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Variational analysis of the disc-loaded waveguide slow-wave structures*

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The variational method is applied to calculate the dispersion characteristics of disc-loaded waveguide slow-wave structures. The parameters describing the waveguide discontinuities in disc-loaded waveguide are calculated by the variational method. Then the dispersion characteristics of slow-wave structures are obtained using lossless microwave quadrupole theory. Good agreement was observed between results of the variational method and those of field matching method and high frequency structure simulator. In the case of broad band, results of the variational method are better than those of field matching method.

Keywords: variational method, disc-loaded waveguide, slow-wave structures, dispersion characteristics

PACC: 7170G, 7145L, 7110

1. Introduction

The disc-loaded waveguide slow-wave structure is widely used in the linear accelerator and high power travelling wave tubes, due to its simple structure, big size and excellent thermal dissipation.[1−6] The dispersion characteristics of slow-wave structures are usually analysed by the field matching method.[6−8] In this paper, a new method is put forward to calculate the dispersion characteristics of the disc-loaded waveguide slow-wave structure, which is a variational method introduced by Schwinger.[9]

The disc-loaded waveguide slow-wave structure is illustrated in Fig.1. Here a is the external radius of the disc, b is its internal radius, L is the space periodic length of disc-loaded waveguide, d is the spacing between two neighbouring discs. Figure 2 is one section of the periodic structure, which can be seen as a thick capacitive window. $2l = L - d$ is the total thickness of the window. The equivalent circuit of the window is a symmetrical T network, whose parameters can be calculated by variational methods. Then the dispersion characteristics of the periodic structure can be obtained using lossless microwave quadrupole theory.[10]

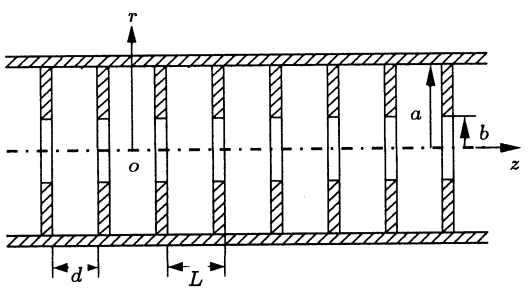


Fig.1. The disc-loaded waveguide slow-wave structure.

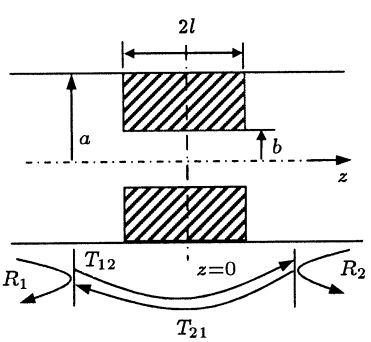


Fig.2. One section of the periodic structure.

2. Variational method for waveguide discontinuities

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In Fig.2, R_1 and R_2 are the reflection coefficients, T_{12} and T_{21} are the transmission coefficients. Because of the symmetry of the structure, $R_1 = R_2$ and $T_{12} = T_{21}$. The dimension a is chosen so that only the TM_{01} mode can propagate. However, b is smaller than a, so that the TM_{01} mode may or may not propagate in the restricted section. The discontinuity is symmetrical about the z=0 plane. If we choose terminal reference planes at $z = \pm l$, the equivalent circuit for the window is a symmetrical T network, as illustrated in Fig.3.

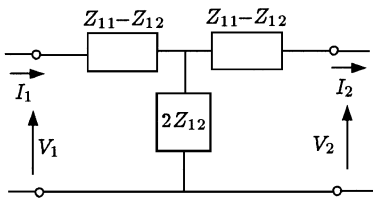


Fig.3. Equivalent circuit for a thick capacitive window.

