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Blind source separation of multichannel electroencephalogram based on wavelet transform and ICA*

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Combination of the wavelet transform and independent component analysis (ICA) was employed for blind source separation (BSS) of multichannel electroencephalogram (EEG). After denoising the original signals by discrete wavelet transform, high frequency components of some noises and artifacts were removed from the original signals. The denoised signals were reconstructed again for the purpose of ICA, such that the drawback that ICA cannot distinguish noises from source signals can be overcome effectively. The practical processing results showed that this method is an effective way to BSS of multichannel EEG. The method is actually a combination of wavelet transform with adaptive neural network, so it is also useful for BBS of other complex signals.

Keywords: blind source separation, electroencephalogram, wavelet transform, independent component analysis

PACC: 0545

1. Introduction

Blind source separation (BSS) was first proposed by Jutten and Herault,[1] and is a technique for estimating original source signals using only observed mixtures. BSS of multichannel electroencephalogram (EEG) has a wide range of applications including clinical diagnosis, focal epilepsies detection, source signal location, and the like. EEG is a complex nonlinear signal with chaotic properties, clinically it is recorded from the multiple electrodes placed over the scalp or intracranial epileptogenic zones of brain: therefore, it is a kind of multichannel signal. Because the brain system is an interconnected neural network, signal crosstalk can present in each channel: therefore, EEG recorded from any channel would be a mixture. Crosstalk is a particular problem with EEG in which one channel may interfere with another. Signal crosstalk is harmful to EEG analysis and clinical diagnosis. By principle component analysis (PCA), initiated by Comon,[2] we can verify the fact that crosstalk exists in multichannel EEGs. In order to perform blind separation of multichannel EEG, we adopt a hybrid method that combines discrete wavelet transform with independent component analysis (ICA).[3−5] Wavelet transform and multiresolution analysis[6,7] has become a popular technique for signal decomposition and reconstruction. Wavelet transform has the advantage of decomposing the original signal into different scales, that is, different levels of fineness (wavelet multiscale decomposition). Therefore, one can remove some decomposed components at lower scales or higher scales, which corresponds to removing the high frequency or low frequency components from the original signal. Based on the remaining components, the original signal can be reconstructed again. With a preprocessing step, some high frequency noises and artifacts are easily removed from the original signals, so that the reconstructed signal could be better for the purpose of ICA. The drawback that ICA cannot distinguish noises from source signals can be overcome effectively.

2. Algorithm of BSS

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After Jutten and Herault's pioneering work,[1] Comon did a nice job to explain why ICA is able to solve the source separation problem.[2] From then on, many approaches of BSS have been developed.[8–10] A good review of this subject has been made by Cardoso.[9]

2.1. Mathematical model

Given m observed scalar signals (or scalar time series) denoted by a matrix $\boldsymbol{x} = (x_1, x_2, \cdots, x_m)^{\mathrm{T}}$, which are assumed to be the mixtures of m unknown independent components (or source signals) and are mutually statistically independent, and zero-mean, denoted by another matrix $\boldsymbol{s} = (s_1,\ s_2,\ \cdots,\ s_m)^{\mathrm{T}}$. It is assumed that each observed series is recorded by one sensor. Therefore, the mathematical model of BSS is given by[3,4]

$$\boldsymbol{x} = \boldsymbol{A}s\ = \sum_{j=1}^{m} a_j s_j + \boldsymbol{n}, \tag{1}$$

where $\boldsymbol{A} = (a_1,\ a_2,\ \cdots,\ a_m)^{\mathrm{T}}$ denotes a $m \times m$ square matrix with real and unknown entries, $\boldsymbol{n}$ denotes the additive noise term. Because it is usually impossible to separate noises from the source signals, it can be omitted from Eq.(1), such that Eq.(1) becomes[4]

