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Generalized path-integral solution and coherent states of the harmonic oscillator in D-dimensions*

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We construct a general form of propagator in arbitrary dimensions and give an exact wavefunction of a time-dependent forced harmonic oscillator in $D(D \geq 1)$ dimensions. The coherent states, defined as the eigenstates of annihilation operator, of the D-dimensional harmonic oscillator are derived. These coherent states correspond to the minimum uncertainty states and the relation between them is investigated.

Keywords: D-dimensional harmonic oscillator, coherent state, path integral, propagator
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1. Introduction

As is well known, the harmonic oscillator and coherent states are of great importance in both quantum optics and quantum fields theory. Coherent states, in the coordinate representation, describe wave packet which remains unchanged in shape for an arbitrary long time without dispersing. Coherent states were first considered by Schrödinger in his work on the harmonic oscillator as early as 1926.[1] The concept of coherent states was later used by Glauber[2] to discuss the photo statistics of radiation fields. After that, they have been widely used in various fields of physics. Nieto and Simmons investigated approximate coherent states in a general one-dimensional potential.[3−6] Coherent states for the multi-dimensional Coulomb problem were constructed by Nouri.[7] The path-integral formalism of Feynman provides a general and profound approach to quantum systems. In this theory, if the exact propagator associated with the classical action of a given system is obtained, the wavefunction of the system can be readily calculated. The original Feynman path-integral can be conveniently transformed into a Gaussian integral and this method was applied to calculate the propagator for the harmonic system.[8−12]

The aim of this paper is twofold. In Section 2 the exact Feynman's propagator for the D-dimensional harmonic oscillator is constructed and the wavefunction of the system is obtained. In Section 3 the generalized coherent states for the D-dimensional harmonic oscillator are studied and the correspondent formulas of the annihilation operator A and creation operator A^+ in arbitrary dimensions are constructed. Finally, the relation between the coherent states and the minimum uncertainty states is investigated. In Section 4 we give a terse conclusion.

2. Path-integral solution for the harmonic oscillator in arbitrary dimensions

The Lagrangian L and classical Hamiltonian H of an isotropic harmonic oscillator in D-dimensions may be expressed as

$$L^D(R, \dot{R}) = \frac{m}{2}\dot{R}^2 - \frac{1}{2}m\omega^2 R^2, \qquad (1)$$

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$$H^D(R,P)=\frac{P^2}{2m}+\frac{1}{2}m\omega^2R^2, \tag{2}$$

where R, P are, respectively, the D-dimensional position vector and momentum, and $|R|=\sqrt{\sum_{i=1}^{D}x_i^2}$,$P^2=\sum_{i=1}^{D}p_i^2$. The Hamiltonian operator $\hat{H}$ corresponding to the classical Hamiltonian can be obtained by making the replacement $p_i\to \mathrm{i}\hbar\frac{\partial}{\partial x_i}$, and we have

$$\hat{H}^D(\hat{R},\hat{P})=-\frac{\hbar^2}{2m}\sum_{i=1}^{D}\frac{\partial^2}{\partial x_i^2}+\frac{1}{2}m\omega^2\sum_{i=1}^{D}x_i^2. \tag{3}$$

The Feynman's path-integral propagator K^D for D-dimensions may be written as

$$K^D(R,t;R_0,t_0)=\int_{R^D}[\mathrm{d}^Dx]\exp\left[\frac{\mathrm{i}}{\hbar}\int_{t_0}^{t}\mathrm{d}tL^D(R,\dot{R},t)\right],$$

$$[\mathrm{d}^Dx]=\lim_{\substack{N\to\infty\\ \varepsilon\to 0}}\left(\frac{m}{2\pi\mathrm{i}\hbar\varepsilon}\right)^{ND/2}\prod_{j=1}^{D}\prod_{k=1}^{N-1}\mathrm{d}x_{jk}. \tag{4}$$

Here, we follow Feynman's path-integral method and adopt a Gaussian type propagator K^D,

$$K^D(R,t;R_0,t_0)=A_0(t)\exp[-a_1(t)R^2-a_2(t)R_0^2-a_3(t)RR_0], \tag{5}$$

where $A_0(t)$ is normalization constant. The propagator contains all information about the system's evolution and satisfies the following equation

$$\left[\hat{H}^D(t)-\mathrm{i}\hbar\frac{\partial}{\partial t}\right]K^D(R,t;R_0,t_0)=-\mathrm{i}\hbar\delta(R-R_0)\delta(t-t_0). \tag{6}$$

