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Stochastic resonance and nonequilibrium dynamic phase transition of Ising spin system driven by a joint external field*

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The dynamic response and stochastic resonance of a kinetic Ising spin system (ISS) subject to the joint action of an external field of weak sinusoidal modulation and stochastic white-noise are studied by solving the mean-field equation of motion based on Glauber dynamics. The periodically driven stochastic ISS shows that the characteristic stochastic resonance as well as nonequilibrium dynamic phase transition (NDPT) occurs when the frequency ω and amplitude h_0 of driving field, the temperature t of the system and noise intensity D are all specifically in accordance with each other in quantity. There exist in the system two typical dynamic phases, referred to as dynamic disordered paramagnetic and ordered ferromagnetic phases respectively, corresponding to a zero- and a unit-dynamic order parameter. The NDPT boundary surface of the system which separates the dynamic paramagnetic phase from the dynamic ferromagnetic phase in the 3D parameter space of h_0-t-D is also investigated. An interesting dynamical ferromagnetic phase with an intermediate order parameter of 0.66 is revealed for the first time in the ISS subject to the perturbation of a joint determinant and stochastic field. The intermediate order dynamical ferromagnetic phase is dynamically metastable in nature and owns a peculiar characteristic in its stability as well as the response to external driving field as compared with a fully order dynamic ferromagnetic phase.

Keywords: Ising spin system, stochastic resonance, dynamic phase transition, dynamical symmetry
PACC: 7510H, 0540, 7530K, 1130N

1. Introduction

Stochastic resonance (SR) is an interesting phenomenon and has attracted much attention from physicists over the last decade.[1−10] Contrary to conventional perception of destructive effect caused by noise, noise in an SR process surely plays a constructive positive role in transmission of noisy signals and patterns forming through the nonlinearly cooperative interaction between random signal and determinant modulation within a system. Generally, the bistable system in a double well model has been fully studied in comparison with that in an Ising spin system. Some efforts, however, have been devoted to the particular studies of SR in the Ising spin system in recent years.[1,2,4,6−13] Among those investigations of SR in an Ising model, most of them are concerned with the practical applications of devices by resorting to SR technology. As for a kinetic Ising spin system, another subject of interest is associated with the nonequilibrium dynamical phase transition (NDPT)[14−22] and the nonlinear resonant excitation of soliton[23] induced by an external driving field. Unfortunately, the investigations on NPDT features in the Ising model have been executed mainly under the condition of either a determinant driving field, e.g. sinusoidal oscillation, or a stochastic impulsive perturbation. To our certain knowledge, there have not yet existed any systematical studies ever reported, even including our recent paper,[24] concerning the NDPT features in a kinetic Ising model subject to a persistent stochastic perturbation plus a determinant periodic modulation. This paper focuses on the NDPT in the kinetic Ising model driven simultaneously by white noise and sinusoidal oscillation and has an insight into the dynamic response of Ising spin system to the joint external field

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as well as possible new features about NDPT with stochastic resonance involved.

2. Brief description of nonequilibrium dynamic phase transition and stochastic resonance

2.1. Nonequilibrium dynamic transition of kinetic Ising spin system subject to an external field

Nonequilibrium dynamic phase transition occurs in a kinetic Ising spin system when a dynamic external field is applied, and the system exists in either dynamic disordered or dynamic ordered state. Considering a kinetic Ising spin system with N interactive spins S subject to an external field $H(\tau)$, Hamiltonian could be expressed as follows:

$$\hat{H} = -\frac{J}{N}\sum_{(i,j)} \boldsymbol{S}_i \cdot \boldsymbol{S}_j - H(\tau)\sum_i S_i, \qquad (1)$$

where J denotes the exchange coupling constant and spin S takes a value either positive or negative. $\sum_{(i,j)}$ and $\sum_i$ stand for the sum of neighboring spin pairs and the sum of all spins, respectively. $H(\tau)$ is a resultant driving field consisting of a determinant periodic modulation (sinusoidal oscillation $h_0 \sin(\omega\tau)$) and a stochastic perturbation (white noise $\xi(\tau)$) as expressed in Eq.(2a). Symbols h_0, ω and τ represent the amplitude and frequency of sinusoidal field, and evolution time, respectively. Gaussian white noise conforms to the zero average and auto-correlativity as listed in Eq.(2b). D is the intensity of white noise. The Kronecker Delta function $\delta(\tau)$ is expressed in Eq.(2c):

$$H(\tau) = h_0 \sin(\omega\tau) + \sqrt{2D}\zeta(\tau), \qquad (2a)$$

$$\langle \xi(\tau) \rangle = 0, \quad \langle \zeta(\tau)\zeta(0) \rangle = 2D\delta(\tau), \qquad (2b)$$

$$\delta(\tau - \tau') = \begin{cases} 1, & \tau = \tau', \\ 0, & \tau \neq \tau'. \end{cases} \qquad 2(c)$$

