substituting Eq.(12) into Eq.(11), we obtain

$$\begin{aligned}\mathrm{d}s^2 =& \hat{g}_{00}\mathrm{d}t^2+2\sqrt{\hat{g}_{00}\left(1-\frac{\Sigma}{\Delta}\right)}\mathrm{d}t\mathrm{d}r \\ & +2\hat{g}_{00}G(r,\theta)\mathrm{d}t\mathrm{d}\theta+\mathrm{d}r^2 \\ & +2\sqrt{\hat{g}_{00}\left(1-\frac{\Sigma}{\Delta}\right)}G(r,\theta)\mathrm{d}r\mathrm{d}\theta \\ & +\left[\hat{g}_{00}G^2(r,\theta)+\Sigma\right]\mathrm{d}\theta^2. \quad (13)\end{aligned}$$

Substituting Eq.(13) into Landau's condition of coordinate clock synchronization

$$\frac{\partial}{\partial x^i}\left(-\frac{g_{0j}}{\hat{g}_{00}}\right) = \frac{\partial}{\partial x^j}\left(-\frac{g_{0i}}{\hat{g}_{00}}\right), \quad (14)$$

we obtain

$$\frac{\partial F(r,\theta)}{\partial\theta} = \frac{\partial G(r,\theta)}{\partial r}. \quad (15)$$

The condition is in accordance with Eq.(10); then the Painlevé coordinate system satisfies Landau's condition of coordinate clock synchronization. Through coordinate transformation in Eq.(9), we obtain the spacetime expressed by line element (13) with the following advantages: there does not exist coordinate singularity; the outer infinite red-shift surface is in coincidence with the event horizon; it satisfies Landau's condition of coordinate clock synchronization. We consider uncharged tunnelling radiation coming from the black hole. From Eq.(13) we can have the radial null geodesics

$$\dot{r} = \frac{\mathrm{d}r}{\mathrm{d}t} = \frac{\pm\Sigma - \sqrt{\Sigma\left(\Sigma - \Delta\right)}}{\sqrt{\left(r^2 + a^2 - 2Dr\right)^2 - \Delta a^2 \sin^2\theta}}, \tag{16}$$

where "±" signs can be identified with outgoing (ingoing) radial motion, namely "+" corresponds to the outgoing geodesic and "–" corresponds to the ingoing geodesic.

3. Quantum tunnelling characteristics

We can consider the picture of a pair of virtual particles spontaneously created inside the horizon, where the positive energy virtual particle can tunnel out and the negative energy particle is absorbed by the black hole. In the following discussion, we consider the particle's self-gravitation and the tunnelling particle as a shell (an ellipsoid shell) of energy ω and angular momentum ωa, the mass and the angular momentum of the black hole will become $(m-\omega)$ and $(m-\omega)a$ respectively; at this critical moment the mass m in the radial null geodesics and the line element (13) will be replaced by $(m-\omega)$, and the event horizon is

$$r'_{\mathrm{H}} = (m-\omega) + \sqrt{(m-\omega)^2 - a^2}, \tag{17}$$

and the dragging angular velocity of the black hole is

$$\Omega'_{\mathrm{H}} = \frac{a}{2(m-\omega)^2 + 2(m-\omega)\sqrt{(m-\omega)^2 - a^2} - 2D(m-\omega) - 2D\sqrt{(m-\omega)^2 - a^2}}. \tag{18}$$

Since the event horizon is in coincidence with the infinite red-shift surface, the geometrical limit is approximately reliable. From WKB approximation can obtain the relation between the action and the tunnelling rate of particles[23,24]

$$\Gamma \sim \mathrm{e}^{-2\mathrm{Im}S}, \tag{19}$$

where S is the action of particles with

$$S = \int_{t_i}^{t_f} L\left(r, \dot{r}, \varphi, \dot{\varphi}, t\right)\mathrm{d}t, \tag{20}$$

Here $L\left(r, \dot{r}, \varphi, \dot{\varphi}, t\right)$ is a Lagrangian function. There is no φ in the dragging coordinate system, so φ is a negligible coordinate. In order to eliminate the freedom of φ, we have

$$S = \int_{t_i}^{t_f} \left(L - P_\varphi \dot{\varphi}\right)\mathrm{d}t. \tag{21}$$

So the imaginary part of the action can be expressed as

$$\begin{aligned}\mathrm{Im}S =& \mathrm{Im}\left[\int_{r_i}^{r_f} P_r \mathrm{d}r - \int_{\varphi_i}^{\varphi_f} P_\varphi \mathrm{d}\varphi\right] \\ =& \mathrm{Im}\left[\int_{r_i}^{r_f}\int_0^{p_r} \mathrm{d}P'_r \mathrm{d}r - \int_{\varphi_i}^{\varphi_f}\int_0^{p_\varphi} \mathrm{d}P'_\varphi \mathrm{d}\varphi\right].\end{aligned} \tag{22}$$