From Eqs.(3), (5) and (6) we can obtain the coefficients in the propagator K^D, as follows:

$$\begin{aligned}a_1(t)&=\frac{m\omega}{2\mathrm{i}\hbar}\cot(\omega t),\\ a_2(t)&=\frac{m\omega}{2\mathrm{i}\hbar}\cot(\omega t),\\ a_3(t)&=-\frac{m\omega}{\mathrm{i}\hbar\sin(\omega t)},\\ A_0(t)&=\left(\frac{m\omega}{2\pi\mathrm{i}\hbar\sin(\omega t)}\right)^{D/2}.\end{aligned} \tag{7}$$

Now, we can write the explicit form of the propagator K^D,

$$\begin{aligned}K^D(R,t;R_0,t_0=0)&=\left(\frac{m\omega}{2\pi\mathrm{i}\hbar\sin\omega t}\right)^{D/2}\exp\left\{\frac{\mathrm{i}m\omega}{2\hbar\sin\omega t}[(R^2+R_0^2)\cos\omega t-2RR_0]\right\}\\ &=\left(\frac{m\omega}{2\pi\mathrm{i}\hbar\sin\omega t}\right)^{D/2}\prod_{j=1}^{D}\exp\left\{\frac{\mathrm{i}m\omega}{2\hbar\sin\omega t}[(x_j^2+x_{0j}^2)\cos\omega t-2x_jx_{0j}]\right\}.\end{aligned} \tag{8}$$

For the $D=1$ case, the K^D is reduced to

$$K(x,t;x_0,t_0=0)=\left(\frac{m\omega}{2\pi\mathrm{i}\hbar\sin\omega t}\right)^{1/2}\exp\frac{\mathrm{i}m\omega}{2\hbar\sin\omega t}[(x^2+x_0^2)\cos\omega t-2xx_0], \tag{9}$$

it turns into the usual formula of the propagator K for a simple harmonic oscillator in one-dimension.

The wavefunction of the system in D-dimensions is readily calculated using the formula

$$\psi^D(R,t)=\int_{R^D}\mathrm{d}^Dx_0K^D(R,t;R_0,t_0)\psi^D(R_0,t_0), \tag{10}$$

where $\psi^D(R_0,t_0)$ is the D-dimensional wavefunction for a simple harmonic oscillator. We can write $\psi^D(R_0,t_0)$ in Cartesian space as[13]

$$\begin{aligned}\psi^D_{N_1,\ldots,N_D}(R_0,t_0)=&\prod_{j=1}^{D}\left(\alpha/\sqrt{\pi}2^{N_j}N_j!\right)^{1/2}\\ &\times\exp\left(-\frac{\alpha^2x_{0j}^2}{2}\right)H_{N_j}(\alpha x_{0j})\\ &\times\exp\left[-\mathrm{i}(N_j+\frac{1}{2})\omega t\right],\end{aligned} \tag{11}$$

where $H_{N_j}(\alpha x_0)$ is the Hermite polynomials of order N_j and $\alpha=(m\omega/\hbar)^{1/2}$.

Now, let us consider a harmonic oscillator driven by a D-dimensional external force $F^D(t) = \sum_{i=1}^{D} f_i(t)$. The Hamiltonian operator $\hat{H}(t)$ of a time-dependent forced harmonic oscillator in D-dimensions may be expressed in coordinate representation as

$$\hat{H}^D(\hat{R},\hat{P},t) = -\frac{\hbar^2}{2m}\sum_{i=1}^{D}\frac{\partial^2}{\partial x_i^2} + \frac{1}{2}m\omega^2\sum_{i=1}^{D}x_i^2 - \sum_{i=1}^{D}x_i f_i(t), \tag{12}$$

where the external driving force $f_i(t)$ is assumed to be turned on at time $(t = 0)$, and for $t \le 0$ the system is described by the simple harmonic oscillator wavefunction. It is conveniently to adopt a Gaussian type propagator K^D,

$$K^D(R,t;R_0,t_0) = A_0(t)\exp\Big[-a_1(t)\sum_{j=1}^{D}x_j^2 - \sum_{j=1}^{D}a_{2j}(t)x_j - a_3(t)\sum_{j=1}^{D}x_{0j}^2 - a_4(t)\sum_{j=1}^{D}x_j x_{0j} - \sum_{i=1}^{D}a_{5j}(t)x_{0j}\Big]. \tag{13}$$