$$\boldsymbol{x} = \boldsymbol{A}s\ = \sum_{j=1}^{m} a_j s_j. \tag{2}$$

In order to separate the source signals from observed mixtures, it is necessary to solve or estimate the matrix $\boldsymbol{A}$. However, the estimation of $\boldsymbol{A}$ can usually be simplified by performing a prewhitening of the observed signal $\boldsymbol{x}$. This means that its elements x_i are mutually uncorrelated and all have unit variance.[11] The correlation matrix of $\boldsymbol{x}$ equals unity, that is $E\{\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}\} = \boldsymbol{I}$, and $\boldsymbol{A}$ is an orthogonal matrix. After prewhitening $\boldsymbol{x}$, from Eq.(2) we have

$$\boldsymbol{s} = \boldsymbol{A}^{-1}\boldsymbol{x} = \boldsymbol{A}^{\mathrm{T}}x. \tag{3}$$

Therefore, the BSS of observed data becomes the problem of estimating the matrix $\boldsymbol{A}$. If the ith column of $\boldsymbol{A}$ is denoted by $\boldsymbol{a}_i$, then the ith independent component can be obtained (or estimated) from $\boldsymbol{s}_i = \boldsymbol{a}_i^{-\mathrm{T}}\boldsymbol{x}$.

2.2. Decomposition and reconstruction of original EEG

EEG is usually a super-Gaussian or sub-Gaussian signal which is mixed with noises, artifacts and crosstalks. Because noises and artifacts cannot be separated from the source signal by ICA, Eq.(2) rather than Eq.(1) is used for ICA. Therefore, removing some high frequency noises and artifacts from the original EEG before ICA is an ideal way to the BSS of multichannel EEG. This can be done by wavelet multiscale decomposition. The theoretical description of wavelet multiscale decomposition can be found in Refs.[6, 7, 12–14].

Usually, the global EEG activity is decomposed into four component frequency bands (β, α, θ, δ). The frequencies of these components are very low (in the range of 0.5Hz to 40Hz). Clinically, spike-and-slow-wave complexes are common waves in epileptic patients, its frequency is only 3Hz in seizure. Those significant components of EEG are all low frequency signals. It means that high frequency components of mixed EEG can be removed by wavelet decomposition. Although noises and artifacts are removed, the significant components of EEG are still reserved for reconstruction. In this algorithm, the wavelet decomposition is given by

$$d_k^j = \sum_n h(n-2k)x_n^{j-1}. \tag{4}$$

The wavelet reconstruction of signal $\boldsymbol{x}$ is given by

$$x_n^{j-1} = \sum_k g(n-2k)x_k^j + \sum_k h(n-2k)d_k^j, \tag{5}$$

where j is the scale index, $h(\cdot)$ is the lowpass filter coefficient, $g(\cdot)$ is bandpass filter coefficient, $n = 1, 2 \ldots\ldots N$, $k = 1, 2, \ldots . K$.

Figure 1 shows an original epileptic EEG (upper panel), and its reconstruction (lower panel) with two high frequency components removed at scales j=1, 2, i.e. reconstructed using only three low frequency components at scales j=3–5. Figure 2 shows the decomposition of this EEG at scales j=1–5.

Besides the wavelet base function, the frequency width of wavelet transform also depends on scale factor. In the case of binary wavelet multiscale decomposition, both the central frequency and frequency width are decreased with the increase of scale j.[6] Since the significant components of EEG are all low frequency signals, the feature components of EEG can be effectively reserved when the maximum scale j=5. And at scale j=5, the frequency width is about 40Hz, at scale j=4, the frequency width is about 80Hz. Similarly, j=3 corresponds to 160Hz and j=2 to 320Hz.

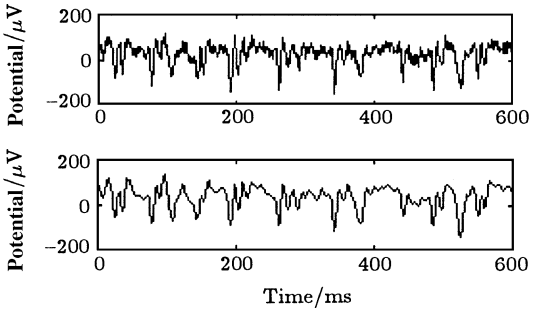


Fig.1. An original epileptic EEG (upper panel) and its reconstruction (lower panel) with two high frequency components removed.

