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Observer-based passive control for uncertain linear systems with delay in state and control input

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This paper deals with the robust passivity synthesis problem for a class of uncertain linear systems with time-varying delay in state and control input. The parameter uncertainties are norm-bounded and allowed to appear in all matrices of the model. The problem aims at designing an observer-based dynamic output-feedback controller that robustly stabilizes the uncertain systems and achieves the strict passivity of closed-loop systems for all admissible uncertainties. By converting the problem at hand into a class of strictly passive control problem for a parameterized system, the explicit solution is established and expressed in terms of a linear matrix inequality. A numerical example is provided to demonstrate the validity of the proposed approach.

Keywords: passive control, time-varying delay, observer, parametric uncertainties, linear matrix inequality (LMI)
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1. Introduction

Passivity theory roots in circuit theory[1] and plays an important role in both electrical networks and control systems.[2] It has received a lot of attention from the control community since the 70s.[3−6] The fact that passivity and stability are closely related[2,3,7] supplies a new approach to solving stabilization problems. Moreover, passivity-based control has important robustness properties. It is well known that when a passive system is connected with some uncertainty or nonlinearity which can be characterized by a passive system, in a negative-feedback configuration, the closed-loop system is guaranteed to be stable,[2] which may simplify many robust control problems. In addition, passivity theory also provides an energy-based analysis approach,[4] which is easier to perform when dealing with electrical circuits and mechanical systems.

On the other hand, time delay and parameter uncertainties often cause instabilities in various engineering systems. Especially, the existence of input delay may be the source of instability or serious deterioration in the performance of the closed-loop systems. The issues of stabilization and H_∞ control design for input-delay systems have been considerable interest over the past years[8−13] and the references cited therein. However, the passive control of time-delayed systems has not been fully investigated. For systems with constant state-delay and parameter uncertainties, or only including parameter uncertainties, robust passivity results related via state feedback and output feedback have been presented,[14,15] but they did not deal with parameter uncertain systems with time-varying delay in state and control input.

Inspired by Ref.[14], in this paper we deal with the observer-based passive control problem for a class of uncertain linear time-delayed systems with time-varying delay in state and control input.The parameter uncertainties are norm-bounded. The problem aims at designing a robust observer-based dynamic output-feedback controller such that the closed-loop system is strongly robustly stable with strictly passive for all admissible parameter uncertainties. Firstly, we address the problem of strongly robust stability with strict passivity (SP) for a class of linear time-delayed systems subject to parameter uncertainty, and convert it into a class of strictly passive control problem with scaling parameters; and then we investigate the

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problem of designing robustly passive observer-based dynamic controller for the above class of uncertain time-delayed systems. Again a scaling parameterization approach is developed to solve this problem, and an uncertainty-independent solution is established and expressed in a linear matrix inequality. Finally, an example is given to illustrate the developed theory.

Fact 1 (Schur complement) Given a constant symmetric matrix

$$S = \begin{pmatrix} S_{11} & S_{12} \\ S_{12}^{\mathrm{T}} & S_{22} \end{pmatrix},$$

where S_{11} is $r \times r$ dimensional, the following statements are equivalent:

$$S < 0, \tag{1}$$

$$S_{11} < 0, \quad S_{22} - S_{12}^{\mathrm{T}} S_{11}^{-1} S_{12} < 0, \tag{2}$$

$$S_{22} < 0, \quad S_{11} - S_{12} S_{22}^{-1} S_{12}^{\mathrm{T}} < 0. \tag{3}$$

Fact 2 (Xie 1992) For any real matrices Σ_1, Σ_2 and Σ_3 with appropriate dimensions, such that $\Sigma_3 = \Sigma_3^{\mathrm{T}}$, it follows that

$$\Sigma_3 + \Sigma_1 \Delta(t) \Sigma_2 + \Sigma_2^{\mathrm{T}} \Delta(t)^{\mathrm{T}} \Sigma_1^{\mathrm{T}} < 0, \tag{4}$$

$$\forall t, \forall \Delta : \Delta(t)^{\mathrm{T}} \Delta(t) \le I, \tag{5}$$

if and only if for some $\varepsilon > 0$,

$$\Sigma_3 + \begin{pmatrix} \varepsilon^{-1} \Sigma_2^{\mathrm{T}} & \varepsilon \Sigma_1 \end{pmatrix} \begin{pmatrix} \varepsilon^{-1} \Sigma_2 \\ \varepsilon \Sigma_1^{\mathrm{T}} \end{pmatrix} < 0. \tag{6}$$

