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Effect of external noise on the dynamical localization of two coupling electrons in quantum dot array*

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The effect of external noise, which is characterized by an Ornstein–Uhlenbeck process, on the dynamical localization of two coupling electrons in a quantum dot array under the action of an ac electric field is studied. A numerical solution of the stochastic equations is obtained by averaging over stochastic trajectories. The results show that the external noise may destroy the dynamical localization, but the anti-noise capacity of the system is stronger when the two electrons are localized at the ends of the quantum dot array.

Keywords: quantum dot, dynamical localization, external noise
PACC: 7335, 7340L

1. Introduction

In recent years, with the development of fabrication technology of semiconductor nanostructures, there have been considerable progress both in theoretical and experimental investigations of electron dynamics in a mesoscopic system driven by an ac electric field.[1−5] Grossmann *et al*[6] first noted that an electron in a symmetric double-well potential can be localized in one of the wells when the system is driven by an ac field, i.e. the dynamical localization. This phenomenon provides an effective way to dynamically control the quantum states, and this has important applications in the quantum computation. For this reason, much work has been done on the dynamical localization effect of many kinds of quantum devices, such as double quantum dot (QD) molecule,[7−11] square QD molecule,[12,13] QD array [14,15] and the like. In practice, the application of electric field will inevitably induce external fluctuations (noise). Within a two-band tight-binding model, Duan and Zhao[16] have studied the effect of external noise on Zener tunnelling and Bloch oscillations and found that the external noise may destroy the Zenner resonance, the Bloch oscillation, and the miniband localization. With a single-band tight-binding model, Zhang *et al*[17] have studied the effect of external noise on the propagation of a charged particle in a one-dimensional system with an impurity. They found that the dynamical localization of the charged particle and the resonant oscillations between the impurity and its nearest-neighbouring or next-nearest-neighbouring sites could be destroyed by the external noise.

In this paper, we study the effect of external noise on the dynamical localization of two interacting electrons moving in a one-dimensional QD array in the simplest case that the statistical properties of the external field can be assumed to be independent of the characteristic of the system. That is, both the systematic and fluctuating components of the external field influence the system but the system itself does not affect the properties of the external field as Duan and Zhao stated in Ref.[16]. So we assume that the noise is characterized by an Ornstein–Uhlenbeck (OU) process.[18] This will permit us to investigate the role of the strength and decay constant on the delocalization of the system separately. By solving the time

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evolution equation for the probability amplitudes numerically, we have found that the dynamical localization of the system may be destroyed by the external noise, but it can stand stronger noise intensity when the two interacting electrons are localized initially at the boundaries of the QD array, as compared with other dots.

2. Model and method

We consider a QD array that confines two electrons. Using a Hubbard model of a single-band system in an ac electric field $E(t)$, the model Hamiltonian of this system can be written as[14,15]

$$H = W\sum_{\sigma,m}(a^{+}_{m,\sigma}a_{m+1,\sigma} + a^{+}_{m+1,\sigma}a_{m,\sigma}) - edE(t)\sum_{\sigma,m} m a^{+}_{m,\sigma}a_{m,\sigma} + U\sum_{m} N_{m,\uparrow}N_{m,\downarrow}, \quad (1)$$

where $a^{+}_{m,\sigma}(a_{m,\sigma})$ creates(annihilates) an electron with spin σ on the mth site, W describes the coupling between the nearest-neighbouring dots, d is the lattice constant, and U is the Coulomb interaction of the two electrons in one dot. The external electric field $E(t)$ is composed of two components, an ac electric field $E_1(t) = E_0\cos\omega t$ and a stochastic part $F(t)$ fluctuating around $E_1(t)$ due to the circuitry thermal fluctuations, i.e.

$$E(t) = E_0\cos\omega t + F(t), \quad (2)$$

where E_0 and ω are the amplitude and frequency of the ac field $E_1(t)$, respectively.