Let TM_{01} modes of amplitude a_1 be incident on the diaphragm from the left and right simultaneously. For this particular choice of incident modes, we have $V_1=(1+R_1)a_1+T_{21}a_1=(1+R_2)a_1+T_{12}a_1 = V_2$, since, from the symmetry of the structure, $R_1 = R_2$ and $T_{12} = T_{21}$. In view of the symmetrical excitation, the electric field E_r must be symmetrical about z=0 and the magnetic field H_θ must be antisymmetrical about z=0 and, in particular, equal to zero on the symmetry plane. A magnetic wall may, therefore, be placed at z=0. The circuit equations for the T network are

$$V_1 = Z_{11}I_1 - Z_{12}I_2, \tag{1a}$$

$$V_2 = Z_{12}I_1 - Z_{11}I_2, \tag{1b}$$

and, when $V_1 = V_2$, we have I_1=$-I_2$. The input impedance to the network with an open circuit (magnetic wall) in the symmetry plane is

$$(Z_{\text{in}})_{\text{oc}} = \frac{V_1}{I_1} = Z_{11} + Z_{12}. \tag{2}$$

If we choose incident waves with amplitudes a_1 and $-a_1$, we will have E_r equal to zero on the symmetry plane because of the antisymmetrical method of excitation. In this case the input impedance is given by

$$(Z_{\text{in}})_{\text{sc}} = \frac{V_1}{I_1} = Z_{11} - Z_{12}. \tag{3}$$

Hence, in order to find $Z_{11} + Z_{12}$, we solve the problem with a magnetic wall in the symmetry plane, while, in order to find Z_{11}–Z_{12}, we solve the problem with a short circuit in the symmetry plane. From the two solutions we may then obtain Z_{11} and Z_{12} by addition and subtraction.

The field in the regions z≤$-l$ and $-l \le z \le 0$ will be given by an expansion in terms of an infinite set of TM_{0n} modes. At z= $-l$, the transverse fields must be continuous. So we have

$$(1 - R_1 + T_2)\, a_1 \Phi_1 - \sum_{n=2}^{\infty} a_n \Phi_n = \sum_{n=1}^{\infty} b_n \Psi_n \cosh(\gamma_n l), \tag{4a}$$

$$(1 + R_1 - T_2)\, Y_1 a_1 \Phi_1 + \sum_{n=2}^{\infty} Y_n a_n \Phi_n = \sum_{n=1}^{\infty} Y_{0n} b_n \Psi_n \sinh(\gamma_n l), \tag{4b}$$

where $(1 - R_1 + T_2)\, a_1 = V_1$, $(1 + R_1 - T_2)\, Y_1 a_1 = I_1 = (Z_{11} + Z_{12})^{-1} V_1$, $Y_n = \mathrm{j}\frac{\omega\varepsilon_0}{\Gamma_n}$, $\Gamma_n^2 = k_{cn}^2 - k_0^2$, $k_{cn} = \frac{p_{0n}}{a}$, $k_0 = \mu_0\varepsilon_0\omega^2$, $Y_{0n} = \mathrm{j}\frac{\omega\varepsilon_0}{\gamma_n}$, $\gamma_n^2 = k_{c0n}^2 - k_0^2$, $k_{c0n} = \frac{p_{0n}}{b}$, $\Phi_n = \frac{J_1(k_{cn}r)}{\sqrt{2\pi\int_0^a J_1(k_{cn}r)^2\, r\mathrm{d}r}}$, $\Psi_n = \frac{J_1(k_{c0n}r)}{\sqrt{2\pi\int_0^b J_1(k_{c0n}r)^2\, r\mathrm{d}r}}$, P_{0n} is the nth root of the Bessel function of order 0, J_1 is the Bessel function of order 1.

Let the transverse electric field in the aperture at z= $-l$ be $\varepsilon(r)$. By Fourier analysis, we obtain

$$V_1 = 2\pi\int_0^b \varepsilon(r)\Phi_1 r\mathrm{d}r, \tag{5a}$$

$$a_n = 2\pi\int_0^b \varepsilon(r)\Phi_n r\mathrm{d}r, \tag{5b}$$

$$b_n = \frac{2\pi}{\cosh(\gamma_n l)}\int_0^b \varepsilon(r)\Psi_n r\mathrm{d}r. \tag{5c}$$

A variational expression for $(Z_{11}+Z_{12})^{-1}$ may be obtained by substituting the integral expressions for the amplitude coefficients into Eq.(4b), multiplying Eq.(4b) by $\varepsilon(r)$, integrating over the aperture, and, finally, dividing both sides by $\left(2\pi\int_0^b \varepsilon(r)\Phi_1 r\mathrm{d}r\right)^2$.