2. Strongly robust stability with SP of time-delayed systems

The class of linear time-delayed systems under consideration are represented by

$$\begin{aligned} (\Sigma_\Delta) : \dot{x}(t) =& (A_{1\Delta}(t))x(t) + (A_{2\Delta}(t))x(t - d(t)) \\ &+ (B_{1\Delta}(t))u(t - d(t)) \\ &+ (B_{2\Delta}(t))w(t), \end{aligned} \tag{7}$$

$$\begin{aligned} z(t) =& (C_{11\Delta}(t))x(t) + (C_{12\Delta}(t))x(t - d(t)) \\ &+ (D_{11\Delta}(t))u(t - d(t)) \\ &+ (E_{11\Delta}(t))w(t), \end{aligned} \tag{8}$$

$$x(t) = 0, t \le 0, \tag{9}$$

$$y(t) = C_{21} x(t), \tag{10}$$

where $x \in \Re^n$ is the state, $w \in \Re^q$ is the exogenous input, $z \in \Re^q$ is the output, $u \in \Re^m$ is the control input and $y \in \Re^r$ is the measured output and $d(t)$ is an arbitrary differentiable function satisfying

$$0 \le d(t) \le \infty, \quad \dot{d}(t) \le m < 1. \tag{11}$$

In addition, The uncertain matrices are given by

$$\begin{aligned} &\begin{pmatrix} A_{1\Delta} & A_{2\Delta} & B_{1\Delta} & B_{2\Delta} \\ C_{11\Delta} & C_{12\Delta} & D_{11\Delta} & E_{11\Delta} \end{pmatrix} \\ &= \begin{pmatrix} A_1 & A_2 & B_1 & B_2 \\ C_{11} & C_{12} & D_{11} & E_{11} \end{pmatrix} \\ &\quad + \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} F(t) \begin{pmatrix} E_1 & E_2 & E_3 & E_4 \end{pmatrix}, \end{aligned} \tag{12}$$

where $A_1, A_2, B_1, B_2, C_{11}, C_{12}, D_{11}$ and E_{11} are constant matrices with appropriate dimensions which represent the nominal plant, H_1, H_2, E_1, E_2, E_3 and E_4 are structural matrices of uncertainties, and $F \in \Re^{\alpha \times \beta}$ is an unknown time variable matrix with Lebesgue measurable elements satisfying

$$F^{\mathrm{T}}(t) F(t) \le I, \forall t. \tag{13}$$

The unforced nominal systems of systems (7)–(9) are

$$(\Sigma_0) : \dot{x}(t) = A_1 x(t) + A_2 x(t - d(t)) + B_2 w(t), \tag{14}$$

$$z(t) = C_{11} x(t) + C_{12} x(t - d(t)) + E_{11} w(t), \tag{15}$$

$$x(t) = 0, t \le 0. \tag{16}$$

Assumption $\lambda(A_1) \in C^-$.

While there are various concepts of passivity in literature, we shall adopt the following definition.

Definition1:[14] The dynamical system Σ_0 is called a strictly passive system if $E_{11} + E_{11}^{\mathrm{T}} > 0$ and

$$\int_0^\infty z(t)^{\mathrm{T}} w(t)\, \mathrm{d}t > \beta, \forall w \in L_2[0, \infty], \tag{17}$$

where β is a constant depending on the initial condition of the system.

We have the following delay-independent stability result concerning the system Σ_0.

Lemma Under the above assumption, the time-delayed system (Σ_0) with $w = 0$ is globally asymptotically stable independent-of-delay if there exist matrices $0 < P = P^{\mathrm{T}} \in \Re^{n \times n}$ and $0 < R = R^{\mathrm{T}} \in \Re^{n \times n}$ satisfying the linear matrix inequality (LMI)

$$\begin{pmatrix} PA_1 + A_1^{\mathrm{T}} P + R & PA_2 \\ A_2^{\mathrm{T}} P & -(1 - m)R \end{pmatrix} < 0. \tag{18}$$

The proof is similar to that in Ref.[11], so it is omitted here.

Our task is to establish the conditions under which system (Σ_Δ) are strongly robustly stable with strictly passive for all admissible uncertainties. Firstly, we introduce the following results for the system (Σ_0).

