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Generation of maximally entangled states in a two-component Bose-Einstein condensate

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We study analytically the generation of maximally entangled states (MESs) formed by a two-component Bose–Einstein condensate (BEC) trapped in an adiabatically driven single potential well. Under the condition of the linear interaction controlled by a driven field being much stronger than the effective nonlinear interaction between the components, MESs, as some particular cases of superpositions of spin coherent states (SSCS), may emerge periodically along with not only time evolution but also the equidifferent change of the linear coupling strength at a particular time.

Keywords: maximally entangled state, spin coherent states, Bose-Einstein condensates, nonlinearity
PACC: 0365, 7450, 0530J, 4660D

1. Introduction

MESs are of great importance in fundamental physics as well as in applications in quantum teleportation.[1−3] The two-component atomic BEC has manifested its outstanding advantages such as its intrinsic nonlinearity[4,5] but coherently controlled linearity, extremely long coherent time, initially pure and separable states prepared, etc.[6−12] MESs are very closely related to atom-macroscopic quantum superposition states. It has been proved that the quantum superposition of two macroscopically distinct coherent states is actually a maximally entangled state (MES),[13] which implies that MESs may be easily produced by using the same schemes devised to generate some coherent superpositions of different quantum states.[8,14,15] In this paper, we describe a method to generate a SSCS with periodicity formed by a two-component atomic BEC trapped in a single potential well consisting of two internal states which are coupled by a spatially uniform radiation field, then show explicitly that MESs, as some particular cases of SSCS, may appear periodically along with not only time evolution but also the equidifferent change of the linear coupling strength at a particular time, if the linear interaction is strong enough compared with the effective nonlinear interaction between the components.

2. Model and solution

We consider for simplicity a so-called two-model describing the coupled two-component (A and B) condensate trapped in a single potential well. Its second quantized Hamiltonian is($\hbar = 1$)[7,10,16]

$$\begin{aligned}\hat{H} =&\omega_a \hat{a}^+\hat{a} + \omega_b \hat{b}^+\hat{b} \\ &+\frac{\eta_a}{2}\hat{a}^+\hat{a}^+\hat{a}\hat{a} + \frac{\eta_b}{2}\hat{b}^+\hat{b}^+\hat{b}\hat{b} + \chi\hat{a}^+\hat{a}\hat{b}^+\hat{b} \\ &+G\left(\hat{a}^+\hat{b}\mathrm{e}^{\mathrm{i}\varphi(t)} + \hat{a}^+\hat{b}\mathrm{e}^{-\mathrm{i}\varphi(t)}\right),\end{aligned} \tag{1}$$

where a and b are the annihilation operators for components A and B, respectively,

$$\omega_i = \int \mathrm{d}^3 r \phi_i^*(r)\left[-\frac{\hbar^2}{2m}\Delta^2 + V_i(r)\right]\phi_i(r) \quad (i = a, b) \tag{2}$$

is the single-particle energy of mode i, where $V_i(r)$ is the trapping potential, $\phi_i(r)$ represents the condensate wave function for mode i, η_i denotes the coupling constant of two-atom collision in each atom species, and χ is the coupling constant of atomic interspecies collisions. Finally, G and $\varphi(t)$ describe the strength and phase of the linear coupling between components, respectively, which can be controlled by a driven field. Here the phase $\varphi(t)$ is assumed to be

$$\varphi(t) = \begin{cases} \varphi_0 & t = 0, \\ 0 & t \neq 0. \end{cases} \tag{3}$$

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Obviously, the adiabatic condition, i.e. $\dfrac{\mathrm{d}\varphi}{\mathrm{d}t}=0$ and $\dfrac{\mathrm{d}\lambda}{\mathrm{d}t}=0$ is tenable.

