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The autocorrelation of speckles in deep Fresnel diffraction region and characterizations of random self-affine fractal surfaces*

Teng Shu-Yun(滕树云)[a)†], Cheng Chuan-Fu(程传福)[a)b)], Liu Man(刘 曼)[a)], Gui Wei-Ling(桂维玲)[a)], and Xu Zhi-Zhan(徐至展)[b)]

[a)] *Department of Physics, Shandong Normal University, Jinan 250014, China*

[b)] *Shanghai Institute of Optics and Fine Mechanics, Chinese Academy of Sciences, Shanghai 201800, China*

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This paper studies the correlation properties of the speckles in the deep Fresnel diffraction region produced by the scattering of rough self-affine fractal surfaces. The autocorrelation function of the speckle intensities is formulated by the combination of the light scattering theory of Kirchhoff approximation and the principles of speckle statistics. We propose a method for extracting the three surface parameters, i.e. the roughness w, the lateral correlation length ξ and the roughness exponent α, from the autocorrelation functions of speckles. This method is verified by simulating the speckle intensities and calculating the speckle autocorrelation function. We also find the phenomenon that for rough surfaces with $\alpha = 1$, the structure of the speckles resembles that of the surface heights, which results from the effect of the peak and the valley parts of the surface, acting as micro-lenses converging and diverging the light waves.

Keywords: speckles, deep Fresnel diffraction, random self-affine fractal surface
PACC: 4225F, 4225H

1. Introduction

Speckle fields are the random intensity distributions produced when the incident light waves are scattered from random surfaces. They appear in many optical phenomena related to light propagations[1,2] and have found wide applications in a variety of scientific and technical fields.[3−5] In reviewing the studies of speckles, we may divide the speckle fields into different categories according to the position of the observation plane in the following diffraction regions: the near-field region,[6] specifying a region within a distance smaller than a few wavelengths from the random surface and with the evanescent waves inducing interesting phenomena,[7] the far-field regions including the Fresnel and the Fraunhofer diffraction regions, and the deep Fresnel diffraction region located in between the near-field region and the Fresnel diffraction region, defined by Wu *et al*[8] in the studies of coherent polychromatic speckles. At present, most of the papers in the literature are focused on the speckles in the Fresnel and the Fraunhofer diffraction regions. The structures of the speckle fields in these two regions have been well understood to depend sensitively on the structure of the surfaces, but in the statistical descriptions of the speckle structures, the autocorrelation function of intensities are only related to the scattering aperture,[1] thus the information of the random surfaces is missed. In recent researches[9] with a qualitative analysis, the speckles in the deep Fresnel region have been shown to have a correlation function containing the information of the random surface and then the average scattering grain sizes of the objects are successfully extracted. In the descriptions of practical surfaces, more parameters such as surface roughness w, lateral correlation length ξ (or average scattering grain size, as it is called) and fractal dimension α are needed. The extraction of all these parameters would be of great significance in the characterization of random surfaces; however, it has not been tackled by using the correlation functions of the speckle fields.

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†E-mail: tengshuyun@yahoo.com.cn

Combining the scattering theories of random rough surfaces and the statistical theories of speckles, we will derive in this paper the rigorous analytical expression for the autocorrelation function of the speckle fields in the deep Fresnel diffraction region. In contrast to the autocorrelation function of the far-field speckles, it has the form with the scattering intensity profile taking the place of the aperture of the scattering object, and hence it is related explicitly and quantitatively to the surface parameters. Then we propose the method for extracting the surface parameters by fitting the data of the speckle autocorrelation function to the obtained theoretical results. This method is validated by the simulational measurement of the three surface parameters: i.e., the roughness w, the lateral correlation length ξ and the roughness exponent α of the self-affine fractal surface samples. In the simulations, we generate the random surface samples, and calculate the corresponding speckle intensities and the data of their autocorrelation functions, with which the fits are performed. Additionally, by analysis of the calculated speckle intensities, we can find the local fluctuations in the speckle structure in deep Fresnel diffraction region, and reveal the obvious resemblances between the structure of the speckle intensities in the observation plane at an appropriate distance and the variation of the surface heights for the random surface with $\alpha = 1$, which originates from the peaks and the valleys of the surface, converging and diverging the light waves like the micro-lenses.