According to the Hamilton equation we have

$$\dot{r} = \left.\frac{\mathrm{d}H}{\mathrm{d}P_r}\right|_{(r;\varphi;p_\varphi)} = \frac{\mathrm{d}\left(m-\omega\right)}{\mathrm{d}p_r} = -\frac{\mathrm{d}\omega}{\mathrm{d}p_r}, \tag{23}$$

$$\dot{\varphi} = \left.\frac{\mathrm{d}H}{\mathrm{d}P_\varphi}\right|_{(\varphi;r,p_r)} = a\,\Omega'_{\mathrm{H}}\frac{\mathrm{d}\left(m-\omega\right)}{\mathrm{d}p_\varphi} = -a\,\Omega'_{\mathrm{H}}\frac{\mathrm{d}\omega}{\mathrm{d}p_\varphi}. \tag{24}$$

Substituting Eqs.(23) and (24) into Eq.(22), we have

$$\begin{aligned}\mathrm{Im}S =& \mathrm{Im}\left[\int_m^{m-\omega}\int_{r_i}^{r_f}\frac{\mathrm{d}r}{\dot{r}}\mathrm{d}\left(m-\omega'\right) - \int_m^{m-\omega}\int_{r_i}^{r_f} a\,\Omega'_{\mathrm{H}}\frac{\mathrm{d}r}{\dot{r}}\mathrm{d}\left(m-\omega'\right)\right] \\ =& \mathrm{Im}\left[\int_m^{m-\omega}\int_{r_i}^{r_f}\left(1 - a\,\Omega'_{\mathrm{H}}\right)\frac{\sqrt{\left(r^2+a^2-2Dr\right)^2 - \Delta' a^2\sin^2\theta}}{\Sigma - \sqrt{\Sigma\left(\Sigma - \Delta'\right)}}\mathrm{d}r\mathrm{d}\left(m-\omega'\right)\right] \\ =& \pi\left[m^2 - (m-\omega)^2 + m\sqrt{m^2-a^2} - (m-\omega)\sqrt{(m-\omega)^2 - a^2} + 2D\sqrt{(m-\omega)^2 - a^2}\right. \\ & \left. -2D\sqrt{m^2-a^2} + 2D(m-\omega) - 2Dm\right],\end{aligned} \tag{25}$$

where

$$\begin{aligned}
&\Delta' = r^2 + a^2 - 2\left(m - \omega'\right) r = \left(r - r'_{\mathrm{H}}\right)\left(r - r'_{-}\right), \\
&r_i = r_{\mathrm{H}} = m + \sqrt{m^2 - a^2}, \\
&r_f = r'_{\mathrm{H}} = (m - \omega) + \sqrt{(m - \omega)^2 - a^2}, \\
&r'_{-} = (m - \omega) - \sqrt{(m - \omega)^2 - a^2}.
\end{aligned} \tag{26}$$

then the tunnelling rate is

$$\begin{aligned}
\Gamma \sim & \mathrm{e}^{-2\mathrm{Im}S} \\
= & \mathrm{e}^{-2\pi[m^2-(m-\omega)^2+m\sqrt{m^2-a^2}-(m-\omega)\sqrt{(m-\omega)^2-a^2}+2D\sqrt{(m-\omega)^2-a^2}-2D\sqrt{m^2-a^2}+2D(m-\omega)-2Dm]} \\
= & \mathrm{e}^{\Delta S_{\mathrm{BH}}},
\end{aligned} \tag{27}$$

in which $\Delta S_{\mathrm{BH}} = S_{\mathrm{BH}}(m-\omega) - S_{\mathrm{BH}}(m)$; and $S_{\mathrm{BH}}(m-\omega)$, $S_{\mathrm{BH}}(m)$ represent the Bekenstein–Hawking entropies of the black hole after and before radiation.

4. Conclusion

The radiation of black hole causes the spacetime background geometry to be not constant. Because of the energy conservation and angular momentum conservation, the event horizon of a black hole will contract when some particles tunnel out. The two turning points, for the pre-contraction and post-contraction occurrence, are the two turning points of the radial barrier. The tunnelling rate of particles is related to m, a and D. When $D = a = 0$, the tunnelling rate $\Gamma \sim \exp\left[-8\pi m\omega\left(1 - \dfrac{\omega}{2m}\right)\right]$ is the quantum-tunnelling rate of the Schwarzschild black hole, where only when the item ω^2 is ignored can we obtain a consistent pure thermal spectrum of Hawking radiation in the Schwarzschild black hole. The tunnelling radiation characteristics of Einstein–Maxwell–Dilaton–Axion black hole are investigated through the above-mentioned methods and the result not only consists of the exact expression of tunnelling rate, but also works as a good amendment to Hawking pure thermal spectrum.

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