For the Ising model with nearest-neighbour interactive spins as described in Eq.(1), the mean-field equation of motion based on Glauber dynamics is given as[14]

$$\frac{\partial m(\tau)}{\partial \tau} = K\left(\mathrm{Tanh}\left[\frac{m(\tau) + H(\tau)}{t}\right] - m(\tau)\right), \qquad (3)$$

where the static order parameter $m(\tau) = < S_i >$ is an ensemble average over all spins. Symbols t and K are the temperature of the system and a phenomenological constant, respectively. For simplicity, all physical variables in Eq.(3) are reduced to the dimensionless form. The dynamic order parameter Q is defined below to describe quantitatively the dynamical response of the system to the driving field as well as the characteristics of the relevant NDPT:

$$Q = \frac{\omega}{2\pi}\oint m(\tau)\mathrm{d}\tau. \qquad (4)$$

The dynamic order parameter Q is defined as the period-average of the static order parameter m. The two states when $Q = 0$ and $Q \neq 0$ correspond to the dynamic disordered and dynamic ordered states, and both of them are generally referred to as dynamical paramagnetic phase (P) and dynamical ferromagnetic phase (F), respectively. The decrease in Q parameter from non-zero to zero means that the dynamic symmetry-breaking vanishes and the system evolves into dynamic symmetry (i.e. dynamic disorder) finally while $Q = 0$. Figure 1 exhibits a typical NDPT boundary of the kinetic Ising spin system driven simply by a sinusoidal field at zero noise $D = 0$. The NDPT boundary is plotted on the plane consisting of amplitude h_0 and temperature t, and on the dynamic boundary curve situated are the critical points of amplitude h_{0c} and temperature t_c. Dynamic paramagnetic phase ($Q = 0$) and dynamic ferromagnetic phase ($Q \neq 0$) occupy the two regions above and below the boundary in the h_0-t phase diagram, respectively. The two insets in Fig.1 illustrate the time evolutions

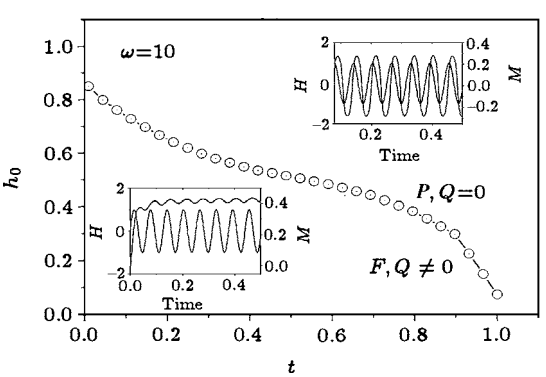


Fig.1. The illustrative boundary of dynamic phase transition at zero noise intensity D=0. The boundary of dynamic phase transition separates dynamical ordered ferromagnetic phase (F, $Q \neq 0$) from the disordered paramagnetic (P, Q=0) phase. Two insets show the evolutions of sinusoidal field (H) and magnetization (M) versus time within F and P regions, respectively.

of static order parameter $m(\tau)$ and driving field $H(\tau)$. The static order parameter $m(\tau)$ oscillates symmet-

rically in accordance with driving field $H(\tau)$ within the region P, resulting in a dynamic symmetry with $Q = 0$. In contrast, a dynamic symmetry-breaking with Q≠0 occurs because the static order parameter $m(\tau)$ always holds in either positive or negative status, leading to a dynamic ferromagnetic ordering.