2. Theoretical analysis of the speckle formation in deep Fresnel diffraction region

For simplicity, we consider the one-dimensional geometry of speckle formation in deep Fresnel diffraction region as shown in Fig.1. The random screen is placed on the object plane with coordinate x_0, and the random screen is illuminated by a parallel laser beam with wavelength λ and amplitude U_0. The observation plane with coordinate x at a distance z from the object plane is in the deep Fresnel diffraction region, which means that z is so small that the area on the object plane, from which the scattered light waves contribute to an observation point, is smaller than the area of illuminating beam. The formula of the scattered light field in Kirchhoff approximation is given by Eq.(4.17) in Ref. [10]. With the terms of surface height derivatives neglected and the transmitting object taken into account and by letting $y = 0$, we can obtain the expression for the scattered light field at point x for the one-dimensional case:

$$U(x) = \int \exp[\mathrm{i}2\pi(n-1)h(x_0)/\lambda] \times \frac{\exp(\mathrm{i}kr)}{r}\cos\theta \mathrm{d}x_0, \quad (1)$$

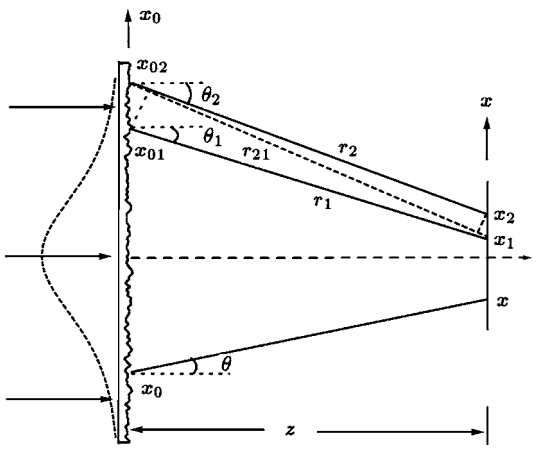


Fig.1. Schematic diagram for generating the speckles in the deep diffraction region.

where $h(x_0)$ is the surface height distribution of the screen, $\cos\theta$ is the inclination factor, n is the refractive index of the screen, $k = 2\pi/\lambda$ is the magnitude of the wave vector, and $r = [z^2 + (x - x_0)^2]^{1/2}$ is the distance between the object point and the observation point. To obtain the autocorrelation of speckle intensities, we need to calculate the autocorrelation function of the light field first, which is defined as follows:

$$\begin{aligned} R_U(x_1, x_2) =& \langle U(x_1)U^*(x_2)\rangle \\ =& \iint \langle \exp\{\mathrm{i}2\pi(n-1)[h(x_{01}) - h(x_{02})]/\lambda\}\rangle \\ & \times \frac{\exp(\mathrm{i}kr_1)}{r_1}\frac{\exp(-\mathrm{i}kr_2)}{r_2} \\ & \times \cos\theta_1\cos\theta_2 \mathrm{d}x_{01}\mathrm{d}x_{02}, \end{aligned} \quad (2)$$

where Eq.(1) has been used, $\langle\ \rangle$ denotes the ensemble average, and r_1 and r_2 are the distances of the observation points x_1, x_2 from the object points x_{01}, x_{02}, respectively. Assuming that the height distributions of the random screen have Gaussian joint probability density, then we have the following relation:

$$\begin{aligned} &\langle \exp\{\mathrm{i}2\pi(n-1)[h(x_{01}) - h(x_{02})]/\lambda\}\rangle \\ =& \exp\{-[2\pi(n-1)]^2[w^2 - R_h(x_{01} - x_{02})]\}, \end{aligned} \quad (3)$$

where w is the root-mean-square deviation roughness, $R_h(x_{01} - x_{02}) = R_h(x_{01}, x_{02}) = \langle h(x_{01})h(x_{02})\rangle$ is the

autocorrelation function of the surface height. Usually $R_U(x_1, x_2)$ and $R_h(x_{01} - x_{02})$ are narrow functions with respect to the correlation separations $x_1 - x_2$ and $x_{01} - x_{02}$, respectively, indicating that the range of $x_{01} - x_{02}$ for the integrand on the right-hand side of Eq.(2) to be significant is very small (in the order of correlation length of the random surface, i.e., about a few micrometers). Then we only need to consider the case with x_2 close to x_1 and x_{02} close to x_{01}, and we have $\theta_1 \approx \theta_2$. Defining r_{21} as the distance between points x_{02} and x_1 as shown in Fig.1, we can obtain in first-order approximation:

$$r_{21} \approx r_2 + (x_2 - x_1)\sin\theta_1, \tag{4}$$

$$r_{21} \approx r_1 + (x_{02} - x_{01})\sin\theta_1, \tag{5}$$

where we have used $\sin\theta_2 \approx \sin\theta_1 = (x_{01} - x_1)/r_1$. Then we have:

$$r_1 - r_2 \approx (\Delta x_0 - \Delta x)\sin\theta_1, \tag{6}$$

where $\Delta x_0 = x_{02} - x_{01}$ and $\Delta x = x_2 - x_1$. With these relations and the variable substitutions of $x_{02} = x_{01} + \Delta x_0$ and $k_q = \sin\theta_1/\lambda = (x_{01} - x_1)/(\lambda r_1)$ in the integral, Eq.(2) can be simplified as

$$R_U(\Delta x) = \lambda \iint \exp\{-[2\pi(n-1)/\lambda]^2[w^2 - R_h(\Delta x_0)]\} \times \frac{\exp[\mathrm{i}2\pi k_q(\Delta x_0 - \Delta x)]}{r_1}\mathrm{d}k_q\mathrm{d}\Delta x_0, \tag{7}$$

where $R_U(\Delta x) = R_U(x_1, x_2)$, and we have used $\cos\theta_2 \approx \cos\theta_1$ and $\mathrm{d}k_q = (\cos^2\theta_1)dx_{01}/(r_1\lambda)$ and $r_1 \approx r_2$ in the denominator.

The integral with respect to Δx_0 in Eq.(7) is actually the Fourier transform $FT^{-1}[F(\Delta x_0)]$ of the function $F(\Delta x_0) = \exp\{-[2\pi(n-1)/\lambda]^2[w^2 - R_h(\Delta x_{01})]\}$, and it is transformed to the function of k_q, where the respective integral is in essence carried out over the random screen. From the results of light scattering theory,[10,11] we know that $FT^{-1}[F(\Delta x_0)]$ is the scattered intensity profile of k_q. Then Eq.(7) has the obvious significance in physics: the correlation function of the light wave in the deep Fresnel diffraction region is the integral with respect to k_q, the integrand being the scattering profile reciprocally projected onto the screen surface. This is illustrated by the bell-shaped curve on the screen in Fig.1. In Eq.(7), $r_1 = z/\cos\theta_1$ is varied in the integral with respect to k_q, and it can be written as

$$R_U(\Delta x) = (\lambda/z) \iint \exp\{-[2\pi(n-1)/\lambda]^2 \times [w^2 - R_h(\Delta x_{01})]\} \times \exp[\mathrm{i}2\pi k_q(\Delta x_{01} - \Delta x)]\cos\theta_1\mathrm{d}k_q\mathrm{d}\Delta x_{01}. \tag{8}$$

If the inclination factor $\cos\theta_1$ does not vary considerably within the significant part of the scattered intensity profiles, the approximation $\cos\theta_1 \approx 1$ can be made, as in most of the literature for diffraction theory.[9] The integral on the right-hand side of Eq.(8) simply reduces to the pair of Fourier transform and the inversion of the $\exp\{-[2\pi(n-1)/\lambda]^2[w^2 - R_h(\Delta x_{01})]\}$. The correlation function of the light field turns out to be

$$R_U(\Delta x) = (\lambda/z)\exp\{-[2\pi(n-1)/\lambda]^2[w^2 - R_h(\Delta x)]\}. \tag{9}$$

Based on the definition of the autocorrelation function of intensity, we have $R_I(x_1, x_2) = \langle I(x_1)I(x_2)\rangle$. From the speckle theory, we know that when the surface is rough, $U(x)$ is a circular complex Gaussian variable,[1] and with the Gaussian moment theorem, we have

$$R_I(x_1, x_2) = \langle I\rangle^2 + |\langle U(x_1)U^*(x_2)\rangle|^2, \tag{10}$$

where $\langle I\rangle = \langle I(x_1)\rangle = \langle I(x_2)\rangle$ is the average speckle intensity. It is usually more convenient to use the normalized autocorrelation function of the speckle intensity defined as

$$\gamma_I(x_1, x_2) = [R_I(x_1, x_2) - \langle I\rangle^2]/\langle I\rangle^2. \tag{11}$$

Using Eqs.(9)–(11) and noticing $\langle I\rangle = \langle R_U(\Delta x = 0)\rangle$ and $w^2 = R_h(\Delta x = 0)$, we finally obtain

$$\gamma_I(\Delta x) = \exp\{-2[2\pi(n-1)/\lambda]^2[w^2 - R_h(\Delta x)]\}. \tag{12}$$

Compared with the speckle intensity autocorrelation function $\gamma_I(\Delta x)$ in the far-field,[1] which is related to the Fourier transform of the squared modulus of the scattering aperture function, the above equation is the result that the scattering intensity profile reciprocally projected on the object plane takes the place of the aperture function. Rigorously speaking, the scattering aperture function and the reciprocated scattering intensity profile determine $\gamma_I(\Delta x)$ altogether. Because the scattering intensity profile is narrowed when the observation plane is close to the object plane, its range is much smaller than the scattering aperture, and then it dominates $\gamma_I(\Delta x)$. As a result, Eq.(12) is obtained.