By these procedures, we obtain

$$(Z_{11}+Z_{12})^{-1} = -\frac{\mathrm{j}\int_0^b\int_0^b \varepsilon(r)\,\varepsilon(r')\,G_2(r\,|r'\,)\,rr'\mathrm{d}r\mathrm{d}r'}{\left(\int_0^b \varepsilon(r)\,\Phi_1 r\mathrm{d}r\right)^2}, \quad (6)$$

where

$$-\mathrm{j}G_2(r\,|r'\,) = \sum_{n=2}^{\infty} Y_n\,\Phi_n(r)\,\Phi_n(r') + \sum_{n=1}^{\infty} Y_{0n}\,\Psi_n(r)\,\Psi_n(r')\tanh(\gamma_n l). \quad (7)$$

In the case of an electric wall at z=0, the transverse electric field for the nth mode in the region $-l \le z \le 0$ is taken proportional to $\psi_n \sinh(\gamma_n z)$ in order that it should vanish at z=0. By an analysis similar to the above, we obtain in the following the variational expression for $(Z_{11}-Z_{12})^{-1}$:

$$(Z_{11}-Z_{12})^{-1} = -\frac{\mathrm{j}\int_0^b\int_0^b \varepsilon(r)\,\varepsilon(r')\,G_1(r\,|r'\,)\,rr'\mathrm{d}r\mathrm{d}r'}{\left(\int_0^b \varepsilon(r)\,\Phi_1 r\mathrm{d}r\right)^2}, \quad (8)$$

where

$$-\mathrm{j}G_1(r\,|r'\,) = \sum_{n=2}^{\infty} Y_n\,\Phi_n(r)\,\Phi_n(r') + \sum_{n=1}^{\infty} Y_{0n}\,\Psi_n(r)\,\Psi_n(r')\coth(\gamma_n l). \quad (9)$$

Equations (6) and (8) have the property that $(Z_{11}\pm Z_{12})^{-1}$ are stationary for small arbitrary variations in the electric field distribution about their correct values. It, therefore, follows that a first-order approximation to the electric field distribution will yield a second-order approximation to the input admittance. The expressions for $(Z_{11}\pm Z_{12})^{-1}$ are homogeneous in the sense they do not depend on the amplitude of the electric field distribution $\varepsilon(r)$, but only on its functional form. So we can choose

$$\varepsilon(r) = \sum_{m=1}^{M} b_m \psi_m(r). \quad (10)$$

The parameter M must be chosen according to the accuracy required. Then Eq.(6) becomes

$$(Z_{11}+Z_{12})^{-1} P_{r1} \sum_{s=1}^{M} P_{s1} b_s = -\mathrm{j}\sum_{s=1}^{M} b_s g_{rs}, \quad (r=1,2,\ldots,M), \quad (11)$$

where

$$P_{mn} = 2\pi\int_0^b \psi_m\,\Phi_n \mathrm{d}r, \quad (12)$$

$$-\mathrm{j}g_{rs} = \sum_{n=1}^{\infty} Y_n P_{sn} P_{rn} + \sum_{m=1}^{\infty} Y_{0n}\delta_{sm}\delta_{rm}\tanh(\gamma_m l), \quad (13)$$

with $\delta_{mn} = \begin{cases} 0, & m \ne n, \\ 1, & m = n. \end{cases}$

For a solution, the determinant of this set of equations must vanish. We have

$$(Z_{11}+Z_{12})^{-1} \begin{vmatrix} 1 & 1 & \cdots & 1 \\ \dfrac{g_{21}}{P_{11}P_{21}} - \dfrac{g_{11}}{P_{11}^2} & \dfrac{g_{22}}{P_{21}^2} - \dfrac{g_{12}}{P_{11}P_{21}} & \cdots & \dfrac{g_{2M}}{P_{21}P_{M1}} - \dfrac{g_{1M}}{P_{11}P_{M1}} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{g_{M1}}{P_{11}P_{M1}} - \dfrac{g_{11}}{P_{11}^2} & \cdots & \cdots & \dfrac{g_{MM}}{P_{M1}^2} - \dfrac{g_{1M}}{P_{11}P_{M1}} \end{vmatrix}$$