The amplitude h_0 and frequency ω of driving field as well as the temperature t are vital to the nonequilibrium dynamic phase transition of a kinetic Ising system. When a cooperatively interacting spin system with numerous degrees of freedom of motion is driven by an external field, the thermodynamic response of the system usually lags behind the applied field due to the intrinsic relaxation delay, giving rise to a nonequilibrium dynamic hysteresis.[17] This hysteresis is dynamical in origin and disappears in the quasistatic limit. When the time period of the external driving field $2\pi/\omega$ becomes comparable in quantity with the typical relaxation time λ of the thermodynamic system, the dynamic phase arises spontaneously out of dynamically broken symmetries due to the competition between the above two time scales in such a nonequilibrium driven system.[18,20] Studying the matching between the two competing time scales under a variety of conditions is one of the most interesting topics on NDPT.[15−21]

2.2. Stochastic resonance

Consider a simple case in which a Brownian particle agitated by a fluctuational force, e.g. noise, moves in a symmetric double-well potential. The fluctuational force causes the transition of the particle between the neighboring potential wells at a rate given by the famous Kramers rate[1]

$$\gamma_k = \gamma_0 \exp(-\Delta V/D). \tag{5}$$

The prefactor γ_0 is a positive constant. ΔV and D are the height of the potential barrier separating the two minima and the noise intensity, respectively. D is directly proportional to temperature T as $D = k_{\rm B}T$. The intensity of noise D has a twofold effect on the jump capability of moving particle over the energy barrier as well as the jump frequency, and this differs significantly from the amplitude of determinant driving field. When the stochastic field and determinant periodic driving field are applied simultaneously, there appear two characteristic time scales in competition: the vibrating period of determinant driving field τ_ω and the average transition lifetime of moving particle jumping twice over double-well energy barrier $\tau_k(D)$. Apparently,

$$\tau_\omega = 2\pi/\omega, \quad \text{and} \quad \tau_k(D) = 1/\gamma_k. \tag{6}$$

When the two competing time scales above match exactly each other in quantity, the original independent stochastic signal and determinant driving modulation interact with each other in synchronization and the stochastic resonance comes into being. It is easy to derive the matching conditions suitable for the occurrence of stochastic resonance as below

$$\tau_\omega = 2\tau_k(D), \quad \text{and} \quad \omega \sim \pi\gamma_k. \tag{7}$$

The matching of above conditions can be attained through tuning either the frequency ω of determinant driving field or the intensity of noise D. In this paper, we concentrate on the variation of intensity of noise D and take D as the variable for stochastic resonance. The dynamic order parameter Q defined in Eq.(4) serves as a characteristic indicator for NDPT resulting from the stochastic field. The distinctive intensity of noise D is marked as $D_{\rm SR}$, the characteristic value optimal for stochastic resonance, at which dynamic order parameter Q peaks. Actually, the NDPT owing to the stochastic resonance is in nature different from the NDPT caused simply by a determinant periodic oscillation.

The statement above makes it clear that there exist three kinds of competing time scales in the Ising spin system driven by a determinant modulation and stochastic perturbation jointly. They are the intrinsic relaxation time λ of a thermodynamic system, the time period of an external driving field $2\pi/\omega$ and the average transition lifetime of moving particle jumping twice over a double-well energy barrier $\tau_k(D)$ derived from the Kramers rate, respectively. Of the three time scales, the matching between λ and $2\pi/\omega$ brings about the conventional NDPT while the matching of $2\pi/\omega$ with $\tau_k(D)$ gives rise to the stochastic resonance. Could a new NDPT characterizing the stochastic resonance be attained if the three competing time scales befit each other in a certain way?

3. Computational result and analysis

As elaborated above, the dynamic order parameter Q of the Ising spin system driven by an external field changes either from zero to nonzero or vice versa when the system undergoes a nonequilibrium dynamic phase transition. We select the amplitude h_0 and the

frequency ω of external field, the intensity of noise D and temperature t as the variables to investigate the dependence of parameter Q on them in this paper. On the consideration of the linear response, the amplitude is limited to $h_0 \leq 1$ while the frequency ω and the intensity of noise D take values in a wide range. Below are some of typical results.

