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Digital communication of two-dimensional messages in a chaotic optical system*

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The digital communication of two-dimensional messages is investigated when two solid state multi-mode chaotic lasers are employed in a master-slave configuration. By introducing the time derivative of intensity difference between the receiver (carrier) and the transmittal (carrier plus signal), several signals can be encoded into a single pulse. If one signal contains several binary bits, two-dimensional messages in the form of a matrix can be encoded and transmitted on a single pulse. With these improvements in secure communications using chaotic multi-mode lasers, not only the transmission rate can be increased but also the privacy can be enhanced greatly.

Keywords: digital communication, multi-mode laser, two-dimensional messages, chaotic synchronization
PACC: 0545, 4265, 4250

1. Introduction

Recently, much attention has been paid to the synchronization between chaotic oscillators due to the potential applications in various areas, especially in secure communication.[1−32] Chaotic synchronization[1−11] and its application to secure communications[12−32] have been demonstrated in electronic circuits and optical systems. Using two synchronized chaotic oscillators, encoding and decoding messages can be achieved, where the chaotic spiky pulses serve as a natural masking background. Meanwhile, the investigation of spatiotemporal chaos in nonlinear systems has revealed interesting phenomena[33,34]. The synchronization of spatiotemporal chaotic systems[35−37] can also be used to encode and decode messages, and even be applied to multi-channel communication.[38−40] The communication using chaos can be relatively easily achieved by well-developed Chua circuits. By using chaotic laser systems, the bandwidth can be extended from several tens of KHz to hundreds of MHz in electronic circuits.

In most of the previous investigations of chaotic communications, each pulse was only used to encode and decode one signal of either bit "1" or bit "0". At the same time, the chaotic synchronization and mode dynamics of multi-mode lasers were studied.[41,42] In this paper, the digital communication of two-dimensional messages encoded on a single pulse in a multi-mode laser system is analysed when the two lasers are in the form of master–slave configuration. In Section 2, the characteristics of communication using multi-mode lasers are investigated. The method of encoding and decoding two-dimensional messages on a single pulse is presented in Section 3. A conclusion is given in Section 4.

2. Chaotic synchronization and communication of multi-mode lasers

The theoretical model of two-multi-mode solid state Nd:YAG lasers in the form of master–slave configuration can be described as follows:[19−22,39−42]

$$\frac{\mathrm{d}E_{\mathrm{T}}^{j}}{\mathrm{d}t}=\gamma_{\mathrm{c}}(G_{\mathrm{T}}^{j}-\alpha_{\mathrm{T}}^{j})E_{\mathrm{T}}^{j}+\mathrm{i}\omega_{\mathrm{T}}^{j}E_{\mathrm{T}}^{j}+\sqrt{\varepsilon_{\mathrm{T}}^{j}}\eta_{\mathrm{T}}^{j}, \tag{1}$$

$$\frac{\mathrm{d}G_{\mathrm{T}}^{j}}{\mathrm{d}t}=\gamma_{\mathrm{f}}\left[P_{\mathrm{T}}^{j}-G_{\mathrm{T}}^{j}\left(1+|E_{\mathrm{T}}^{j}|^{2}+\sum_{k\neq j}^{N}\beta_{jk}|E_{\mathrm{T}}^{k}|^{2}\right)\right], \tag{2}$$

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$$\frac{\mathrm{d}E_{\mathrm{R}}^{j}}{\mathrm{d}t} = \gamma_{\mathrm{c}}(G_{\mathrm{R}}^{j} - \alpha_{R}^{j})E_{\mathrm{R}}^{j} + \mathrm{i}\omega_{\mathrm{R}}^{j}E_{\mathrm{R}}^{j} + \sqrt{\varepsilon_{\mathrm{R}}^{j}}\eta_{\mathrm{R}}^{j} - \kappa A_{\mathrm{b}}\gamma_{\mathrm{c}}\left(E_{\mathrm{R}}^{j} - \frac{A_{\mathrm{s}}^{j}}{A_{\mathrm{b}}}E_{\mathrm{T}}^{j}\right), \quad (3)$$