From Eqs.(6), (12) and (13), we can obtain the coefficients in the propagator K^D as follows:

$$\begin{aligned}
a_1(t) &= \frac{m\omega}{2\mathrm{i}\hbar}\cot(\omega t),\\
a_{2j}(t) &= \frac{m}{\mathrm{i}\hbar}[-\omega\cot(\omega t)\xi_j(t) + \dot{\xi}_j(t)],\\
a_3(t) &= \frac{m\omega}{2\mathrm{i}\hbar}\cot(\omega t),\\
a_4(t) &= -\frac{m\omega}{\mathrm{i}\hbar\sin(\omega t)},\\
a_{5j}(t) &= \frac{m\omega}{\mathrm{i}\hbar\sin(\omega t)}\xi_j(t),\\
\xi_j(t) &= \int_0^t \frac{f_j(\tau)}{m\omega}\sin[\omega(t-\tau)]\mathrm{d}\tau,\\
A_0(t) &= \left(\frac{m\omega}{2\pi\mathrm{i}\hbar\sin(\omega t)}\right)^{D/2} \times \prod_{j=1}^{D}\exp\left\{\frac{m}{2\mathrm{i}\hbar}\int_0^t[-\omega\cot(\omega\tau)\xi_j(\tau) + \dot{\xi}_j(\tau)]\mathrm{d}\tau\right\}.
\end{aligned} \tag{14}$$

Now, we can write the explicit form of the propagator K^D,

$$\begin{aligned}
&K^D(R,t;R_0,t_0=0)\\
&= \left(\frac{m\omega}{2\pi\mathrm{i}\hbar\sin\omega t}\right)^{D/2}\exp\frac{\mathrm{i}m\omega}{2\hbar\sin\omega t}\\
&\times\left\{\left(\sum_{j=1}^{D}x_j^2 + \sum_{j=1}^{D}x_{0j}^2\right)\cos\omega t - 2\sum_{j=1}^{D}x_j x_{0j}\right.\\
&+\frac{2}{\omega}\sum_{j=1}^{D}[x_j\dot{\xi}_j(t)\sin\omega t - \omega x_j\xi_j(t)\cos\omega t]\\
&+2\sum_{j=1}^{D}x_{0j}\xi_j(t) + \sum_{j=1}^{D}\int_0^t\frac{\omega}{\sin\omega t}\\
&\times\left.\left[\xi_j(\tau)\cos(\omega\tau) - \frac{\dot{\xi}_j(\tau)\sin\omega t}{\omega}\right]^2\mathrm{d}t\right\}.
\end{aligned} \tag{15}$$

If we let the external driving force $f(t) \equiv 0$, the propagator K^D will turn into Eq.(8), which is the formula of the propagator K^D for a simple harmonic oscillator in D-dimensions.

From Eqs.(10) and (11) we can obtain the wavefunction of the time-dependent system in D-dimensional Cartesian space as

$$\begin{aligned}
\psi^D_{N_1,\cdots,N_D}(R,t) &= \int_{R^D}\mathrm{d}^D x_0 K^D(R,t;R_0,t_0)\psi^D_{N_1,\cdots,N_D}(R_0,t_0)\\
&= \int_{R^D}\mathrm{d}^D x_0 K^D(R,t;R_0,t_0)\prod_{j=1}^{D}\left(\alpha/\sqrt{\pi}2^{N_j}N_j!\right)^{1/2}\\
&\times\exp\left(-\frac{\alpha^2x_j^2}{2}\right)H_{N_j}(\alpha x_j)\exp\left[-\mathrm{i}\left(N_j+\frac{1}{2}\right)\omega t\right].
\end{aligned} \tag{16}$$