Theorem 1 System (Σ_0) satisfying the above assumption is asymptotically stable with strictly passive independent-of-delay if there exist matrices $0 < P = P^{\mathrm{T}} \in \Re^{n\times n}$ and $0 < R = R^{\mathrm{T}} \in \Re^{n\times n}$ satisfying the linear matrix inequality (LMI)

$$\begin{pmatrix} PA_1 + A_1^{\mathrm{T}}P + R & PA_2 & PB_2 - C_{11}^{\mathrm{T}} \\ A_2^{\mathrm{T}}P & -(1-m)R & -C_{12}^{\mathrm{T}} \\ B_2^{\mathrm{T}}P - C_{11} & -C_{12} & -(E_{11} + E_{11}^{\mathrm{T}}) \end{pmatrix} < 0. \tag{19}$$

Proof Introduce the following Lyapunov functional:

$$V(x(t)) = x^{\mathrm{T}}Px + \int_{t-d(t)}^{t} x(s)^{\mathrm{T}}Rx(s)\,\mathrm{d}s, \tag{20}$$

where $0 < P = P^{\mathrm{T}} \in \Re^{n\times n}$ and $0 < R = R^{\mathrm{T}} \in \Re^{n\times n}$. By differentiating $V(x)$ along with the solution of Eq.(14) and using Eq.(15), we have

$$\begin{aligned} \dot{V} - 2z^{\mathrm{T}}w \leq & x^{\mathrm{T}}(PA_1 + A_1^{\mathrm{T}}P + R)x - (1-m)x(t-d(t))^{\mathrm{T}}Rx(t-d(t)) \\ & - 2w^{\mathrm{T}}E_{11}w + 2x^{\mathrm{T}}(PB_2 - C_{11}^{\mathrm{T}})w + 2x^{\mathrm{T}}PA_2x(t-d(t)) \\ & - 2x(t-d(t))^{\mathrm{T}}C_{12}^{\mathrm{T}}w. \end{aligned} \tag{21}$$

In terms of the augmented state vector $X(t) = \begin{pmatrix} x(t)^{\mathrm{T}} & x(t-d(t))^{\mathrm{T}} & w(t)^{\mathrm{T}} \end{pmatrix}$, we have

$$\dot{V} - 2z^{\mathrm{T}}w \leq X(t) \begin{pmatrix} PA_1 + A_1^{\mathrm{T}}P + R & PA_2 & PB_2 - C_{11}^{\mathrm{T}} \\ A_2^{\mathrm{T}}P & -(1-m)R & -C_{12}^{\mathrm{T}} \\ B_2^{\mathrm{T}}P - C_{11} & -C_{12} & -(E_{11} + E_{11}^{\mathrm{T}}) \end{pmatrix} X(t)^{\mathrm{T}}. \tag{22}$$

The inequality (19) implies the inequality (18), so the system (Σ_0) is asymptotically stable when $w = 0$; if the inequality (19) holds, then

$$\int_0^\infty z(t)^{\mathrm{T}}w(t)\,\mathrm{d}t > -\frac{1}{2}V(x(0)), \tag{23}$$

from which and by definition 1 it follows that the system (Σ_0) is strictly passive .

When the inequality (19) holds, the system (Σ_0) is said to be strongly stable with the strictly passive.

Next, we introduce the following definition of strongly robust stability with SP for systems (7)–(9).

Definition 2 The uncertain time-delayed systems (7)–(9) with $u = 0$ are said to be strongly robustly stable with strictly passive if there exist matrices $0 < P = P^{\mathrm{T}} \in \Re^{n\times n}$ and $0 < R = R^{\mathrm{T}} \in \Re^{n\times n}$ satisfying the linear matrix inequality (LMI) for all admissible uncertainties:

$$\begin{pmatrix} PA_{1\Delta} + A_{1\Delta}^{\mathrm{T}}P + R & PA_{2\Delta} & PB_{2\Delta} - C_{11\Delta}^{\mathrm{T}} \\ A_{2\Delta}^{\mathrm{T}}P & -(1-m)R & -C_{12\Delta}^{\mathrm{T}} \\ B_{2\Delta}^{\mathrm{T}}P - C_{11\Delta} & -C_{12\Delta} & -(E_{11\Delta} + E_{11\Delta}^{\mathrm{T}}) \end{pmatrix} < 0. \tag{24}$$

3. Parameterization

In order to solve the problem of strongly robust stability with SP of unforced systems (7)–(9), we introduce the following parameterized linear time-invariant system:

$$\begin{aligned}(\Sigma_\mu): \dot{x}(t) =& A_1x(t) + A_2x(t-d(t)) \\ &+ B_{2\mu}\tilde{w}(t), \qquad (25)\\ \tilde{z}(t) =& C_{11\mu}x(t) + C_{12\mu}x(t-d(t)) \\ &+ E_{11\mu}\tilde{w}(t), \qquad (26)\end{aligned}$$

where

$$B_{2\mu} = \begin{pmatrix} B_2 & 0 & \mu H_1 \end{pmatrix},$$

$$C_{11\mu} = \begin{pmatrix} C_{11} \\ -\mu^{-1}E_1 \\ 0 \end{pmatrix},$$

$$C_{12\mu} = \begin{pmatrix} C_{12} \\ -\mu^{-1}E_2 \\ 0 \end{pmatrix},$$

$$E_{11\mu} = \begin{pmatrix} E_{11} & -\mu^{-1}E_4^{\mathrm{T}} & \mu H_2 \\ 0 & 2^{-1}I & 0 \\ 0 & 0 & 2^{-1}I \end{pmatrix}. \qquad (27)$$

The first result establishes the relationship between the strong robust stability with SP of unforced systems (7)–(9) and the strong stability with SP of the system (Σ_μ).