Figure 2 presents the various curves of reduced $Q/Q_{\max}$ versus D at $t = 0.8$ and $h_0 = 0.5$ with different frequencies ω. All curves take on a peak-shaped trend, and the parameter Q attains the maximum at a certain D value, denoted as $D_{\rm SR}$, a characteristic of the stochastic resonance. Only single resonant peak or valley is observed in a wide range of D from 10^{-3} to 10^3. The increase in frequency causes the shift of resonant peak to small values of D, accompanying the change of the shape of curve from peak to valley. The change of the shape of curve above indicates the transformation of the system from a steady state of preferentially dynamic symmetry to a steady state dominated by dynamic symmetry-breaking.

In addition to the intensity of noise D, frequency

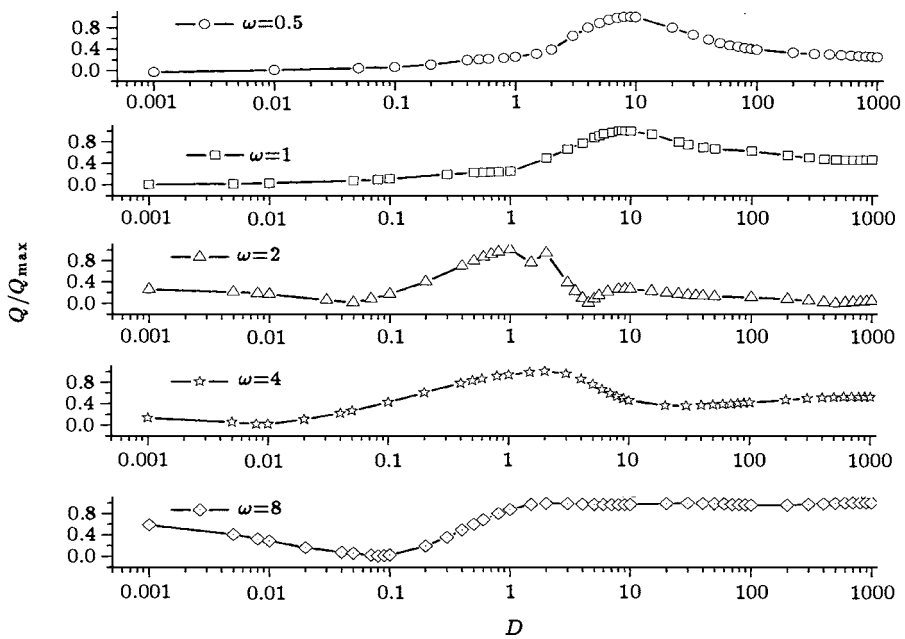


Fig.2. Dynamic order-parameter Q versus intensity of noise D at several frequencies with a high temperature t=0.8 and the amplitude h_0=0.5 of driving field.

ω is another key factor affecting the NDPT, which has been identified by numerous studies in the absence of noise.[14–20] The frequency dependence of NDPT in the presence of noise, however, remains yet to be studied. Under the condition free of noise, the determinant periodic modulation acts as just a factor triggering the NDPT of system. It is of theoretical significance to investigate the possible change, if any, in the feature of NDPT when a stochastic field is involved. Figure 3 displays the trend of $Q/Q_{\max}$ versus both h_0 and D at a low temperature $t = 0.3$ and high temperature $t = 0.8$ with the frequency fixed at ω=1. The stable regions of dynamic ferromagnetic (Q≠0) and paramagnetic phases ($Q = 0$) exist in a parameter space of h_0-D. At a low temperature, the system always stays in a dynamic ferromagnetic state with a terrace at $Q = 1$ if both h_0 and D keep small ($h_0 < 0.3$, $D < 0.1$), which is quite similar to the situation of symmetry-breaking in a conventional zero-noise case. When h_0 and D take larger values ($h_0 > 0.5$, $D > 0.1$), however, Q data surface transforms into a peak shape with $D_{\rm SR} = 1$, a new evidence of NDPT characterizing the stochastic resonance. In contrast, at high temperatures the terrace region of common NDPT shrinks substantially while the system-breaking region of stochastic resonance extends, and the characteristic intensity of noise $D_{\rm SR}$ shifts also from the preceding $D_{\rm SR} = 1$ at $t = 0.3$ to the current $D_{\rm SR} \geq 10$ at $t = 0.8$.