$$\frac{\mathrm{d}G_{\mathrm{R}}^{j}}{\mathrm{d}t} = \gamma_{\mathrm{f}}\left[P_{\mathrm{R}}^{j} - G_{\mathrm{R}}^{j}\left(1 + |E_{\mathrm{R}}^{j}|^{2} + \sum_{k\neq j}^{N}\beta_{jk}|E_{\mathrm{R}}^{k}|^{2}\right)\right]. \quad (4)$$

Here, the superscript j denotes the jth mode in the lasers, the subscript T represents the transmitting laser while R the receiving laser, E is the complex, slowly varying amplitude of the electric field, G is the gain of the active medium, $\tau_{\mathrm{c}} = \dfrac{1}{\gamma_{\mathrm{c}}}$ is the cavity round trip time, $\tau_{\mathrm{f}} = \dfrac{1}{\gamma_{\mathrm{f}}}$ is the decay time of the upper lasing level, ω is the detuning of the laser frequency from the nearest empty cavity mode, P is the pump parameter, N is the number of modes, ϵ is the spontaneous emission noise strength, and η is the complex Gaussian white noise term of zero mean and in the form of correlation $\langle\eta^{j}(t)\eta^{*k}(t')\rangle = 2\delta_{jk}\delta(t-t')$. The sinusoidal loss modulation is given by $\alpha^{j} = \alpha_{0}^{j} - \alpha_{1}^{j}\cos(\Omega t)$ where $\alpha_{1}^{j} \ll \alpha_{0}^{j}$. The last terms on the right-hand sides in Eqs.(2) and (4) are responsible for the coupling between modes. The last term on the right-hand side in Eq.(3) is responsible for the coupling between the transmitter and the receiver lasers, which also accounts for any losses in the transmission process. To achieve chaotic synchronization, the parameter should be set at $\alpha_{0\mathrm{R}} = \alpha_{0\mathrm{T}} + \kappa A_{\mathrm{b}}$, where κ is the coupling coefficient. It is through A_{s}^{j} that the message is added to the output of the transmitter. When there is no signal, $A_{\mathrm{s}}^{j} = A_{\mathrm{b}}$. While a signal is encoded, $A_{\mathrm{s}}^{j} = (1 + m * \xi)A_{\mathrm{b}}$, where ξ is the signal strength, $m = 1$ corresponds to the signal "1" while $m = -1$ corresponds to the signal "0".

By numerical integrations of Eqs.(1)–(4), the relations between the output laser intensities of the receiver and transmitter (carrier) [or transmittal (carrier plus message)] can be obtained. For simplicity, it is assumed that there are three modes in each laser with $N = 3$. In two multi-mode lasers in the form of master–slave configuration, the total intensity of either master or slave is the sum of the intensities distributed in each mode. If the total intensity between two lasers is chaotically synchronized, the intensities in the corresponding cavity modes between two lasers are also chaotically synchronized. However, there is almost no chaotic synchronization between intensities of different modes.[41,42]

The intensity relations of the corresponding modes in two multi-mode lasers with $N = 3$ are plotted in Fig.1 when the two lasers are employed in digital communications. Both lasers are operated in a chaotic regime. In Fig.1(a), it is seen that the relation between the receiver and the transmitter looks like a swallowtail. The upper branch corresponds to bits "1" while the lower branch to bits "0". In Fig.1(b), it is evident that good synchronization is obtained between the receiver and transmittal. However, when a bit "1" is encoded on the transmitter, it needs a short period of time to recover the chaotic synchronization between the receiver and transmittal [marked by "1" in Fig.1(b)]. A similar phenomenon is observed when a bit "0" is encoded (marked by "0" in Fig.1(b)).