Theorem 2 The unforced systems (7)–(9) satisfying the inequalities (12) and (13) are strongly robustly stable with strictly passive if and only if there exists such a scalar $\mu \neq 0$ that (Σ_μ) is strongly stable with strictly passive.

Proof By Theorem 1, the system (Σ_μ) is strongly stable with strictly passive if there exist matrices $0 < P = P^{\mathrm{T}} \in \Re^{n\times n}$ and $0 < R = R^{\mathrm{T}} \in \Re^{n\times n}$ such that

$$\begin{pmatrix} PA_1 + A_1^{\mathrm{T}}P + R & PA_2 & PB_{2\mu} - C_{11\mu}^{\mathrm{T}} \\ A_2^{\mathrm{T}}P & -(1-m)R & -C_{12\mu}^{\mathrm{T}} \\ B_{2\mu}^{\mathrm{T}}P - C_{11\mu} & -C_{12\mu} & -(E_{11\mu} + E_{11\mu}^{\mathrm{T}}) \end{pmatrix} < 0. \qquad (28)$$

Substituting the inequality (27) into the inequality (28) leads to

$$\begin{pmatrix} \Pi_1 & \Pi_2 \\ \Pi_2^{\mathrm{T}} & \Pi_3 \end{pmatrix} < 0, \qquad (29)$$

where

$$\Pi_1 = \begin{pmatrix} PA_1 + A_1^{\mathrm{T}}P + R & PA_2 & PB_2 - C_{11}^{\mathrm{T}} \\ A_2^{\mathrm{T}}P & -(1-m)R & -C_{12}^{\mathrm{T}} \\ B_2^{\mathrm{T}}P - C_{11} & -C_{12} & -(E_{11} + E_{11}^{\mathrm{T}}) \end{pmatrix},$$

$$\Pi_2^{\mathrm{T}} = \begin{pmatrix} \mu^{-1}E_1 & \mu^{-1}E_2 & \mu^{-1}E_4 \\ \mu H_1^{\mathrm{T}}P & 0 & -\mu H_2^{\mathrm{T}} \end{pmatrix}, \quad \Pi_3 = \begin{pmatrix} -I & 0 \\ 0 & -I \end{pmatrix}. \qquad (30)$$

Applying Fact 1 and Fact 2, the inequality (29) with notation (30) holds if and only if

$$\Pi_1 + \begin{pmatrix} PH_1 \\ 0 \\ -H_2 \end{pmatrix} F(t) \begin{pmatrix} E_1 & E_2 & E_4 \end{pmatrix} + \begin{pmatrix} E_1^{\mathrm{T}} \\ E_2^{\mathrm{T}} \\ E_4^{\mathrm{T}} \end{pmatrix} F(t)^{\mathrm{T}} \begin{pmatrix} H_1^{\mathrm{T}}P & 0 & -H_2^{\mathrm{T}} \end{pmatrix} < 0. \qquad (31)$$

Rearranging the inequality (31) gives

$$\begin{pmatrix} PA_{1\Delta} + A_{1\Delta}^{\mathrm{T}}P + R & PA_{2\Delta} & PB_{2\Delta} - C_{11\Delta}^{\mathrm{T}} \\ A_{2\Delta}^{\mathrm{T}}P & -(1-m)R & -C_{12\Delta}^{\mathrm{T}} \\ B_{2\Delta}^{\mathrm{T}}P - C_{11\Delta} & -C_{12\Delta} & -(E_{11\Delta} + E_{11\Delta}^{\mathrm{T}}) \end{pmatrix} < 0, \qquad (32)$$

which in view of definition 2 implies that systems (7)–(9) are strongly robustly stable with strictly passive.