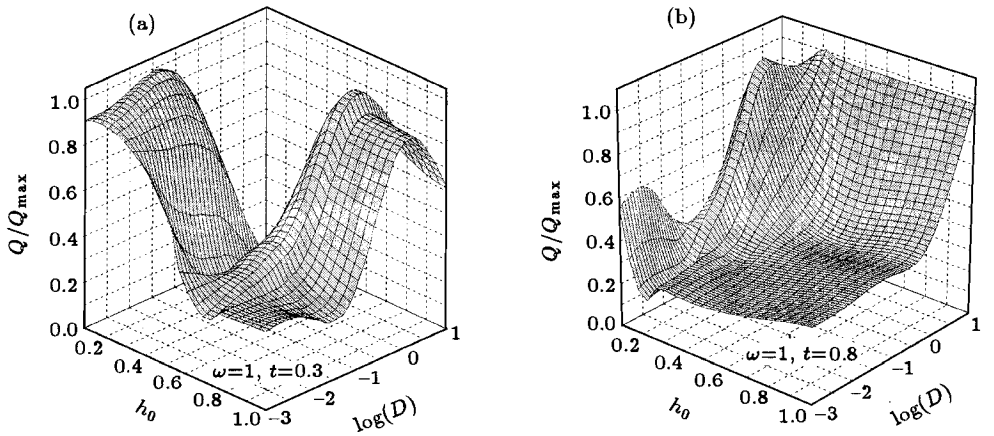


Fig.3. The data surface of dynamic order-parameter Q in a coordinate system consisting of noise intensity D and driving-field amplitude h_0, while driving-field frequency ω=1 and temperatures t=0.3 and t= 0.8 respectively.

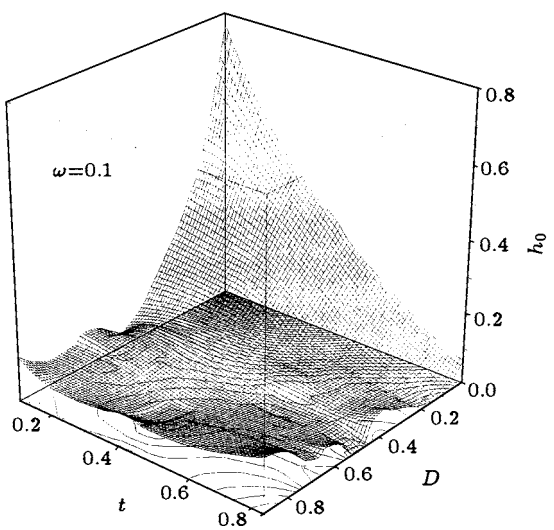


Fig.4. The dynamic boundary surface separating the dynamical paramagnetic phase from the dynamical ferromagnetic phase in a parameter space consisting of amplitude h_0 of driving field, the temperature t of system and noise intensity D.

Figure 4 visualizes the boundary surface of dynamic phase transition in a parameter space of h_0, t and D through extruding a noise axis from the two-dimensional h_0-t boundary as shown in Fig.1 by computation. The regions above and below the boundary surface correspond to a dynamic symmetric paramagnetic state ($Q = 0$) and a dynamic symmetry-breaking ferromagnetic state ($Q \neq 0$), respectively. The boundary surface neither fluctuates randomly nor declines monotonously with the increase of temperature and intensity of noise. The dynamic ferromagnetic phase augments regularly within some specific regions of t and D, as profiled by the enlarged zone of dynamic ferromagnetic phase under the boundary surface, implying that a certain dynamic ferromagnetic order is induced by noise. The intersection of the boundary surface with the plane $D = 0$ is the common boundary of dynamic phase transition as exhibited in Fig.1.