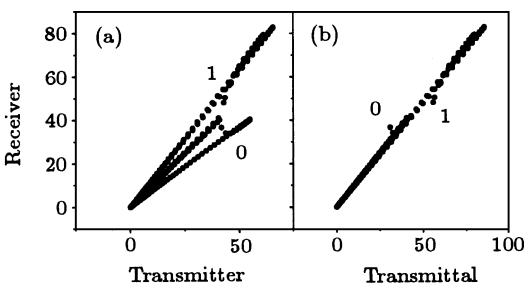


Fig.1. The intensity relations between the receiver and transmitter (carrier) and between the receiver and transmittal (carrier plus message) in two multi-mode lasers.

In digital communications, the chaotic synchronization of total intensities in two multi-mode lasers depends on the chaotic synchronization of each corresponding mode. The communication scheme well developed in a single-mode laser system needs to be modified when two multi-mode lasers are used.

In chaotic communications using two multi-mode lasers, the signals can be decoded by the time derivative D of the difference between the transmittal $|A_{\mathrm{s}}E_{\mathrm{T}}|^{2}$ and the carrier $|A_{\mathrm{b}}E_{\mathrm{R}}|^{2}$

$$D = \frac{\mathrm{d}(|A_{\mathrm{s}}E_{\mathrm{T}}|^{2} - |A_{\mathrm{b}}E_{\mathrm{R}}|^{2})}{\mathrm{d}t}. \quad (5)$$

The whole process of encoding and decoding a signal is shown in Fig.2 through numerical simulations. The signal strength is 1.0% of the threshold value and the threshold is 2.0. In Fig.2(a), the signal of bit "1" is encoded on the transmitter at about 157.8 μs. The profile of the intensity pulse of the transmitter is shown in Fig.2(b). Since the signal strength is very small, the curve of the intensity pulse of the transmittal is almost the same as that of transmitter. The intensity difference is within the width of the curve shown in Fig.2(b). As soon as the intensity

pulse of the transmitter is increased above the threshold, a signal of bit "1" can be encoded. Figure 2(c) shows the signal decoded in real time by the quantity D. In Fig.2(c), there are two pulses with a very narrow width. One pulse (labelled A) at about 157.8μs corresponds to the signal. The second pulse (labelled B) at about 159.8μs is located at the end of the pulse of transmitter. The sign of the second pulse is opposite to that of the signal. It seems that the second pulse appears to balance the encoded signal. At first glance, the second pulse provides wrong information of decoded signals. However, it can be used to check the correctness of signals transmitted since the sum of the total pulses decoded is zero. After checking the whole signals decoded correctly, the last pulse among the decoded signals can be dropped.

3. Transmission of a matrix on a single pulse

From Fig.2, it is seen that if the width of the signal is much narrower than that of the pulse of the transmitter, the signal can be encoded at any location on the pulse when the height of the pulse is above the threshold. If one digit is encoded on one pulse at any location, most parts of the pulse are useless and wasted. To increase the efficiency of utilization of the pulse, several digits instead of one digit can be encoded on a single pulse. If several signals with different height can represent a group of different binary bits, a two-dimensional matrix can be encoded and decoded on a single pulse of the transmitter. Thus, multi-channel communications using multimode chaotic lasers can be achieved. The transmission of a two-dimensional matrix can be obtained through the quantity $A_{\rm s}$ and Eq.(5). The quantity $A_{\rm s}$ can be defined as follows.

$$A_{\rm s}^{j} = \left(1 + \frac{m * \xi}{I_j + I_0}\right) A_{\rm b}. \qquad (6)$$

Here, superscript j denotes the jth mode, $\dfrac{m * \xi}{I_j + I_0}$ represents the strength of message, I_j is the intensity in the jth mode when the signal is encoded, I_0 is required to reduce the threshold and transmit a large amplitude of matrix element, ξ is a parameter to adjust the strength of the message. The different values of m correspond to different heights of the signal with multiple unit length.