We assume that the noise $F(t)$ is characterized by an Ornstein–Uhlenbeck process, in which it has zero average and correlation function

$$\langle F(t)F(s)\rangle = \Delta^2\exp(-|t-s|/\tau_c), \quad (3)$$

where Δ and τ_c are, respectively, the strength and decay constant of the noise. We assume that the noise is outside the system, and therefore not necessarily thermal in character, Δ and τ_c can be varied in a controlled manner, and are not restricted by the physical properties of the material. Since there are two electrons in the system, we can write the Hamiltonian (1) in the space spanned by the two-particle basis vectors as[7–9]

$$H(t) = \begin{bmatrix} H_3 & 0 \\ 0 & H_1 \end{bmatrix}, \quad (4)$$

where H_3 corresponds to the triplet subspace and H_1 describes the singlet subspace. Since Eq.(1) contains no spin-flip terms, there is no mixture of singlet and triplet subspace in the evolution equation. Therefore, if the spin of the initial state is located in the singlet subspace, the system will preserve this property in the process of evolution and we can focus our attention only on this subspace. In this subspace, we use $|k,l\rangle$ to describe the two-particle basis vectors, which means one electron in the kth dot and the other one in the lth. By expressing the particle state $|\psi(t)\rangle$ as a linear combination of the two-particle basis vectors $|k,l\rangle$,

$$|\psi(t)\rangle = \sum_{k,l} c_{k,l}(t)|k,l\rangle, \quad (5)$$

where $c_{k,l}(t)$ are the time-dependent amplitudes $\langle k,l|\psi(t)\rangle$, one can obtain from Eq.(1) the following evolution equation for the amplitudes $c_{k,l}(t)$ (we set $\hbar = 1$ throughout this paper)

$$\begin{aligned} \mathrm{i}\frac{\mathrm{d}}{\mathrm{d}t}c_{k,l}(t) =& 2^{\frac{\delta_{k+1,l}}{2}}Wc_{k+1,l} + 2^{\frac{\delta_{k,l-1}}{2}}Wc_{k,l-1} \\ & - (k+l)edE(t)c_{k,l} \\ & + 2^{\frac{\delta_{k,l}}{2}}(2-2^{\delta_{k-1,l}})Wc_{k-1,l} \\ & + 2^{\frac{\delta_{k+1,l+1}}{2}}(2-2^{\delta_{k,l+1}})Wc_{k,l+1} \\ & + \delta_{k,l}Uc_{k,l}, \end{aligned} \quad (6)$$

with different initial condition

$$c_{k,l} = \delta_{k,l}\delta_{k,n}, \quad (7)$$

where $n = 1, 2\ldots$, which means the two electrons are both located in the same nth quantum dot initially. Since there is no scheme for solving these stochastic equations analytically, we turn to numerical solutions. In solving Eqs.(6) and (7), we consider a QD array of 7 coupling quantum dots. The procedure to integrate the stochastic equations is as follows. A stochastic term is added at each step with its statistical properties described by an OU process. The OU process is generated by solving a Langevin equation with a delta-correlated noise term. This will ensure that the correlation function of $F(t)$ has the desired statistical property given by Eq.(3). The procedure is carried out for a sufficiently large number of trajectories to yield the desired average behaviour. The details of the method can be found in the Appendix of Ref.[18]. Once we obtain the probability amplitudes $c_{k,l}(t)$, the time evolution of the occupation probability $P_{k,l}(t) = |c_{k,l}(t)|^2$ and the mean-square displacement $\langle k^2+l^2\rangle = \sum_{k,l}(k^2+l^2)P_{k,l}(t)$ can be obtained. In the following we will investigate the two observables

$P_{k,l}(t)$ and $\langle k^2+l^2\rangle$ separately when the two electrons are located initially in the nth ($n=1,2,4$) QD.

3. Results and discussion

In this section, we will analyze the influence of external noise on the dynamical localization of the system. We set ω as the unit and define $\mu = edE_0/\omega$. Here we consider the case of pure ac electric field with parameters $W/\omega = 0.25$ and $U/\omega = 2.3$.

First consider the case of two electrons in the ac electric field without external noise ($ed\Delta/\omega = 0$) in order to find whether the dynamical localization occurs with the parameters we have chosen. The parameter μ is chosen by using the long-time averaged occupation probability (LAOP) method which Jiang *et al* have introduced in Ref.[19]. In the following section, all the relevant quantities are plotted as functions of dimensionless time t/T. The highest three curves in Figs.1(a), 1(b) and 1(c) correspond to the dynamical evolution of the occupation probability $P_{|n,n\rangle,|n,n\rangle}$ ($n=1,2,4$), showing that the initial and final states are both $|n,n\rangle$ of the two electrons in the 7-QD array in the absence of noise with an appropriate parameter μ. It can be seen from the three curves that by choosing appropriate parameter μ, the two electrons can be localized in the nth ($n=1,2,4$) dot forever, i.e. the dynamical localization will be perfectly stable.