In addition to the typical dynamic paramagnetic phase ($Q = 0$) and dynamic ferromagnetic phase ($Q = 1$) existing in the kinetic Ising spin system, there appears intriguingly, probably due to the involvement of stochastic field, another new intermediate-order dynamic phase (IODP) at $Q/Q_{\max} = 0.66$ nearly invariable. The region where IODP exists steadily is sensitively dependent upon other parameters, particularly D and ω. In Fig.5 plotted is the curve of reduced dynamic order $Q/Q_{\max}$ against h_0 in the cases of several noise intensities D. The influence of white noise on the transition from the dynamic order state $Q = 1$ to the dynamic disorder state $Q = 0$ is also demonstrated in Fig.5. At zero noise, the smooth curve of $Q/Q_{\max}$–h_0 presents simply a two-state trend, and the critical amplitude h_{0c} declines with temperature increasing, as indicated in Fig.1. The involvement of white noise brings in not only the fluctuation of original dynamic disorder phase within the range of $Q/Q_{\max} = 0 - 0.3$, but also the presence of the new IODP. Note that the IODP initiates at $Q/Q_{\max}$=0.66 when D=0.1 and the range in which IODP exists steadily (the flat extension of Δh_0 at $Q/Q_{\max}$=0.66) extends with the increase of D as shown in Fig.6(a). The temperature dependence of IODP existence, however, is complicated in that two distinctive trends of Δh_0 versus t are observed at a weak and a strong noise, respectively. Figure 6(b) indicates that the monotonous drop of Δh_0 versus t in larger D values turns into a peak-shaped trend in small D values, and the greater the D value is, the faster the Δh_0 drops. The optimal temperature at which Δh_0 peaks, if any, shifts to a low temper-

ature with the increase of D. The influence of frequency ω on Δh_0 is quite prominent as displayed in Fig.7. According to Fig.7, the flat extension of Δh_0 comes into view only when frequency ω takes even number, such as 2 or its doubled frequency 4 as indicated by the solid curve in the figure, and the extension shrinks once frequency rises. A slight deviation of frequency from 2, e.g. ω=1.8 and ω=2.2 in the figure, causes completely the disappearance of the flat extension of Δh_0 finally. And this very sensitivity to the frequency also discloses the metastable nature of IODP at $Q/Q_{\max}$=0.66 in contrast to the stable dynamic ferromagnetic phase at $Q/Q_{\max}$=1.