From Eq.(12), we know that $\gamma_I(\Delta x)$ depends only on the properties of the random surface, such as its roughness w and the autocorrelation function $R_h(\Delta x)$ of its height. In this paper, we will consider the random self-affine fractal surfaces which can describe a variety of practical surfaces ranging from growth fronts to ground interfaces.[12,13] The height autocorrelation function of this kind of surfaces is phenomenologically given by

$$R_h(\rho) = w^2 \exp[-(\rho/\xi)^{2\alpha}], \tag{13}$$

where $\rho = |\Delta x_0|$ is the correlation separation, ξ the lateral correlation length and α the roughness exponent related with the fractal dimension d_f by $d_f = d - \alpha$, with d the embedded dimension.[14] When $\alpha = 1$, the random surfaces reduce to those with Gaussian correlation.

3. Numerical simulation of the speckle field scattered by the random self-affine fractal surface

Next, we simulate the speckle field produced by the scattering of random self-affine fractal surface. We generate the numerical height distribution $h(x_0)$ in surface samples with arbitrary values of parameters by using the speckle-autocorrelation-function-analogy algorithm, which has been proposed in Ref.[15]. Based on Eq.(1), we calculate the light field $U(x)$ in the deep Fresnel diffraction region with the generated height $h(x_0)$ of a surface sample and the corresponding intensity distribution $I(x)$, which is the quantity that can be actually measured with the detector in practical experiment. In practical calculations, the wavelength of the illuminating light is set at 632.8nm, corresponding to that of the He-Ne laser. The refractive index of the screen is that of glass, i.e. $n = 1.532$.

Figure 2(a) gives the speckle intensity distributions of 256×256 pixels in the deep Fresnel diffraction region produced by a random surface sample with the parameters w =0.2μm and ξ =3.0μm, and $\alpha = 0.8$. The range of distance from the observation plane to the object plane is $[5\lambda, (5+255)\lambda]$ with a distance increment λ. We can see that there are structures of local fluctuations in the speckle intensities, which are induced by the fractal characteristics of the random surface, and can rarely be found in the far-field speckles except in the deliberately arranged optical systems.[16] This figure also shows how the speckle intensities evolve in the deep Fresnel diffraction region. Figure 2(b) shows the intensity distributions at the distances 60, 80, 100 and 120λ, respectively, produced by a surface sample of Gaussian correlation ($\alpha = 1.0$) with w =0.2μm and ξ =4.5μm, as shown in the lower part of this figure. Here proper background values have been added in the intensity curves except for the curve at the distance 60λ. We may see that the peak positions in the speckle intensity curves are almost the same as those in the surface height. This means there are obvious resemblances between the speckle intensities and the surface heights. These resemblances originate from the local smoothness of the surface, of which the peaks and the valleys act as convex and concave micro-lenses, respectively. Obviously, the convex ones converge the light and the bright spots with larger intensities are formed behind them, while the concave ones diverge light with dark area formed behind. By careful comparison of the speckle intensity curves and the surface height, we may find that the intensity distributions at the distance 100λ resembles the heights most, indicating that this distance is close to the average focal length of the convex micro-lenses.

It should be noted that the light waves scattered from a point on the random surface propagate in a cone-like region with a certain angular range behind the surface. Correspondingly, the light field at a point on the observation plane is the addition of the light waves scattered from the area on the random surface subtending an angular range of the same size at this point. In the deep Fresnel diffraction region, this angular range does not change with distance of the observation plane from the object surface, and for most random surface scattering problems, this range does not extend too large and the approximation of $\cos\theta \approx 1$ will not result in significant error.[9]