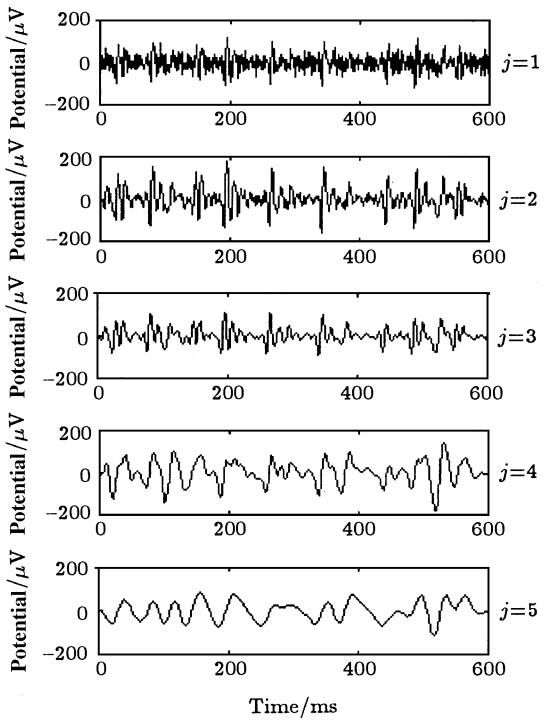


Fig.2. Decomposition of the epileptic EEG at scales j=1–5.

2.3. Algorithm realization

In the field of nonlinear signal processing, methods based on the fourth-order cumulant or kurtosis of signals have been most commonly used for BSS in recent years. For a zero-mean and prewhitened signal $\boldsymbol{x}$, $E\{\boldsymbol{x}\boldsymbol{x}^{\mathrm{T}}\} = \boldsymbol{I}$. Let us search for a linear transform of $\boldsymbol{x}$ by a normalized weight $\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}$, i.e. $\boldsymbol{w}$ ($||\boldsymbol{w}|| = 1$), such that it has maximal or minimal kurtosis. The fourth-order cumulant or kurtosis of the $\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}$ is defined by[4]

$$\begin{aligned}\mathrm{Cum}(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}) &= E\{(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^4\} - 3\,[E\{(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^2\}]^2 \\ &= E\{(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^4\} - 3\,||\boldsymbol{w}||^4. \end{aligned} \quad (6)$$

Let $\boldsymbol{w}$ be a vector formed by one column of $\boldsymbol{A}$ in Eq.(2). All columns of $\boldsymbol{A}$ would be obtained by searching for the appropriate $\boldsymbol{w}$ one by one in an iteration process. After obtaining $\boldsymbol{A}$, we can obtain one component of the mixed $\boldsymbol{x}$ from Eq.(3) as follows:

$$\hat{\boldsymbol{s}}_i = \boldsymbol{w}^{\mathrm{T}}\boldsymbol{x} = \boldsymbol{a}_{\mathrm{i}}^{-\mathrm{T}}\boldsymbol{x}, \quad (7)$$

where $\hat{\boldsymbol{s}}_i$ is the ith estimation or approximation of the original source signals due to $\boldsymbol{w}$ being an estimated vector. If we consider $\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}$ the input of a neural network, $\boldsymbol{w}$ the weight vector, then $\hat{\boldsymbol{s}}_i$ is the output of the network. Generally, Eq.(6) has a number of local maximal or minimal values under the condition of $||\boldsymbol{w}|| = 1$.