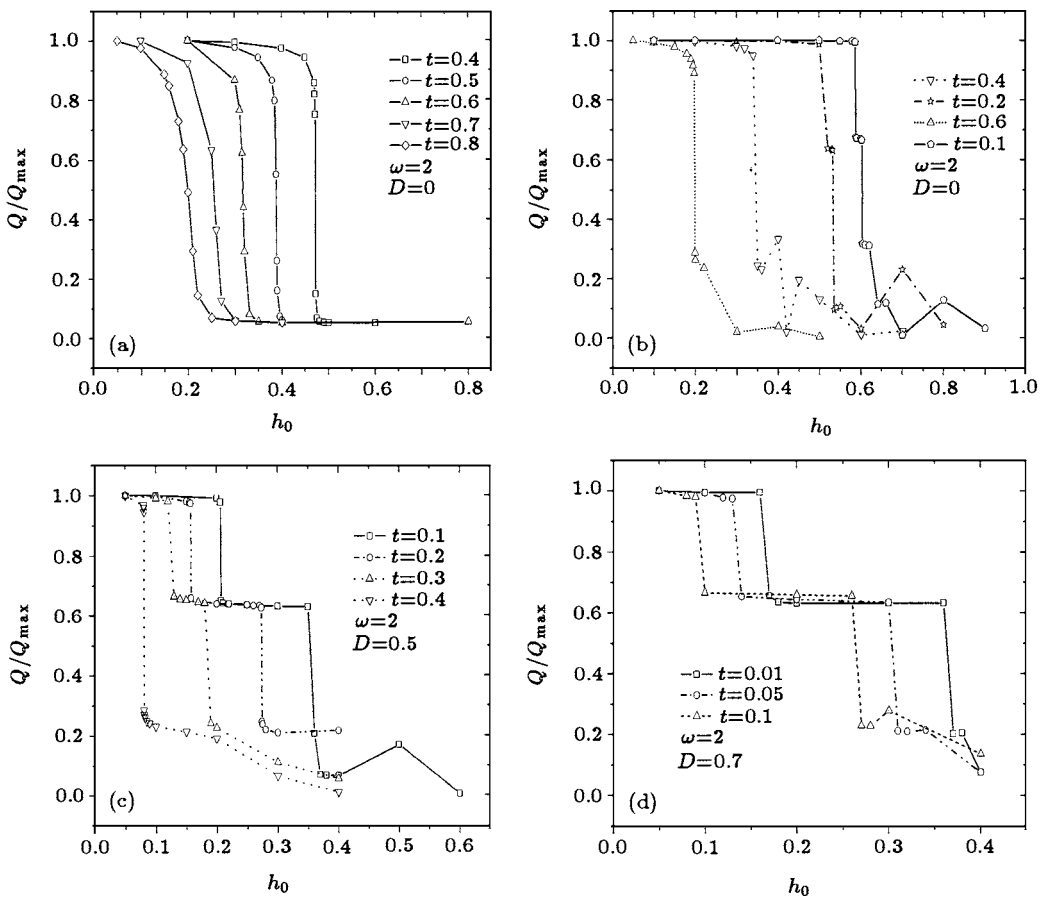


Fig.5. The dependence of reduced dynamic order-parameter $Q/Q_{\max}$ upon the amplitude h_0 of driving field at various temperatures t and different noise intensities D.

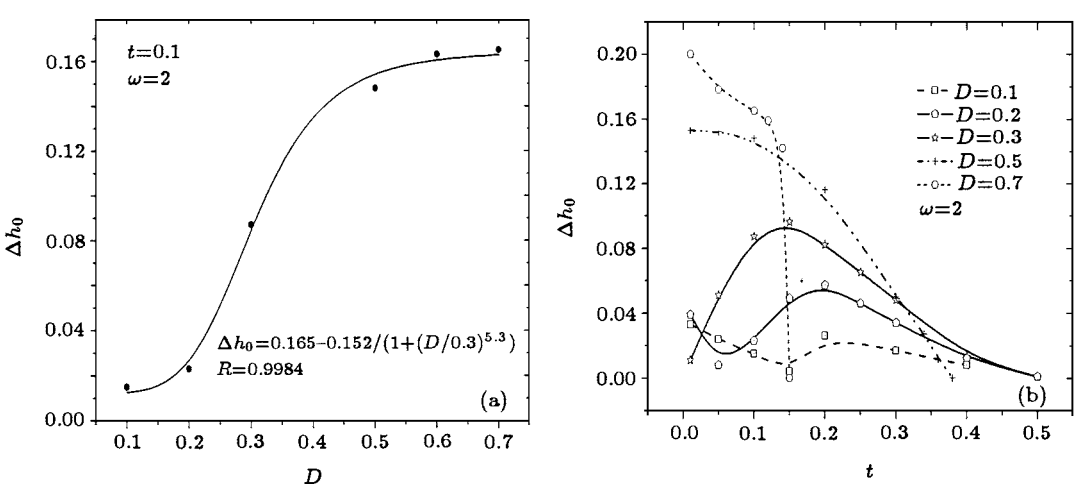


Fig.6. The amplitude extension of driving field Δh_0, corresponding to a stable intermediate-order dynamic ferromagnetic phase, versus noise intensity D (a) and temperature t (b).

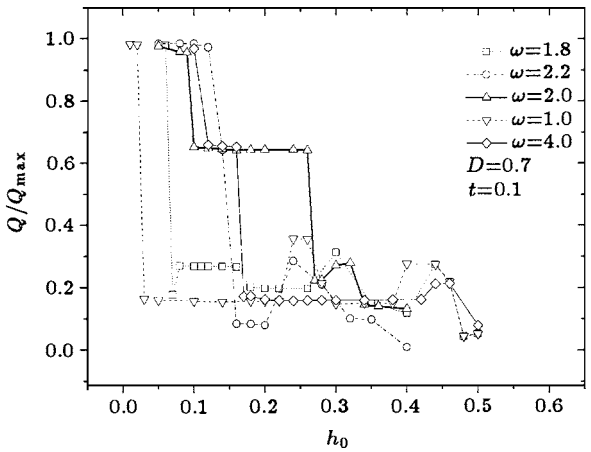


Fig.7. The dependence of reduced dynamic order-parameter $Q/Q_{\max}$ upon the amplitude h_0 of driving field at different frequencies ω of driving field.