Theorem 2 provides an assessment of strongly robust stability with SP for uncertain systems (7)–(9), which involves solving a linear matrix inequality. By letting $\epsilon = \mu^{-2}$ and manipulating the inequality (29), we have

$$\begin{pmatrix} \Omega_1 & PA_2 & \Omega_2 & \epsilon E_1^{\mathrm{T}} & PH_1 \\ A_2^{\mathrm{T}}P & -\tilde{R} & -C_{12}^{\mathrm{T}} & \epsilon E_2^{\mathrm{T}} & 0 \\ \Omega_2^{\mathrm{T}} & -C_{12} & \Omega_3 & \epsilon E_4^{\mathrm{T}} & -H_2 \\ \epsilon E_1 & \epsilon E_2 & \epsilon E_4 & -\epsilon I & 0 \\ H_1^{\mathrm{T}}P & 0 & -H_2^{\mathrm{T}} & 0 & \epsilon I \end{pmatrix} < 0, \quad (33)$$

where

$$\begin{aligned} \Omega_1 &= PA_1 + A_1^{\mathrm{T}}P + R, \\ \Omega_2 &= PB_2 - C_{11}^{\mathrm{T}}, \\ \Omega_3 &= -(E_{11} + E_{11}^{\mathrm{T}}), \\ \tilde{R} &= (1-m)R. \end{aligned} \quad (34)$$

Using the LMI-Control Toolbox, one can obtain (ϵ, P, R) such that the inequality (33) holds.

4. Observer-based control design

When state variables are not available for the feedback, it is necessary to construct an observer-based controller. Now, we carry out building this observer-based dynamical controller that robustly stabilizes the system Σ_Δ and achieves SP for all admissible uncertainties. We introduce the following observer-based controller in the form

$$\dot{\bar{x}}(t) = A_1\bar{x}(t) + L(y(t) - C_{21}\bar{x}(t)), \quad (35)$$

$$u(t) = K\bar{x}(t), \quad (36)$$

where L and K are constant matrices to be selected. By applying the observer-based controller to the uncertain system Σ_Δ, we obtain the closed-loop system in the following form

$$\begin{aligned} (\Sigma_{\Delta o}): \dot{\xi}(t) =& \hat{A}_{1\Delta}\xi(t) + \hat{A}_{2\Delta}\xi(t-d(t)) \\ &+ \hat{B}_{2\Delta}w(t), \quad (37) \\ z(t) =& \hat{C}_{11\Delta}\xi(t) + \hat{C}_{12\Delta}\xi(t-d(t)) \\ &+ \hat{E}_{11\Delta}w(t), \quad (38) \\ x(t) =& 0, t \le 0, \quad (39) \end{aligned}$$

where

$$\xi(t) = \begin{pmatrix} x(t) \\ x(t) - \bar{x}(t) \end{pmatrix}, \quad (40)$$

and the uncertain matrices are given by

$$\begin{aligned} \hat{A}_{1\Delta} &= \hat{A}_1 + \hat{H}_1 F \hat{E}_1, \\ \hat{A}_{2\Delta} &= \hat{A}_2 + \hat{H}_1 F \hat{E}_2, \\ \hat{C}_{11\Delta} &= \hat{C}_{11} + \hat{H}_2 F \hat{E}_1, \\ \hat{C}_{12\Delta} &= \hat{C}_{12} + \hat{H}_2 F \hat{E}_2, \\ \hat{B}_{2\Delta} &= \hat{B}_2 + \hat{H}_1 F \hat{E}_4, \\ \hat{E}_{11\Delta} &= \hat{E}_{11} + \hat{H}_2 F \hat{E}_4, \\ \hat{A}_1 &= \begin{pmatrix} A_1 & 0 \\ 0 & A_1 - LC_{21} \end{pmatrix}, \\ \hat{A}_2 &= \begin{pmatrix} A_2 + B_1K & -B_1K \\ A_2 + B_1K & -B_1K \end{pmatrix}, \\ \hat{C}_{11} &= \begin{pmatrix} C_{11} & 0 \end{pmatrix}, \\ \hat{C}_{12} &= \begin{pmatrix} C_{12} + D_{11}K & -D_{11}K \end{pmatrix}, \\ \hat{B}_2 &= \begin{pmatrix} B_2 \\ B_2 \end{pmatrix}, \hat{E}_{11} = E_{11}, \\ \hat{H}_1 &= \begin{pmatrix} H_1 \\ H_1 \end{pmatrix}, \\ \hat{H}_2 &= H_2, \hat{E}_1 = \begin{pmatrix} E_1 & 0 \end{pmatrix}, \\ \hat{E}_2 &= \begin{pmatrix} E_2 + E_3K & -E_3K \end{pmatrix}, \\ \hat{E}_4 &= E_4. \end{aligned} \quad (41)$$