In order to solve this constrained problem, we introduce the Lagrange multiplier λ and Lagrange function $L(\boldsymbol{w}, \lambda)$. The constrained problem is then transformed to the problem of solving the extreme values of $L(\boldsymbol{w}, \lambda)$, which is defined by

$$L(\boldsymbol{w}, \lambda) = E\{(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^4\} - 3\,||\boldsymbol{w}||^4 + \lambda||\boldsymbol{w}||^2, \quad (8)$$

where $\lambda||\boldsymbol{w}||^2$ is the penalty item of Eq.(6). Now let $\partial L(\boldsymbol{w}, \lambda)/\partial \boldsymbol{w} = 0$, we have

$$4E\{\boldsymbol{x}(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^3\} - 12||\boldsymbol{w}||^2\boldsymbol{w} + 2\lambda\boldsymbol{w} = 0.$$

Let $c = -2/\lambda$, then we have

$$\boldsymbol{w} = c(E\{\boldsymbol{x}(\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x})^3\} - 3\boldsymbol{w}),$$

where c is obviously an insignificant scalar constant. Let $c = 1$, we obtain the algorithm of $\boldsymbol{w}$ as

$$\boldsymbol{w}(k+1) = E\{\boldsymbol{x}(k)[\boldsymbol{w}(k)^{\mathrm{T}}\boldsymbol{x}(k)^3]\} - 3\boldsymbol{w}(k), \quad (9)$$

where $k = 1, 2, \ldots, K$, which denotes time index.

In order to increase the robustness of the algorithm under the influence of estimation of $\boldsymbol{w}$, we combine Eq.(9) with another adaptive neural algorithm as follows:

$$\boldsymbol{w}_1(n+1) = \boldsymbol{w}_1(n) + \alpha(n)[\hat{\boldsymbol{s}}(n-1) - \boldsymbol{w}_1^{\mathrm{T}}(n)\boldsymbol{x}(n)], \quad (10)$$

where α is the learning factor and set from 0.05 to 0.5. After obtaining the convergent value of $\boldsymbol{w}$ in Eq.(9), we bring it to Eq.(10) iteratively until the $\boldsymbol{w}_1$ converges too.

The introduction of Eq.(10) is based on the fact that each $\hat{\boldsymbol{s}}_i$ is only an estimation or approximation of the original source signals. Therefore, it is necessary that $\hat{\boldsymbol{s}}_i$ is as close to $\boldsymbol{w}^{\mathrm{T}}\boldsymbol{x}$ as possible.

3. Illustration of BSS

In this paper, a total of 200 samples of EEG time series are used, which are recorded from 200 epileptic subjects with sampling frequency 173.61Hz, 12bits A/D transformation, and sampling duration 23.6s. The length of each time series is 4096. Figure 3 shows the plots of 4-channel EEGs recorded from one epileptic patient. They are the mixtures of four source signals with noises and artifacts.

Figure 4 shows the plots of the 4-channel separated (or estimated) signals of epileptic EEG, which is separated from the mixed EEG plotted in Fig.3 by using Eqs.(7), (9), and (10). Figure 4 also shows that the features of epileptic EEG is apparent. The differences among four plots are more apparent than the plots in Fig.3. In view of this fact, we can consider that our method is effective for BSS.

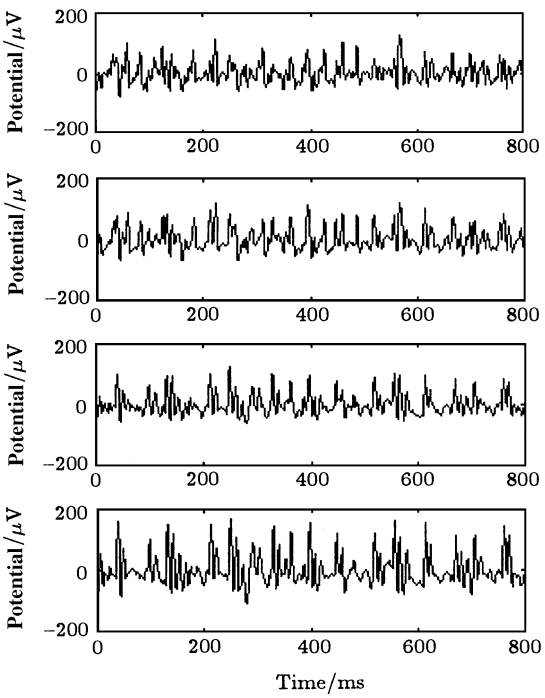


Fig.3. Plots of 4-channel EEGs recorded from one epileptic patient, they are the mixtures of four source signals with noises and artifacts.