$$= -\mathrm{j}\begin{vmatrix} \dfrac{g_{11}}{P_{11}^2} & \cdots & \cdots & \dfrac{g_{1M}}{P_{11}P_{M1}} \\ \dfrac{g_{21}}{P_{11}P_{21}} - \dfrac{g_{11}}{P_{11}^2} & \cdots & \cdots & \dfrac{g_{2M}}{P_{21}P_{M1}} - \dfrac{g_{1M}}{P_{11}P_{M1}} \\ \cdots & \cdots & \cdots & \cdots \\ \dfrac{g_{M1}}{P_{11}P_{M1}} - \dfrac{g_{11}}{P_{11}^2} & \cdots & \cdots & \dfrac{g_{MM}}{P_{M1}^2} - \dfrac{g_{1M}}{P_{11}P_{M1}} \end{vmatrix}. \quad (14)$$

A similar analysis may be used for determining $(Z_{11}-Z_{12})^{-1}$ and results in an equation similar to Eq.(14), with g_{rs} replaced by h_{rs}, where

$$-\mathrm{j}h_{rs} = \sum_{n=1}^{\infty} Y_n P_{sn} P_{rn} + \sum_{m=1}^{\infty} Y_{0n}\delta_{sn}\delta_{rn}\coth(\gamma_n l). \tag{15}$$

3. Dispersion equation

The periodic structure of the disc-loaded waveguide can be seen as a cascade connection of an infinite number of lossless microwave quadrupoles separated by sections of waveguide or transmission lines of sufficient length to ensure that there is no interaction between sections due to higher-order evanescent modes, as Fig.4.

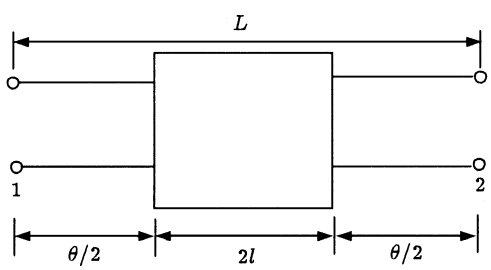


Fig.4. The lossless microwave quadrupole.

Let the wave matrix describing the discontinuity be

$$[A'] = \begin{bmatrix} A'_{11} & A'_{12} \\ A'_{21} & A'_{22} \end{bmatrix}. \tag{16}$$

The wave matrix describing the structure between terminal planes 1 and 2 is

$$[A] = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}. \tag{17}$$

The relations between $[A]$ and $[A']$ are $A_{11} = A'_{11}\mathrm{e}^{\mathrm{j}\beta_0\theta}$, $A_{12} = A'_{12}$, $A_{21} = A'_{21}$ and $A_{22} = A'_{22}\mathrm{e}^{-\mathrm{j}\beta_0\theta}$, where $\beta_0 = -\mathrm{j}\Gamma_0$.

Due to the infinite periodic structure, the following solution can be obtained:

$$\cosh(\mathrm{j}\beta L) = \frac{A_{11}+A_{22}}{2}. \tag{18}$$

According to Ref.[10], A'_{11} and A'_{22} are

$$A'_{11} = \frac{Y_0^2(Z_{11}Z_{22}-Z_{12}^2)+1+Y_0(Z_{11}+Z_{12})}{2Y_0Z_{12}}, \tag{19}$$

$$A'_{22} = \frac{-Y_0^2(Z_{11}Z_{22}-Z_{12}^2)+Y_0(Z_{11}+Z_{12})}{2Y_0Z_{12}}. \tag{20}$$

So, the dispersion equation is

$$\cos\beta L = \cos\beta_0(L-2l)\frac{Z_{11}}{Z_{12}} + \sin\beta_0(L-2l) \times \frac{Y_0^2(Z_{11}^2-Z_{12}^2)+1}{2Y_0Z_{12}}. \tag{21}$$

4. Simulation results and discussion

We use the above variational method to analyse the structures designed in Ref.[6], which were analysed by the field matching method. The parameters of the slow-wave structures are listed in Table 1.

Table 1. The slow-wave-structure parameters of $K\alpha$ band.