The analysis of Eq.(1) is greatly simplified by introducing the following angular momentum operators

$$\begin{aligned}\hat{J}_x&=\frac{1}{2}\left(\hat{a}^+\hat{b}+\hat{b}^+\hat{a}\right),\\ \hat{J}_y&=\frac{1}{2\mathrm{i}}\left(\hat{a}^+\hat{b}-\hat{b}^+\hat{a}\right),\\ \hat{J}_z&=\frac{1}{2}\left(\hat{a}^+\hat{a}-\hat{b}^+\hat{b}\right),\end{aligned}\tag{4}$$

and the Casimir invariant

$$\hat{J}^2=\frac{\widehat{N}}{2}\left(\frac{\widehat{N}}{2}+1\right),\tag{5}$$

where $\hat{N}=\hat{a}^+\hat{a}+\hat{b}^+b^+$ is the total number operator, which is a conserved quantity. Thus the Hamiltonian may be rewritten as[16]

$$\hat{H}=\omega_0\hat{J}_z+q\hat{J}_z^2+G(t)\left(\hat{J}_+\mathrm{e}^{\mathrm{i}\varphi(t)}+\hat{J}_-\mathrm{e}^{-\mathrm{i}\varphi(t)}\right),\tag{6}$$

with $\omega_0=\omega_a-\omega_b+(N-1)(\eta_a-\eta_b)/2$, the effective nonlinear interaction strength being $q=(\eta_a+\eta_b)/2-\chi$, and $\widehat{J}_\pm=\widehat{J}_x\pm\mathrm{i}\widehat{J}_y$.

The time evolution of the quantum state is governed by the Schrödinger equation

$$\mathrm{i}\mathrm{d}/\mathrm{d}t\,|\,\Psi(t)\rangle=\hat{H}\,|\,\Psi(t)\rangle.\tag{7}$$

Considering the $\{j,m\}$ subspace spanned by $|m\rangle=|j,m\rangle$ with j denoting the quantum number of angular momentum for the N-atom BEC, we have $j=\dfrac{N}{2}$ and $m=\left\{-\dfrac{N}{2},-\dfrac{N}{2}+1,...,\dfrac{N}{2}-1,\dfrac{N}{2}\right\}$. In this paper, we choose N as an even number, that is to say, j is an integer.

Since the two-component BEC is experimentally created by coupling two hyperfine states with a radiation field, it has been shown that the prepared initial state can be a particular case of spin coherent states (SCS) which is defined as[17]

$$\begin{aligned}|\theta,\phi\rangle&=\hat{\Omega}(\theta,\phi)\,|j,j\rangle\\ &=\sum_{m=-j}^{j}\left(\begin{array}{c}2j\\ j+m\end{array}\right)^{1/2}\left(\cos\frac{\theta}{2}\right)^{j+m}\\ &\quad\times\left(\sin\frac{\theta}{2}\right)^{j-m}\mathrm{e}^{\mathrm{i}(j-m)\phi}\,|j,m\rangle,\end{aligned}\tag{8}$$

where

$$\hat{\Omega}=\exp[\theta/2\left(\widehat{J}_-\mathrm{e}^{\mathrm{i}\phi}-\widehat{J}_+\mathrm{e}^{-\mathrm{i}\phi}\right)].\tag{9}$$

In order to obtain the solution of Eq.(7), we adopt a generalized gauge transformation in terms of the time-dependent unitary transformation[18] which is defined as

$$\hat{R}=\exp[\lambda/2\left(\hat{J}_-\mathrm{e}^{-\mathrm{i}\varphi(t)}-\hat{J}_+\mathrm{e}^{\mathrm{i}\varphi(t)}\right)].\tag{10}$$

Then the time-dependent Schrödinger equation is covariant under the gauge transformation so that

$$\mathrm{i}\mathrm{d}/\mathrm{d}t\,|\,\Psi'(t)\rangle=\hat{H}'\,|\,\Psi'(t)\rangle,\tag{11}$$

where

$$\hat{H}'=\hat{R}\hat{H}\hat{R}^+,\tag{12}$$

and

$$|\,\Psi'(t)\rangle=\hat{R}(t)\,|\,\Psi(t)\rangle.\tag{13}$$

For a single trap a reasonable condition is $\omega_0=0$ $\left(\lambda=-\dfrac{\pi}{2}\right)$.[16] With the help of time-dependent unitary transformation (10), Eq.(12) can be simplified into the form

$$\hat{H}'=2G\hat{J}_x-q/2\hat{J}_z^2,\tag{14}$$

where the two-photon transition terms (proportional to $\hat{J}_\pm^2\mathrm{e}^{\pm\mathrm{i}2\varphi(t)}$) have been neglected, which is good enough inaccuracy for the condition $q<<G$.[16] The state (13) can be expressed as follows:

$$|\,\Psi'(t)\rangle=\exp[\mathrm{i}t\left(-2G\hat{J}_z+\frac{q}{2}\hat{J}_z^2\right)]R\left(-\frac{\pi}{2},0\right)|\,\Psi(0)\rangle,\tag{15}$$

where $|\,\Psi(0)\rangle=|\theta,\phi\rangle$ as mentioned above.