Substituting Eqs.(11) and (15) into Eq.(16), and performing the integration, we obtain

$$\psi^{D}_{N_1,\ldots,N_D}(R,t)=\prod_{j=1}^{D}\left(\alpha/\sqrt{\pi}2^{N_j}N_j!\right)^{1/2}H_{N_j}\{\alpha[x_j-\xi_j(t)]\}$$
$$\times\exp\Bigg\{-\frac{\alpha^2x_j^2}{2}+\alpha^2x_j\left[\xi_j(t)+\frac{\mathrm{i}\dot{\xi}_j(t)}{\omega}\right]-\frac{\alpha^2}{2}\xi_j^2(t)\Bigg\{1+\mathrm{i}\cot(\omega t)$$
$$+\mathrm{i}\int_0^t\left[\frac{-\omega\cot(\omega t)\xi_j(\tau)+\dot{\xi}_j(\tau)}{\omega\xi_j(\tau)}\right]^2\mathrm{d}\tau\Bigg\}\Bigg\}\exp\left[-\mathrm{i}\left(N_j+\frac{1}{2}\right)\omega t\right]. \tag{17}$$

Thus, we obtain the exact wavefunction of a time-dependent forced harmonic oscillator in D-dimensions.

3. Coherent states for the harmonic oscillator in arbitrary dimensions

In this section we use the preceding results and construct the coherent states for the D-dimensional problem. Coherent state is defined as eigenvector of the annihilation operator, which is a superposition of the energy eigenstates of the harmonic oscillator.

We are able to construct the general formulas of the annihilation operator A and creation operator A^+ for D-dimensions as

$$a_j=\frac{1}{\sqrt{2m\hbar\omega}}(m\omega x_j+\mathrm{i}p_j),$$
$$a_j^+=\frac{1}{\sqrt{2m\hbar\omega}}(m\omega x_j-\mathrm{i}p_j),$$
$$[a_j,a_k]=0,\quad [a_j^+,a_k^+]=0,\quad [a_j,a_k^+]=\delta_{jk},$$
$$A=\sum_{j=1}^{D}a_j,\quad A^+=\sum_{j=1}^{D}a_j^+,$$
$$[A,A^+]=\left[\sum_{j=1}^{D}a_j,\quad\sum_{j=1}^{D}a_j^+\right]=D,$$

$$H^D=\frac{1}{2m}\sum_{j=1}^{D}p_j^2+\frac{m\omega^2}{2}\sum_{j=1}^{D}x_j^2$$
$$=\hbar\omega\left(\sum_{j=1}^{D}a_j^+a_j+\frac{D}{2}\right). \tag{18}$$

Then, the generalized coherent states for D-dimensions may be written as

$$|\alpha_1\rangle\otimes|\alpha_2\rangle\otimes\ldots\otimes|\alpha_D\rangle$$
$$=\exp\left[-\frac{1}{2}(|\alpha_1|^2+|\alpha_2|^2+\ldots+|\alpha_D|^2)\right]$$
$$\times\prod_{j=1}^{D}\sum_{N_j=0}^{\infty}\frac{\alpha_j^{N_j}}{\sqrt{N_j!}}|N_j\rangle, \tag{19}$$

we can also write it in the concise form

$$|[\alpha]\rangle=\exp\left(-\frac{1}{2}|[\alpha]|^2\right)\sum_{[N]=0}^{\infty}\frac{[\alpha]^{[N]}}{\sqrt{[N]!}}|[N]\rangle, \tag{20}$$

where $[\alpha]=(\alpha_1,\alpha_2,\ldots,\alpha_D)$, $[N]=(N_1,N_2,\ldots,N_D)$ and $[N]!=N_1!N_2!\ldots N_D!$. From Eqs.(18), (19) and (20), we can obtain

$$A|[\alpha]\rangle=\sum_{k=1}^{D}a_k|[\alpha]\rangle=\exp\left[-\frac{1}{2}(|\alpha_1|^2+|\alpha_2|^2+\ldots+|\alpha_D|^2)\right]\sum_{k=1}^{D}\left[a_k\prod_{j=1}^{D}\sum_{N_j=0}^{\infty}\frac{\alpha_j^{N_j}}{\sqrt{N_j!}}|N_j\rangle\right]$$
$$=\left(\sum_{k=1}^{D}\alpha_k\right)\exp\left[-\frac{1}{2}(|\alpha_1|^2+|\alpha_2|^2+\ldots+|\alpha_D|^2)\right]\prod_{k=1}^{D}\sum_{N_k=1}^{\infty}\frac{\alpha_k^{N_k-1}}{\sqrt{(N_k-1)!}}|N_k-1\rangle$$
$$=\left(\sum_{k=1}^{D}\alpha_k\right)|[\alpha]\rangle. \tag{21}$$