The matrices $\hat{H}_1, \hat{H}_2, \hat{E}_1, \hat{E}_2$ and $\hat{E}_4$ are constant with appropriate dimensions and F satisfies the inequality (13). In connection with the system (Σ_Δ), we introduce the following parameterized linear time-invariant system:

$$\begin{aligned} (\Sigma_{\mu\mu}): \dot{x}(t) =& A_1x(t) + A_2x(t-d(t)) \\ &+ B_1u(t-d(t)) + B_{\mu\mu}\hat{w}, \quad (42) \\ \hat{z}(t) =& C_{1\mu\mu}x(t) + C_{2\mu\mu}x(t-d(t)) \\ &+ D_{1\mu\mu}u(t-d(t)) + D_{\mu\mu}\hat{w}, \quad (43) \\ y(t) =& C_{21}x(t). \quad (44) \end{aligned}$$

where

$$B_{\mu\mu} = \begin{pmatrix} B_2 & 0 & \mu H_1 \end{pmatrix},$$

$$C_{1\mu\mu} = \begin{pmatrix} C_{11} \\ -\mu^{-1}E_1 \\ 0 \end{pmatrix},$$

$$D_{1\mu\mu}=\begin{pmatrix} D_{11} \\ -\mu^{-1}E_3 \\ 0 \end{pmatrix},$$

$$C_{2\mu\mu}=\begin{pmatrix} C_{12} \\ -\mu^{-1}E_2 \\ 0 \end{pmatrix},$$

$$D_{\mu\mu}=\begin{pmatrix} E_{11} & -\mu^{-1}E_4^{\mathrm{T}} & \mu H_2 \\ 0 & 2^{-1}I & 0 \\ 0 & 0 & 2^{-1}I \end{pmatrix}. \quad (45)$$

The closed-loop systems (35), (36) together with $(\Sigma_{\mu\mu})$ can be expressed as

$$\dot{\xi}(t)=\hat{A}_1\xi(t)+\hat{A}_2\xi(t-d(t))+\hat{B}_{\mu\mu}\hat{w}, \quad (46)$$

$$\hat{z}(t)=\hat{C}_{1\mu\mu}\xi(t)+\hat{C}_{2\mu\mu}\xi(t-d(t))+\hat{D}_{\mu\mu}\hat{w}, \quad (47)$$

where

$$\hat{B}_{\mu\mu}=\begin{pmatrix} B_{\mu\mu} \\ B_{\mu\mu} \end{pmatrix},$$

$$\hat{D}_{\mu\mu}=D_{\mu\mu},$$

$$\hat{C}_{1\mu\mu}=\begin{pmatrix} C_{1\mu\mu} & 0 \end{pmatrix},$$

$$\hat{C}_{2\mu\mu}=\begin{pmatrix} C_{2\mu\mu}+D_{1\mu\mu}K & -D_{1\mu\mu}K \end{pmatrix}. \quad (48)$$

The next theorem provides an connection between the strongly robust stability with SP of the system $(\Sigma_{\Delta_{\mathrm{o}}})$ and the strong stability with SP of the system (46), (47).

Theorem 3 Consider the uncertain system (Σ_{Δ_o}) satisfying relationships (13) and (41). Then an observer-based controller of the form (35), (36) achieves the strongly robust stability with SP for the system $(\Sigma_{\Delta_{\mathrm{o}}})$ for all admissible uncertainties if and only if there exists a scalar $\mu \neq 0$ such that the observer-based controller achieves the strong stability with SP for the system (46), (47).

Proof By Theorem 1, the system (46), (47) is strongly stable with strictly passive independent-of-delay if there exist matrices $0<P=P^{\mathrm{T}}\in\Re^{n\times n}$ and $0<R=R^{\mathrm{T}}\in\Re^{n\times n}$ such that

$$\begin{pmatrix} P\hat{A}_1+\hat{A}_1^{\mathrm{T}}P+R & P\hat{A}_2 & P\hat{B}_{\mu\mu}-\hat{C}_{1\mu\mu}^{\mathrm{T}} \\ \hat{A}_2^{\mathrm{T}}P & -(1-m)R & -\hat{C}_{2\mu\mu}^{\mathrm{T}} \\ \hat{B}_{\mu\mu}^{\mathrm{T}}P-\hat{C}_{1\mu\mu} & -\hat{C}_{2\mu\mu} & -(\hat{D}_{\mu\mu}+\hat{D}_{\mu\mu}^{\mathrm{T}}) \end{pmatrix}<0. \quad (49)$$

By using relationships (45) and (48) into the inequality (49) and following the procedure of Theorem 2, it can be shown that the inequality (49) is equivalent to the following linear matrix inequality:

$$\begin{pmatrix} P\hat{A}_{1\Delta}+\hat{A}_{1\Delta}^{\mathrm{T}}P+R & P\hat{A}_{2\Delta} & P\hat{B}_{2\Delta}-\hat{C}_{11\Delta}^{\mathrm{T}} \\ \hat{A}_{2\Delta}^{\mathrm{T}}P & -(1-m)R & -\hat{C}_{12\Delta}^{\mathrm{T}} \\ \hat{B}_{2\Delta}^{\mathrm{T}}P-\hat{C}_{11\Delta} & -\hat{C}_{12\Delta} & -(\hat{E}_{11\Delta}+\hat{E}_{11\Delta}^{\mathrm{T}}) \end{pmatrix}<0. \quad (50)$$

According to the definition 2, this implies that the system $(\Sigma_{\Delta_{\mathrm{o}}})$ is strongly robustly stable with strictly passive. The proof is completed.