Defining a scaled time $\tau=qt/2$ and a running frequency

$$\theta_m=m^2-\varepsilon m,\tag{16}$$

where $\varepsilon=4G/q(\varepsilon>>1)$, the state (15) can be rewritten as

$$\begin{aligned}|\,\Psi'(t)\rangle=\sum_{m=-j}^{j}&\exp[\mathrm{i}\tau\theta_m]\left(\begin{array}{c}2j\\ j+m\end{array}\right)^{1/2}\left(\cos\frac{\theta_0}{2}\right)^{j+m}\\ &\times\left(\sin\frac{\theta_0}{2}\right)^{j+m}\mathrm{e}^{-\mathrm{i}(j-m)\varphi_0}\,|j,j\rangle,\end{aligned}\tag{17}$$

where $\theta_0=\theta-\pi/2$, $\varphi_0\equiv-\phi$.

We note that the state (17) differs from a conventional spin coherent state (8). According to Kuang's idea,[8,19,20] some states such as generalized coherent states, or generalized squeezed states can always be represented as a continuous sum of entangled coherent states or entangled squeezed states, respectively, because they include a common factor $\mathrm{e}^{\mathrm{i}\varphi}$. Here the

SCS also has such a feature, so the state (17) can also be represented as a continuous sum of SCS

$$|\Psi'(\tau)\rangle = \int_0^{2\pi} \frac{\mathrm{d}\varphi}{2\pi} g(\varphi) |\theta_0, \varphi - \varphi_0\rangle, \tag{18}$$

where the phase function $g(\varphi)$ is given by

$$g(\varphi) = \exp[\mathrm{i}\tau\theta_m - \mathrm{i}(j-m)\varphi]. \tag{19}$$

Here we consider a special situation where ε is taken as a value of nonzero integer that can be chosen by determination of the strength of the linear coupling between the components G controlled by the driven field, and $\varepsilon - j$ is an even number at some particular scaled times $\tau_M = \pi M/j\,(M = 1, 2, \cdots)$. In this case, the phase function $g(\varphi)$ is a periodic function with respect to m with a period j, what is more, the $g(\varphi)$ function is also of double periodicity with respect to both M and ε with the same period N. Then the state (18) can become a discrete SSCS which can be expressed as follows:

$$|\Psi'(\tau_M)\rangle = \sum_{r=1}^{j} C_r |\theta_0, \varphi_r - \varphi_0\rangle, \tag{20}$$

where the running phase φ is defined by

$$\varphi_r = \frac{2\pi r}{j} \quad (r = 1, 2, \cdots, j). \tag{21}$$

From Eqs.(17) and (20) we can find the following equation to determine the coefficients C_r:

$$\sum_{r=1}^{j} C_r \exp\left\{-\frac{\mathrm{i}\pi}{j}[M\theta_m + 2mr]\right\} = 1. \tag{22}$$

Carrying out summations over m on both sides of the above equation from 1 to j, and taking into account the normalization condition $\sum_{r=1}^{j} C_r C_r^* = 1$, we obtain the coefficients C_r given by

$$C_r = \frac{1}{j}\sum_{m=1}^{j} \exp\left\{\frac{\mathrm{i}\pi}{j}[M\theta_m + 2mr]\right\}. \tag{23}$$

Obviously, C_r is of period N with respect to both M and ε. Hence, the state (20) is also a periodic function.