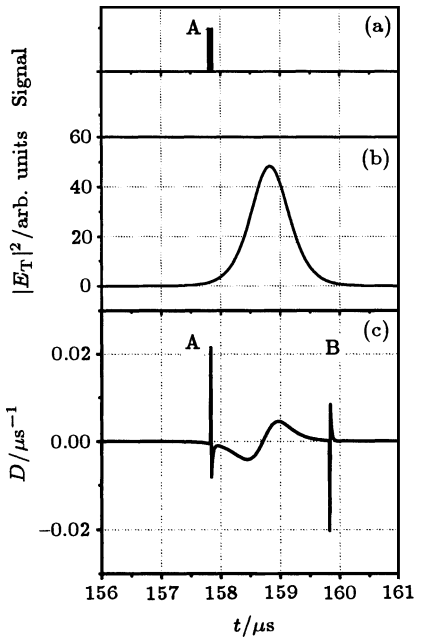


Fig.2. The process of transmitting a digit of bit "1" on a pulse in a three-mode laser system. The parameters are chosen as follows: $\alpha_{0T} = \alpha_{0R} = 0.01$, $\alpha_1 = 2.0 \times 10^{-4}$, Ω=541.6ms^{-1}, $P_{\rm T} = P_{\rm R} = 0.03$, $\epsilon_{\rm T} = \epsilon_{\rm R} = 8.33 \times 10^{-9}$s^{-1}, $\omega_{\rm R} - \omega_{\rm T}$=50000rad/s^{-1}. $\dfrac{1}{\gamma_{\rm c}}$=120ps, $\dfrac{1}{\gamma_{\rm f}} = 240\mu$s, $\beta = 0.66$, $v = 0.01$, $I_{\rm th} = 2.0$. (a) The bit "1" to be encoded into one of the three modes; (b) Intensity output of the mode in transmitter; (c) The signal is decoded by D in Eq.(5). There is an extra shoot (labelled B) at the end of the pulse.

Table 1 is an example of the codes in a two-dimensional matrix $M(2)$. In Table 1, m denotes the height of the signal. There are 16m's that represent 16 groups of binary bits.

Table 1. Example of codes in a two-dimensional matrix.

m	message	m	message
–8	0 0 0 0	1	1 0 0 0
–7	0 0 0 1	2	1 0 0 1
–6	0 0 1 0	3	1 0 1 0
–5	0 0 1 1	4	1 0 1 1
–4	0 1 0 0	5	1 1 0 0
–3	0 1 0 1	6	1 1 0 1
–2	0 1 1 0	7	1 1 1 0
–1	0 1 1 1	8	1 1 1 1

The process of a (55×4) matrix $M(2)$ is encoded and decoded on a single intensity pulse of the transmitter as shown in Fig.3. In Fig.3(a), the messages of a (55×4) matrix are represented by different heights of signals. The 55 signals encoded on a single pulse represent 55 rows of the matrix $M(2)$. In Fig.3(b), it is seen that the intensity profile of the transmittal(carrier plus message) is indistinguishable from that of transmitter(carrier) since the maximum height of

the signal is very small compared with the threshold. In Fig.3(c), the (55×4) matrix is decoded by the quantity D in Eq.(5). The 55 narrow pulses correspond to 55 rows of the matrix $M(2)$. Similarly, the 56th narrow pulse labelled by B at the end of the intensity pulse needs to be dropped after it is used to check the correctness of the decoded signals.

$$M = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$$

If the value of m (In Table 1) can be doubled and the corresponding message remains unchanged, it will be found that the signals encoded on a single pulse are all even numbers. To make sure of the correctness of the decoded signals, an odd number signal is added to the head of the single pulse which will be used to transmit the signals. Following the balance rule that the sum of the total pulses decoded is zero, there will be an odd signal at the end of the pulse. If the head signal and the last odd signal are dropped, the signals will be decoded successfully.