Obviously, the coherent states $|[\alpha]\rangle$ are just the eigenvectors of the annihilation operator $A=\sum_{j=1}^{D}a_j$. We

can also prove the completeness of the coherent states

$$\begin{aligned}\frac{1}{\pi^D}\int |[\alpha]\rangle\langle[\alpha]|\mathrm{d}^2[\alpha] =& \prod_{j=1}^{D}\left[\frac{1}{\pi}\int |\alpha_j\rangle\langle\alpha_j|\mathrm{d}^2\alpha_j\right]\\ =& \prod_{j=1}^{D}\left[\frac{1}{\pi}\sum_{M_j,N_j}\frac{|N_j\rangle\langle M_j|}{\sqrt{N_j!M_j!}}\int \alpha_j^{N_j}\alpha_j^{*M_j}\exp(-|\alpha_j|^2)\mathrm{d}^2\alpha_j\right]\\ =& \prod_{j=1}^{D}\left\{\frac{1}{\pi}\sum_{M_j,N_j}\frac{|N_j\rangle\langle M_j|}{\sqrt{N_j!M_j!}}\int_0^\infty |\alpha_j|^{N_j+M_j+1}\exp(-|\alpha_j|^2)\mathrm{d}|\alpha_j|\int_0^{2\pi}\exp[\mathrm{i}(N_j-M_j)]\mathrm{d}\varphi\right\}\\ =& \prod_{j=1}^{D}\left[\frac{1}{\pi}2\pi\sum_{N_j}\frac{|N_j\rangle\langle N_j|}{N_j!}\int_0^\infty |\alpha_j|^{2N_j+1}\exp(-|\alpha_j|^2)\mathrm{d}|\alpha_j|\right]\\ =& \prod_{j=1}^{D}\left(\sum_{N_j}|N_j\rangle\langle N_j|\right)\\ =& \hat{I},\end{aligned} \tag{22}$$

where a polar coordinate $\mathrm{d}^2\alpha_j = |\alpha_j|\,\mathrm{d}|\alpha_j|\,\mathrm{d}\varphi$ has been used at the intermediate stage of the calculation, and $\hat{I}$ means the unit operator. By using Eqs.(11) and (20), we can construct the generalized coherent states for the harmonic oscillator in arbitrary dimensions in the coordinate representation

$$\begin{aligned}\langle R\,|[\alpha]\rangle =& \exp\left(-\frac{1}{2}|[\alpha]|^2\right)\sum_{[N]=0}^{\infty}\frac{[\alpha]^{[N]}}{\sqrt{[N]!}}\langle R\,|[N]\rangle\\ \Rightarrow \psi^D_{[\alpha]}(R,t) =& \exp\left(-\frac{1}{2}|[\alpha]|^2\right)\\ &\times\sum_{[N]=0}^{\infty}\frac{[\alpha]^{[N]}}{\sqrt{[N]!}}\psi^D_{N_1,\ldots,N_D}(R,t).\end{aligned} \tag{23}$$

If we define $\alpha_j = \sqrt{\frac{m\omega}{2\hbar}}x_{0j} = \frac{\xi_{0j}}{\sqrt{2}}$, and $\sqrt{\frac{m\omega}{\hbar}}x_j = \xi_j$, and substitute Eq.(11) into Eq.(23), then we obtain

$$\begin{aligned}\psi^D_{[\alpha]}(R,t) =& \left(\frac{m\omega}{\pi\hbar}\right)^{D/4}\prod_{j=1}^{D}\exp\left(-\frac{\mathrm{i}\omega t}{2}-\frac{\xi_j^2}{2}-\frac{\xi_{0j}^2}{2}\right)\\ &\times\sum_{N_j}^{\infty}\frac{1}{\sqrt{N_j!}}H_{N_j}(\xi_j)\left[\frac{\xi_{0j}}{2}\exp(-\mathrm{i}\omega t)\right]^{N_j}.\end{aligned} \tag{24}$$