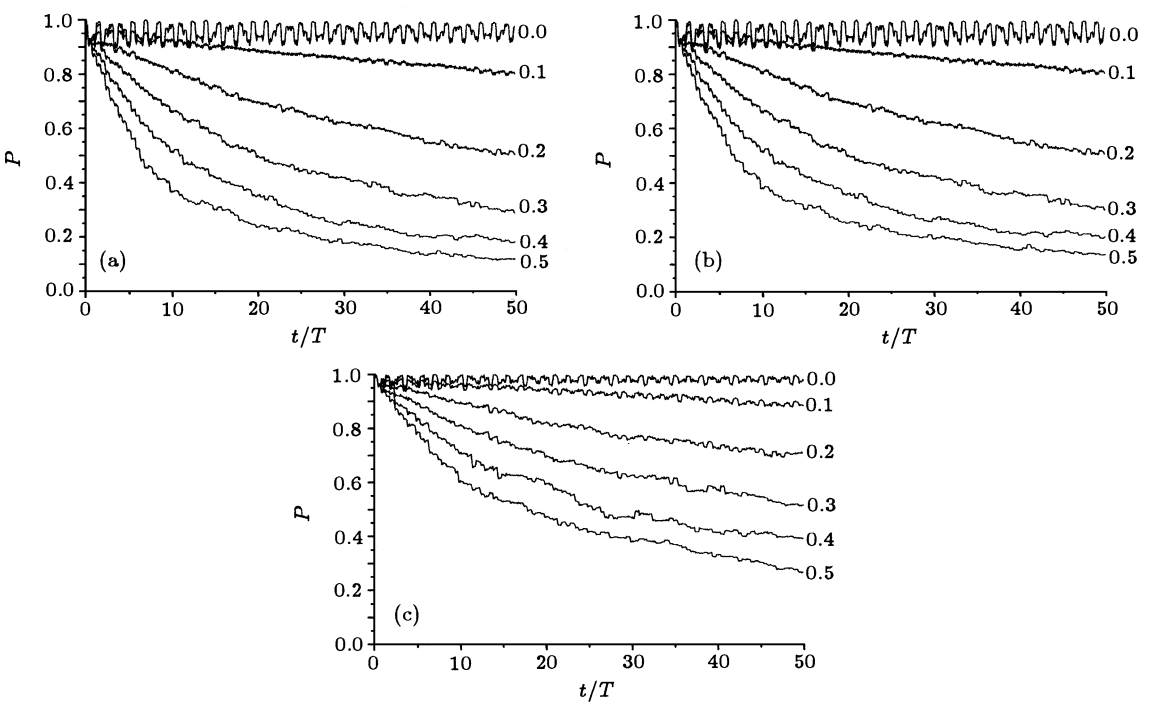


Fig.1. The time evolution of the occupation probability P(t) with and without external noise as functions of dimensionless time t/T with the parameters of appropriate μ and $\omega\tau_c$=1.0 , for $ed\Delta/\omega$=0.0, 0.1, 0.2, 0.3, 0.4, 0.5. (a): $P_{|4,4\rangle,|4,4\rangle}$ with $\mu = 14.8553$. (b): $P_{|2,2\rangle,|2,2\rangle}$ with $\mu = 14.8553$. (c): $P_{|1,1\rangle,|1,1\rangle}$ with $\mu = 14.8551$.

Then the effect of external noise is considered. We see that the noise can destroy the dynamical localization, as shown by the rest of the curves in Fig.1. The other five curves in Fig.1(a) shows the time evolution of the occupation probability of $P_{|4,4\rangle,|4,4\rangle}$ under the action of the ac electric field with external noise of different amplitudes Δ ($ed\Delta/\omega = 0.1, 0.2, 0.3, 0.4, 0.5$), where we set $\omega\tau_c = 1.0$. As can be seen from the figures, when the noise is very weak ($ed\Delta/\omega = 0.1$), the dynamical localization of the system can maintain for nearly 50 cycles. As the amplitude of external noise (Δ) increases, the periods that the dynamical localization can maintain decrease correspondingly. Figures 1(b) and 1(c) also show this behaviour. Figure 1(b) is almost the same as Fig.1(a), indicating that the effect of external noise is much the same, no matter which dot the two electrons are localized in, except for the two boundaries. Figure 1(c) shows the dynamical evolution of occupation probability $P_{|1,1\rangle,|1,1\rangle}$. We see a distinct difference between Fig.1(c) and the other two