Finally, the solution of Schrödinger equation (1) at the time τ_M can be expressed as

$$\begin{aligned}|\Psi(t_M)\rangle =& R^+\left(-\frac{\pi}{2}, \tau_M\right)|\Psi'(\tau_M)\rangle \\ =& \frac{1}{\sqrt{2}}\sum_{r=1}^{j} C_r\left(\left|\theta - \frac{\pi}{2}, \varphi_r - \varphi_0\right\rangle\right. \\ &\left. + \mathrm{e}^{-\mathrm{i}N\varphi_0}\left|\theta + \frac{\pi}{2}, \varphi_0 - \varphi_r\right\rangle\right),\end{aligned} \tag{24}$$

which is the SSCS with a period N with respect to both M and ε.

Now we consider the case of N=8 and ε=100. With the help of the following relations:

$$|j\rangle_z = |0, \varphi\rangle, |-j\rangle_z = |\pi, \varphi\rangle, \tag{25}$$

$$\begin{aligned}|j\rangle_x &= \left|\frac{\pi}{2}, 0\right\rangle = \left|-\frac{\pi}{2}, \pm\pi\right\rangle, \\ |-j\rangle_x &= \left|\frac{\pi}{2}, 0\right\rangle = \left|-\frac{\pi}{2}, \pm\pi\right\rangle,\end{aligned} \tag{26}$$

$$\begin{aligned}|j\rangle_y &= \left|-\frac{\pi}{2}, \frac{\pi}{2}\right\rangle = \left|\frac{\pi}{2}, -\frac{\pi}{2}\right\rangle, \\ |-j\rangle_y &= \left|\frac{\pi}{2}, \frac{\pi}{2}\right\rangle = \left|-\frac{\pi}{2}, -\frac{\pi}{2}\right\rangle,\end{aligned} \tag{27}$$

where $\hat{J}_i|\pm j\rangle_i = j_i|j\rangle_i$, $(i = x, y, z)$, and using Eqs.(16) and (23) we can obtain the results as follows:

(1) At the time $\tau = \pi/j$, we may obtain $c_1 = c_3 = \frac{1}{2}$ and $c_2 = -c_4 = \frac{1}{2}\mathrm{e}^{\mathrm{i}\frac{\pi}{4}}$, resulting in the state

$$|\Psi(\tau_1)\rangle = \frac{1}{\sqrt{2}}\left(|j\rangle_y + |-j\rangle_y\right) \tag{28}$$

for the initial state $|\Psi(0)\rangle = |j\rangle_z$, i.e. $\varphi_0 = \frac{\pi}{2}, \theta = 0$;

(2) At the time $\tau = 2\pi/j$, $c_1 = c_3 = 0$ and $c_2 = c_4^* = \frac{1}{2}(1 - i)$ are obtained, and the state

$$|\Psi(\tau_1)\rangle = \frac{1}{\sqrt{2}}(|j\rangle_x + |-j\rangle_x), \tag{29}$$

for the initial state $|\Psi(0)\rangle = |j\rangle_z$, i.e. $\varphi_0 = 0, \theta = 0$, and

$$|\Psi(\tau_1)\rangle = \frac{1}{\sqrt{2}}(|j\rangle_z + |-j\rangle_z) \tag{30}$$

for the initial state $|\Psi(0)\rangle = |j\rangle_x$ i.e. $\varphi_0 = 0, \theta = \frac{\pi}{2}$ can be produced. These states (28–30) above are called MESs because if one individual spin is found to be aligned along the negative coordinate axis or the positive coordinate axis, so are all other spins. These states are also called Schrödinger cat states since they all come from a coherent superposition of two macroscopically distinct states—one has all the population in component A and the other in component B.[7] On the other hand, due to the periodicity of the wave function in Eq.(24), we may expect that under the condition $q << G$, such MESs can be generated at the time τ_{M+kN} $(k = 1, 2, \cdots)$ for different values of $\varepsilon + kN (k = 1, 2, \cdots)$, that is to say, the MESs may appear periodically not only in the process of time

evolution, but also at a particular time along with the equidifferent increase in the value of ε, i.e. the strength of the linear coupling G controlled by a driven field, if the effective nonlinear interaction strength q remains unchanged. This is the striking feature of this scheme.

3. Conclusion

We have proposed a scheme to generate a periodic SSCS in a two-component BEC trapped in a single potential well under the condition of $q << G$. As some particular cases of SSCS, MESs may appear and present a dual periodicity with respect to both time and the strength of the linear interaction controlled by driven field. This scheme provides a simple and practical way for further applications of quantum MESs.

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