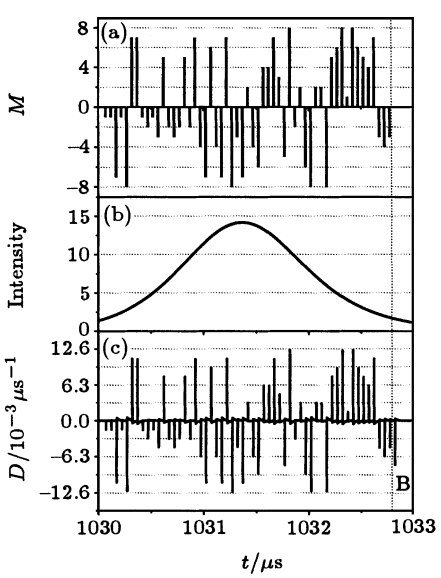


Fig.3. The process of transmitting a two-dimensional matrix on a single pulse. The parameters are chosen as follows: $\xi = 0.001$, $I_{\rm th} = 2.0$. (a) Different heights of signals represent a group of binary bits; (b) The intensity profile of the transmittal; (c) The decoded signals with an extra shoot (labelled B) at the end of signals.

The process of encoding and decoding a two-dimensional (10×4) matrix M(2) on a single pulse with a head signal "5" is plotted in Fig.4. The "5" is chosen as an odd number signal. In Fig.4(a), the values of 10 signals have been doubled. So the signals to be encoded on a single pulse are all even numbers. Figure 4(b) shows the intensity profile of the transmittal. Figure 4(c) is a plot of the decoded signals. The first narrow pulse labelled by A corresponds to the head signal "5". The last shoot labelled by B is an extra odd signal. After dropping both of them, the signals can be decoded correctly.

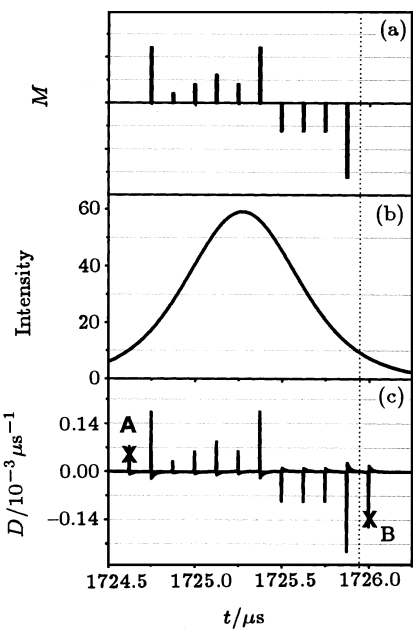


Fig.4. The process of encoding and decoding a two-dimensional (10×4) matrix on a single pulse with a head signal "5". The parameters are chosen as follows: $\xi = 0.02$, $I_{\rm th} = 10.0$. (a) A two-dimensional matrix to be encoded; (b) The intensity profile of the transmittal; (c) The decoded signals with a head signal "5" (labelled A) and an extra shoot (labelled B) at the end of the signals.

Multi-channel communication using chaotic multi-mode lasers allows several two-dimensional matrices to be transmitted simultaneously. It is important to note that the scheme described above can be used to transmit one-dimensional data and two-dimensional information. The process of encoding and decoding a static two-dimensional grey-scale image is shown in Fig.5. The original image [Fig.5(a)] is a square array with 32×32 pixels having a width $dx = 2.0$. The information of the image can be separated into three channels in a given encoding order. The mixture of the information from the three channels following the given encoding order can recover the image [Fig.5(b)]. If the receiver does not decode the signals following the given encoding order and drop

the extra signals, the original image [Fig.5(c)] cannot be recovered. Through the mixture of the matrices in several channels, this communication scheme can enhance the security and increase the rate of transmission.