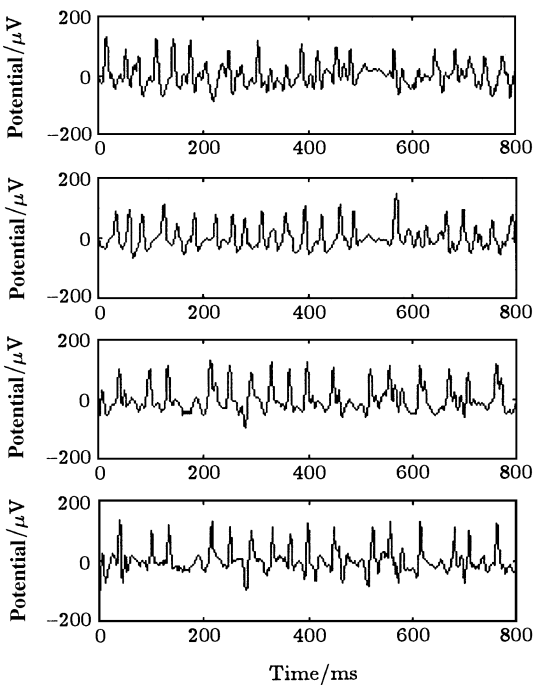


Fig.4. Plots of 4-channel estimated signals of epileptic EEG, which are separated from the mixed EEGs plotted in Fig.3 by using Eq.(9), (10) and (7).

In order to evaluate the efficiency of our algorithm, the performances of our algorithm based on three main wavelets were compared with that without wavelet denoising. The SNR (signal-to-noise ratio) of BSS was introduced as the measure of separation quality. We define the noise of BSS as[15]

$$\boldsymbol{n}(k) = \boldsymbol{x}(k) - \hat{\boldsymbol{s}}(k), \tag{11}$$

then, the SNR of BSS is defined by

$$\mathrm{SNR} = 10\log_{10}\frac{\sum_{k=1}^{N}(\hat{\boldsymbol{s}}(k) - E_{\mathrm{s}})^2}{\sum_{k=1}^{N}(\boldsymbol{n}(k) - E_{\mathrm{n}})^2}\ (\mathrm{dB}), \tag{12}$$

where E_{s} and E_{n} are the mean of the $\hat{\boldsymbol{s}}$ and $\boldsymbol{n}$ respectively. For the four channel EEGs plotted in Fig.3, SNR1=[5.6634, 6.2263, 5.8672, 6.2716]dB, and the mean is 6.0071±0.292dB when only the algorithm in Eq.(9) is employed without denoising. When both Eqs.(9) and (10) are employed with noise removed by wavelet decomposition, SNR2=[7.7642, 7.8268, 8.1256, 8.3242]dB, and the mean is 8.0102±0.2622dB. For 200 time series of EEGs, quadric spline, Morlet and cubic spline wavelets are employed respectively, the statistical results of mean SNR are showed in Table 1. Equations (11) and (12) can be employed to evaluate the effect of wavelet denoising. In the case, the noise is defined by the difference between original and reconstructed signals.

Table 1. Effect comparison of BSS.

Algorithm	SNR/dB
Without noise removal	5.9246±0.2946
Quadric spline wavelet	8.2382±0.2562
Morlet wavelet	7.7252±0.2692
Cubic spline wavelet	7.1446±0.2787

4. Conclusion

Separating the source EEG signals effectively from the observed mixed signals is meaningful in clinical study. In this work, the hybrid method combining discrete wavelet transform with ICA was adopted to remove high-frequency noises and artifacts from the original EEG before performing BSS by ICA.

Table 1 shows that the efficiency of BSS of multichannel EEG based on wavelet denoising is better than that without noise removed. Moreover, test method based on T assumption also verified the conclusion. Although only the BSS of multichannel EEG is studied in this paper, these results suggest that the method combining wavelet transform with ICA has potential advantages for use with the BSS of other complex multichannel signals such as radar signals.

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