By using the expansion of the Hermite polynomial

$$\begin{aligned}&\sum_{N_j=0}^{\infty}\frac{1}{\sqrt{N_j!}}H_{N_j}(\xi_j)\left[\frac{\xi_{0j}}{2}\exp(-\mathrm{i}\omega t)\right]^{N_j}\\ =&\exp\left[-\frac{1}{4}\xi_{0j}^2\exp(-2\mathrm{i}\omega t)+\xi_j\xi_{0j}\exp(-\mathrm{i}\omega t)\right],\end{aligned} \tag{25}$$

the summation in Eq.(24) can be easily carried out

$$\begin{aligned}\psi^D_{[\alpha]}(R,t) =& \left(\frac{m\omega}{\pi\hbar}\right)^{D/4}\prod_{j=1}^{D}\exp\Big[-\frac{1}{2}(\xi_j-\xi_{0j}\cos\omega t)^2\\ &-\mathrm{i}\Big(\frac{1}{2}\omega t+\xi_j\xi_{0j}\sin\omega t\\ &-\frac{1}{4}\xi_{0j}^2\sin 2\omega t\Big)\Big],\end{aligned} \tag{26}$$

$$\begin{aligned}\left|\psi^D_{[\alpha]}(R,t)\right|^2 =& \left(\frac{m\omega}{\pi\hbar}\right)^{D/2}\\ &\times\prod_{j=1}^{D}\exp\left[-\frac{m\omega}{\hbar}(x_j-x_{0j}\cos\omega t)^2\right].\end{aligned} \tag{27}$$

The function given in Eq.(26) is just the wavefunction of coherent states for the D-dimensional isotropic harmonic oscillator in coordinate representation. This coherent state is a displaced ground state wavefunction centred around the $R_0\cos\omega t$ in D-dimensional coordinate space. When $[\alpha]$ is complex, it is a ground wavefunction whose centre is displaced by a complex value. The coherent states correspond to the minimum uncertainty states, and we can obtain the following relations in D-dimensions

$$\langle x_j\rangle = \sqrt{\frac{\hbar}{2m\omega}}\langle\alpha_j|\,(a_j^+ + a_j)\,|\alpha_j\rangle = \sqrt{\frac{\hbar}{2m\omega}}(\alpha_j^* + \alpha_j),$$

$$\langle p_j\rangle = \mathrm{i}\sqrt{\frac{m\omega\hbar}{2}}\,\langle\alpha_j|\,(a_j^+ - a_j)\,|\alpha_j\rangle = \mathrm{i}\sqrt{\frac{m\omega\hbar}{2}}(\alpha_j^* - \alpha_j),$$

$$\langle x_j^2\rangle = \frac{\hbar}{2m\omega}[1+(\alpha_j+\alpha_j^*)^2],$$

$$\langle p_j^2 \rangle = \frac{m\omega\hbar}{2}[1-(\alpha_j^* - \alpha_j)^2],$$

$$\Delta x = \sqrt{\langle x_j^2 \rangle - \langle x_j \rangle^2} = \sqrt{\frac{\hbar}{2m\omega}},$$

$$\Delta p = \sqrt{\langle p_j^2 \rangle - \langle p_j \rangle^2} = \sqrt{\frac{m\omega\hbar}{2}},$$

$$[\Delta x_j \Delta p_j]_{\alpha_j} = \frac{1}{2}\hbar, (j = 1, 2, ..., D). \qquad (28)$$

4. Conclusion

In summary, we have constructed the generalized formulas of Feynman's propagator and the annihilation operator for arbitrary dimensions. And we also obtained the exact wavefunction and the coherent states for the D-dimensional harmonic oscillator, which can be easily reduced to the three-dimensional formula. As a result, in a special three-dimensional case, the correspondent exact wavefunction for a time-dependent forced harmonic oscillator may be used to study the quantum properties of a molecule absorbed on or near a solid surface in a time-dependent external electric field. Moreover, the generalized coherent states can provide an adequate means for a quantum coherency description of the harmonic problem in arbitrary dimensions. In this sense, we think that we have obtained the general formulas.

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