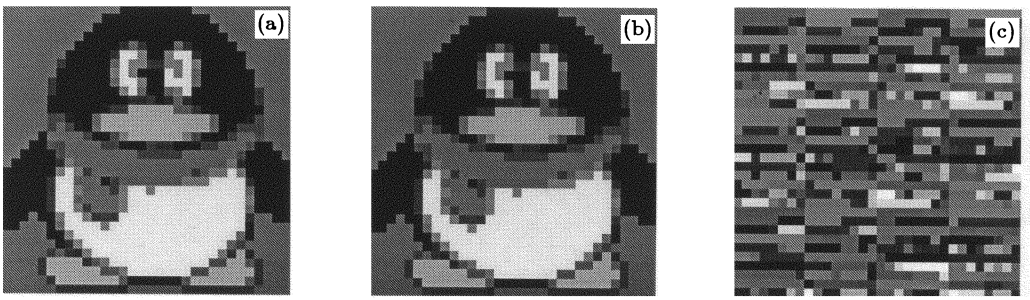


Fig.5. Transmission of 2D static grey-scale image. (a) The original 2D image; (b) The recovered image following the given encoding order; (c) The recovered image not following the given encoding order.

The time trace of the laser intensity is shown in Fig.6. Figures 6(a) and 6(b) are the intensities of the receiver and the transmittal when the two-dimensional matrix $M(2)$ shown in Fig.3(a) is encoded. The parameters are chosen as ξ=1.0×10^{-3}, and $I_j + I_0 = 10$. Since the strength of the messages $\frac{m * \xi}{I_j + I_0} \approx 10^{-4}$ is so weak, the messages are masked in noise-like spiky pulses quite well. In Fig.6(c), it is clear that good chaotic synchronization is obtained between the transmittal and the receiver. Though the messages can be encoded and decoded successfully with very weak signal intensity, the strength of the message is limited by the noise intensity. If the minimum strength of the signal is of one order of magnitude larger than that of noise, the message can be transmitted correctly. Otherwise, the signals are mixed with noise and the transmission fails.

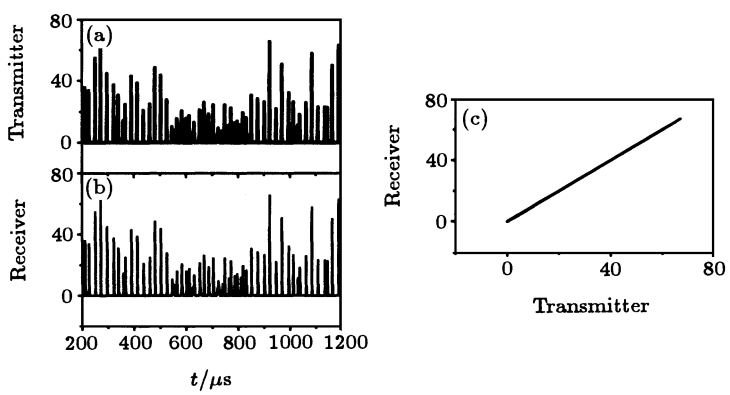


Fig.6. The time series of intensities of receiver and transmittal. (a) The intensity of receiver; (b) intensity of transmittal; (c) relation of receiver and transmittal.

4. Conclusion

The communication using two solid state multimode chaotic lasers with master–slave configuration is investigated. The communication scheme is modified because the chaotic synchronization is quite different between single-mode and multi-mode laser systems. By introducing the time derivative of the difference between the transmittal and the carrier, several digits can be encoded and decoded on a single pulse. If the signals of different height can represent a group of binary bits, two-dimensional messages in the form of a matrix can be transmitted through a single pulse of two multi-mode lasers. With these improvements the transmission rate can be increased thousands of times. Meanwhile, the privacy of communication can be enhanced greatly.

From the intensity profiles shown in Figs.2, 3, and 4, it is seen that there is almost no difference between the receiver and the transmittal as the intensity of

the encoded signal is quite small compared with the threshold. It is also found that the communication scheme does not crucially depend on the shape of the pulse in the transmitter. It is not important whether the pulse is a square wave or not. Only the limitation is that the height of the pulse needs to be above the threshold.

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