The following theorem provides a solution to the problem of designing an observer-based dynamic controller which renders system $(\Sigma_{\Delta_{\mathrm{o}}})$ strongly robustly stable with strictly passive.

Theorem 4 Consider the uncertain time-delayed system $(\Sigma_{\Delta_{\mathrm{o}}})$ satisfying relationships (13) and (41). Then there exists an observer-based controller (35)-(36) for the uncertain system $(\Sigma_{\Delta_{\mathrm{o}}})$ being the strongly robustly stable with the strictly passive if for some $S>0$, there exist a scalar $\gamma>0$ and matrices N, K and M such that the following LMI holds:

$$\begin{pmatrix} \Phi_1 & \Phi_2 & \Phi_3 \\ \Phi_2^{\mathrm{T}} & -\Phi_4 & 0 \\ \Phi_3^{\mathrm{T}} & 0 & -J \end{pmatrix}<0, \quad (51)$$

where

$$\Phi_1=\begin{pmatrix} \Phi_{11} & 0 & \Phi_{13} & -B_1K \\ 0 & \Phi_{12} & \Phi_{14} & -B_1K \\ \Phi_{13}^{\mathrm{T}} & \Phi_{14}^{\mathrm{T}} & -\tilde{R}_1 & 0 \\ -K^{\mathrm{T}}B_1^{\mathrm{T}} & -K^{\mathrm{T}}B_1^{\mathrm{T}} & 0 & -\tilde{R}_2 \end{pmatrix},$$

$$\Phi_2 = \begin{pmatrix} \Phi_{21} & NE_1^{\mathrm{T}} & \gamma H_1 \\ B_2 & 0 & \gamma H_1 \\ \Phi_{22} & \Phi_{23} & 0 \\ K^{\mathrm{T}} D_{11}^{\mathrm{T}} & -K^{\mathrm{T}} E_3^{\mathrm{T}} & 0 \end{pmatrix},$$

$$\Phi_3 = \begin{pmatrix} N & 0 \\ 0 & N \\ 0 & 0 \\ 0 & 0 \end{pmatrix},$$

$$\Phi_4 = \begin{pmatrix} \Phi_{31} & -E_4^{\mathrm{T}} & \gamma H_2 \\ -E_4 & \gamma I & 0 \\ \gamma H_2^{\mathrm{T}} & 0 & \gamma I \end{pmatrix},$$

$$J = \begin{pmatrix} R_1^{-1} & 0 \\ 0 & R_2^{-1} \end{pmatrix},$$

$$\begin{aligned} \Phi_{11} =& A_1 N + N A_1^{\mathrm{T}}, \\ \Phi_{12} =& A_1 N + N A_1^{\mathrm{T}} - M - M^{\mathrm{T}}, \\ \Phi_{13} =& \Phi_{14} = A_2 + B_1 K, \\ \Phi_{21} =& B_2 - N C_{11}^{\mathrm{T}}, \\ \Phi_{22} =& - C_{12}^{\mathrm{T}} - K^{\mathrm{T}} D_{11}^{\mathrm{T}}, \\ \Phi_{23} =& E_2^{\mathrm{T}} + K E_3^{\mathrm{T}}, \\ \Phi_{31} =& (E_{11} + E_{11}^{\mathrm{T}}), \tilde{R}_i = (1-m) R_i, \quad i = 1, 2. \end{aligned} \tag{52}$$

Furthermore, an observer gain is given by the relationship (35) with $L = MN^{-1}C_{21}^{+}$, where $L = MN^{-1}C_{21}^{+}$ express the minimal norm and the least square solution of L.