above. In Fig.1(c) we see that when $ed\Delta/\omega$ is equal to 0.2, the localization behaviour can maintain almost 30 cycles, while it is less than 15 cycles in Figs.1(a) and 1(b). This is partly because when the electrons are localized initially at the boundaries, they have only one direction to escape from this dot as the dynamical localization is destroyed. It reduces the escaping probability of the electrons. Figure 2 shows the mean-square displacement $\langle k^2 + l^2 \rangle$ of different amplitudes Δ when the two electrons are both confined initially in the middle of the QD array. We see from this figure that $\langle k^2 + l^2 \rangle$ rises almost linearly with time just like the Brownian motion,[20] indicating that the two electrons will make diffusive motion under the effect of external noise, but when $ed\Delta/\omega$ is equal to 0.1, the slope coefficient of $\langle k^2 + l^2 \rangle$ is very small and the two electrons still can be seen as located in its initial state for as long as the time for an experiment.

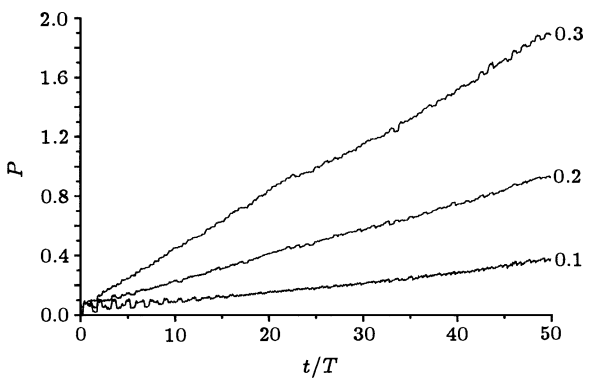


Fig.2. The time evolution of the mean-square displacement $\langle l^2 + k^2 \rangle$ under the action of the external noise of different amplitude Δ as functions of dimensionless time t/T with the parameters of $\mu = 14.8553$ and $\omega\tau_c$=1.0, for $ed\Delta/\omega$=0.1, 0.2, 0.3.

Then we set Δ to be a constant ($ed\Delta/\omega = 0.3$) and consider the role of the decay constant τ_c of the noise. The time evolution of $P_{|n,n\rangle,|n,n\rangle}$ with various decay constant τ_c is very complex and does not have a certain regularity. In the following depiction we just take a rough look at the role of τ_c in a fairly narrow range ($\omega\tau_c$ from 0.1 to 5.0) as shown in Fig.3. From this figure we see that the degree of delocalization is increased as the decay constant τ_c decreases, but the influence of the decay constant τ_c is not as significant as the amplitude Δ of the noise.

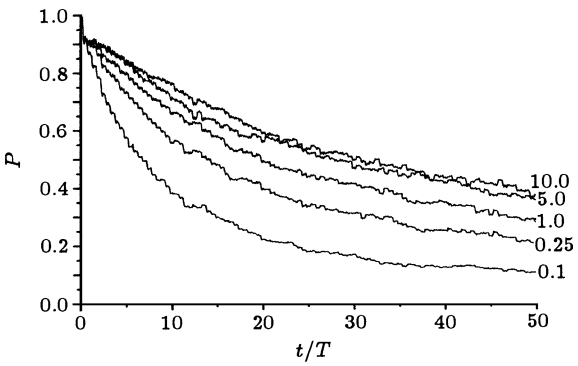


Fig.3. The time evolution of the occupation probability $P_{|4,4\rangle,|4,4\rangle}(t)$ of different decay constant τ_c as functions of dimensionless time t/T with the parameter of $\mu = 14.8553$ and $ed\Delta/\omega$=0.3, for $\omega\tau_c$=10.0, 5.0, 1.0, 0.25, 0.1.

From the discussion above we can conclude that the introduction of the external noise can destroy the dynamical localization and the action of the noise can be considered as a scattering effect. When the external noise is very weak, the localization property can be retained for experimentally long time, especially in the case that the two electrons are confined initially in the boundaries of the QD array. And this phenomenon may be helpful in experimental studies of the behaviour of electrons in QD array.

In conclusion, we have investigated the effect of external noise characterized by an OU process, on the dynamical localization of two interacting electrons in a quantum dot array under the action of an ac electric field. We find that the dynamical localization of the system can be destroyed by the external noise, but it can stand stronger noise intensity when the two electrons are localized initially at the ends of the quantum dot array.

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