Structure	L/mm	d/mm	a/mm	b/mm
A	1.98	0.99	5.0	3.5
B	1.98	0.99	4.4	2.6

(A: broad band, B: narrow band)

Figures 5 and 6 show the dispersion characteristics for structures A and B, separately, where the solid lines are the simulated results of the variational method described in this paper, the dashed lines are the simulated results of the field matching method described in Ref.[6], the square symbols are the simulated results of the high frequency structure simulator (HFSS) code using finite element method. In the low-frequency band, the values of the variational method are in good agreement with those by HFSS, smaller than those of the field matching method. However, in high-frequency band, the values of the variational method are bigger than those of HFSS and the field matching method. In the case of broad band illustrated in Fig.5, the results of the variational method are in better agreement with those of HFSS than in the case of narrow band illustrated in Fig.6.

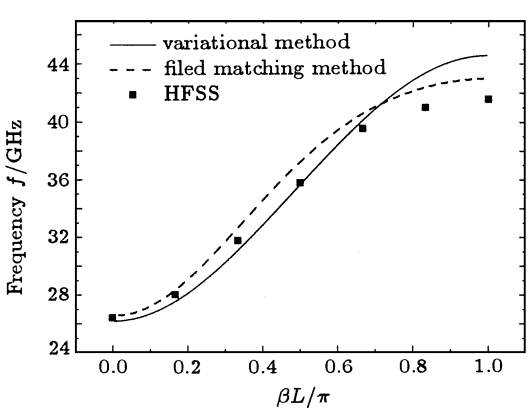


Fig.5. Dispersion characteristics for structure A.

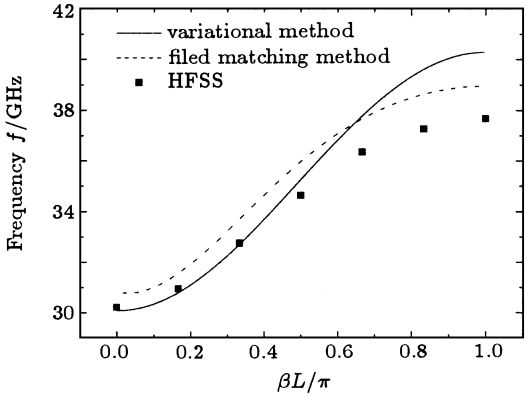


Fig.6. Dispersion characteristics for structure B.

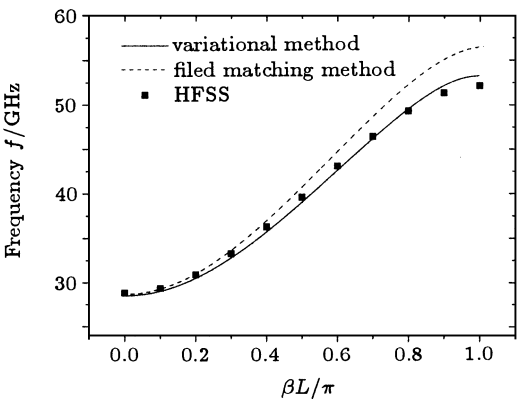


Fig.7. Dispersion characteristics for the structure C.

We also design another structure C, whose parameters are L=2.4mm, d=1.2mm, a=4.4mm, b=3.6mm. Its dispersion characteristics are shown in Fig.7. The bandwidth of structure C is broader than those of structures A and B. From Fig.7, we can see that the results of the variational method are in better agreement with those of HFSS than those of the field matching method.

5. Summary

We have put forward a variational method to analyse the dispersion characteristics of the disc-loaded waveguide slow-wave structure. The variational method for calculating the dispersion characteristics in disc-loaded waveguide slow-wave structures is simpler than the field matching method and faster than HFSS. The simulated results of variational method are in agreement with those of the field matching method and HFSS. In the case of broad band, they are better than the results of field matching method. So the variational method can be used to design this kind of slow-wave structure.

The method presented in this paper can be extended to analyse other slow-wave structure, such as loaded rectangle waveguide, ridge-loaded waveguide, etc. For other kinds of slow-wave structures, only the impedance elements describing the T network are different.

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