Proof We set

$$P = \begin{pmatrix} P_1 & 0 \\ 0 & P_1 \end{pmatrix}, \quad R = \begin{pmatrix} R_1 & 0 \\ 0 & R_2 \end{pmatrix}, \tag{53}$$

where P_1, R_1 and R_2 are symmetric positive-definite matrices. It follows from Theorem 3 that we can focus on the strong stability with SP of the system (46), (47) for some $\mu \neq 0$. Letting $\gamma = \mu^2$, and substituting the relationships (45) and (48) into the inequality (49), we obtain the following inequality that is equivalent to the inequality (49):

$$\begin{pmatrix} \Xi_1 & P\hat{A}_2 & \Xi_2 & \hat{E}_1^{\mathrm{T}} & \gamma P \hat{H}_1 & I \\ \hat{A}_2^{\mathrm{T}} P & -\tilde{R} & -\hat{C}_{12}^{\mathrm{T}} & \hat{E}_2^{\mathrm{T}} & 0 & 0 \\ \Xi_2^{\mathrm{T}} & -\hat{C}_{12} & \Xi_3 & \hat{E}_4^{\mathrm{T}} & -\gamma \hat{H}_2 & 0 \\ \hat{E}_1 & \hat{E}_2 & \hat{E}_4 & -\gamma I & 0 & 0 \\ \gamma \hat{H}_1^{\mathrm{T}} P & 0 & -\gamma \hat{H}_2^{\mathrm{T}} & 0 & -\gamma I & 0 \\ I & 0 & 0 & 0 & 0 & -R^{-1} \end{pmatrix} < 0, \tag{54}$$

where

$$\begin{aligned} \Xi_1 &= P\hat{A}_1 + \hat{A}_1^{\mathrm{T}} P, \\ \Xi_2 &= P\hat{B}_2 - \hat{C}_{11}^{\mathrm{T}}, \\ \Xi_3 &= -(\hat{E}_{11} + \hat{E}_{11}^{\mathrm{T}}), \\ \tilde{R} &= (1-m)R. \end{aligned} \tag{55}$$

Pre-multiplying and post-multiplying the inequality (54) by $\mathrm{diag}\{P^{-1}, I, I, I, I, I\}$ and letting $P_1^{-1} = N$ and $LC_{21}P_1^{-1} = M$, we obtain the LMI (51).

5. Numerical example

In this section, we shall give an example to demonstrate the validity of the proposed method. Consider a linear uncertain time-delayed system Σ_{Δ} with the following parameters:

$$A_1 = \begin{pmatrix} -0.9 & 0 \\ 0 & -0.91 \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0.1 \\ 0 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} -0.03 & 0 \\ 0 & -0.01 \end{pmatrix},$$

$$B_2 = \begin{pmatrix} 0.01 \\ -0.003 \end{pmatrix},$$

$$C_{11} = \begin{pmatrix} -0.01 & 0 \end{pmatrix},$$

$$C_{12} = \begin{pmatrix} -0.01 & 0.01 \end{pmatrix},$$

$$D_{11} = \begin{pmatrix} 0.01 \end{pmatrix},$$

$$E_{11} = \begin{pmatrix} 0.01 \end{pmatrix},$$

$$C_{21} = \begin{pmatrix} 1 & 0 \end{pmatrix}.$$

The uncertain in the system Σ_Δ is assumed to be $d(t) = 2 + 0.5\sin(t)$ satisfying the inequality (11) and

$$H_1 = \begin{pmatrix} -0.02 & 0 \\ 0 & 0.01 \end{pmatrix},$$

$$H_2 = \begin{pmatrix} -0.03 & 0 \end{pmatrix},$$

$$E_1 = \begin{pmatrix} -0.01 & 0 \\ 0 & 0.02 \end{pmatrix},$$

$$E_2 = \begin{pmatrix} -0.01 & 0 \\ 0 & 0.01 \end{pmatrix},$$

$$E_3 = \begin{pmatrix} -0.01 \\ 0.02 \end{pmatrix},$$

$$E_4 = \begin{pmatrix} -0.01 \\ 0.03 \end{pmatrix},$$

$$R_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},$$

$$R_2 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

Now we use Matlab Control Toolbox to solve the LMI (51), and the corresponding solution is obtained as follows:

$$N = \begin{pmatrix} 0.1666 & 0 \\ 0 & 0.1646 \end{pmatrix},$$

$$M = \begin{pmatrix} 0.1667 & 0 \\ 0 & 0.1663 \end{pmatrix},$$

$$\gamma = 0.5767.$$

and the corresponding controller gain and observer gain are respectively

$$K = \begin{pmatrix} 0.1482 & -0.0065 \end{pmatrix},$$

$$L = \begin{pmatrix} 0.9991 \\ -0.0001 \end{pmatrix}.$$

6. Conclusion

In this paper, an observer-based controller for uncertain linear systems with time-varying delay in state and control input is proposed. It not only strongly robustly stabilizes the linear uncertain systems but also renders the closed-loop system strictly passive for all admissible uncertainties. By transforming the problem into a class of strictly passive control problem for a parameterized system, the solution is established and expressed in terms of a linear matrix inequality.

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