4. Summary

We have studied the dynamic response and nonequilibrium dynamic phase transition of a kinetic Ising spin system in the case of stochastic resonance. Noise plays a certain constructive role in inducing the dynamic order of kinetic Ising spin system subject to the jointly driving of both a stochastic field and a determinant periodic modulation at an appropriate temperature. Dynamic order parameter peaks at a specific intensity of noise, a typical feature of stochastic resonance. The jointly driving of stochastic field and determinant modulation gives rise to a steady nonequilibrium dynamic phase transition in the kinetic Ising spin system when some conditions are satisfied. Similar to the situation of only a determinant field applied, there exist the dynamic disordered phase and dynamic ordered phase due to the dynamic symmetry-breaking caused by the dynamic phase transition in the case of stochastic resonance. The properly matching of three kinds of competing time scales, namely the intrinsic relaxation time λ of the thermodynamic system, the time period of the external driving field $2\pi/\omega$ and the average transition lifetime of moving particle jumping twice over the double-well energy barrier $\tau_k(D)$, is vital to the occurrence of the dynamic phase transition. The variations of the amplitude and the frequency of periodic modulation not only affect the intensity of resonant peak of dynamic order parameter, resulting in the shift of the peak, but also bring about the change of the shape of curve from peak to valley, indicating the transformation of the system from a steady state of preferential dynamic symmetry to a steady state dominated by the dynamic symmetry-breaking.

Another peculiar feature of the nonequilibrium dynamic phase transition in the case of stochastic resonance is the development of intermediate-order dynamic phase (IODP) at $Q/Q_{\max}$ =0.66 in the kinetic spin system. The IODP, in contrast with fully dynamic ordered phase at $Q/Q_{\max}$ =1, depends strongly upon the amplitude of periodic field, intensity of noise and temperature. Moreover, the intermediate-order dynamic phase also shows the very sensitivity to the frequency of periodic field and exists only at even number frequencies. By judging the nature of intermediate-order dynamic phase, we regard it as a metastable dynamic ordered phase.

References

[1] Gammaitoni L, Hanggi P, Jung P and Marchesoni F 1998 *Rev. Mod. Phys.* **70** 223
[2] Neda Z 1996 *Phys. Rev.* E **51** 5315
[3] Ruszczynski P, Schimansky-Geier L and Dikshtein I 2000 *Eur. Phys. J.* B **14** 569
[4] Kim B J, Minnhagen P, Kim H J and Choi M Y 2001 *EuroPhysics Lett.* **56** 333
[5] Pikovsky A, Zaikin A and Casa M A 2002 *Phys. Rev. Lett.* **88** 050601-1
[6] Siewert U and Schimansky-Geier L 1998 *Phys. Rev.* E **58** 2843
[7] Xu W, Jin Y F, Li W and Ma S J 2005 *Chin. Phys.* **14** 1077
[8] Leung K T and Neda Z 1999 *Phys. Rev.* E **59** 2730
Leung K T and Neda Z 1998 *Phys. Lett.* A **246** 505
[9] Phillips J C and Schulten K 1995 *Phys. Rev.* E **52** 2473
[10] Mahato M C and Shenoy S R 1994 *Phys. Rev.* E **50** 2503
[11] Vilar J M G and Rubi J M 2001 *Phys. Rev. Lett.* **86** 950
[12] Jeon G S, Lim J S, Kim H J and Choi M Y 2002 *Phys. Rev.* B **66** 024511
[13] Sides S W, Ramos R A, Rikvold P A and Novotny M A 1997 *J. Appl. Phys.* **81** 5597
[14] Tome T and Oliveira M J 1990 *Phys. Rev.* A **41** 4251
[15] Acharyya M 1999 *Phys. Rev.* E **59** 218
Acharyya M 1998 *Phys. Rev.* E **58** 179
[16] Korniss G, Rikvold P A and Novotny M A 2002 *Phys. Rev.* E **66** 056127
[17] Chakrabarti B K and Acharyya M 1999 *Rev. Mod. Phys.* **71** 847
[18] Sides S W, Rikvold P A and Novotny M A 1998 *Phys. Rev. Lett.* **81** 834
[19] Korniss G, White C J, Rikvold P A and Novotny M A 2000 *Phys. Rev.* E **63** 016120
[20] Acharyya M and Chakrabarti B K 1995 *Phys. Rev.* B **52** 6550
[21] Sides S W, Rikvold P A and Novotny M A 1998 *Phys. Rev.* E **57** 6512
[22] Liang G Y 2003 *Chin. Phys.* **12** 377
[23] Li Z D, Li L, Liu W M, Liang J Q and Ziman T 2003 *Phys. Rev.* E **68** 036102
[24] Shao Y Z, Zhong W R, Lin G M and Li J C 2004 *Acta Phys. Sin.* **53** 3157 (in Chinese)