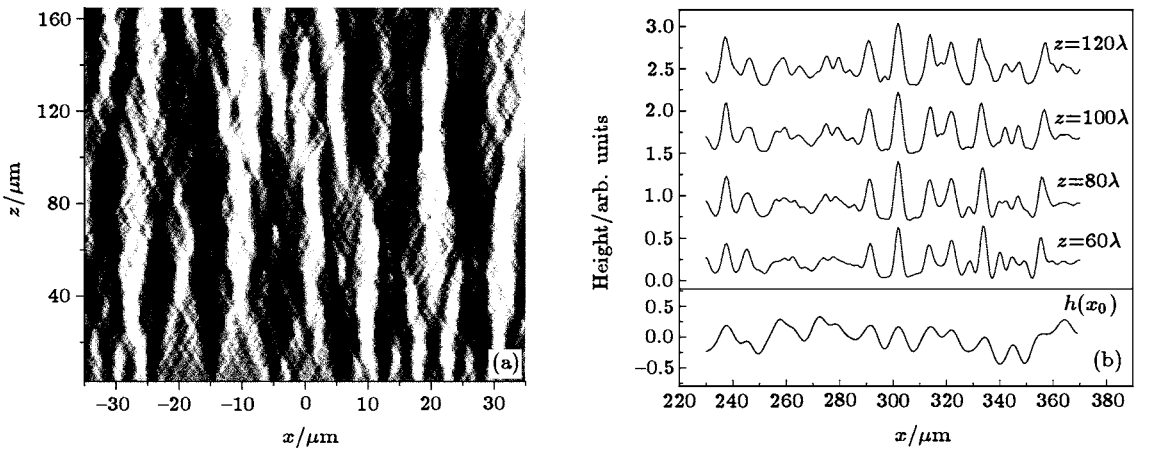


Fig.2. (a) The grey scale map of the speckle intensity distributions in the deep Fresnel diffraction region produced by a random surface sample with w =0.2μm and ξ =3.0μm, and α = 0.8. (b) The intensity distributions at different distances produced by a surface sample of Gaussian correlation (α = 1.0) with w =0.2μm and ξ=4.5μm.

Next, we concentrate on the simulation of the extraction of surface parameters from the autocorrelation function of the speckle intensities using the above theoretical results, in which the speckle fields need to have a Gaussian probability distribution, and require the speckle intensity at the observation plane is contributed from a large number of scatterers on the object. This requires that the reciprocally scattering profile is broad enough, i.e. the distance z of the observation plane in the deep Fresnel region needs to be large enough. Concurrently, the range of the object needs to be larger than that of the reciprocated scattering profile. To ensure these conditions to be satisfied, we set the distance z at 2cm, and the range of the generated surface be 6cm with 2×10^5 sampling points. The size of the observation plane is 8.4mm with 2×10^4 sampling points of the intensity distributions. Again, the speckle fields and the corresponding intensity distributions are calculated with Eq.(1) for the given random surface samples. From the intensity distribution, the correlation function $\langle R_I(\Delta x)\rangle$ and the average intensity $\langle I\rangle$ are numerically calculated. Then $\gamma_I(\Delta x)$ can be calculated based on its definition, i.e. Eq.(11). Finally, by the least squares fit of the $\gamma_I(\Delta x)$ curve to the theoretical result obtained using Eq.(12) combined with Eq.(13), the values of the parameters w, ξ and α of the random surface sample producing the speckle field are obtained. In this process, what is crucial in the extraction of surface parameters is the formulation of Eq.(12), which provides the quantitative foundation for the fit and makes the extraction possible.

Figure 3 shows the data of intensity autocorrelation functions of six generated random surface samples by dots. The corresponding curves fit with Eq.(12) are shown in solid lines. The surface parameters extracted from the fits for samples Nos.1–6 are given in the figure from top to bottom. The values of the parameters set in the generation of these samples are given in Table 1. Comparing the set values and the fit values of the surface parameters, we see that on the whole, results extracted from the intensity correlation function agree well with the set values of the random surfaces. From these curves, we can also see how the autocorrelation function of speckles varies with the parameters of the self-affine fractal surfaces. The values of α determine the decrease of the autocorrelation function in the region of small correlation separation Δx, and the values of ξ and w together determine the size of the speckle correlation range, beyond which the correlation approaches their asymptotical values.

Table 1. Values of the parameters set in the generation of samples Nos.1–6.

Sample number	1	2	3	4	5	6
ω	0.2	0.2	0.3	0.4	0.2	0.2
ξ	3.0	3.0	4.5	4.5	6.0	3.0
α	0.6	0.4	0.8	0.8	1.0	1.0

It should be noted that in the derivation of Eq.(9), i.e. the expression of $R_U(\Delta x)$, we have taken the approximation that inclination factor $\cos\theta_1 \approx 1$, which is shown to be of good accuracy by the above extracted results. The surface parameters can be also extracted rigorously with the following data processing algorithm by numerically eliminating the inclination factor from correlation function based on Eq.(8), especially when the approximation may result in considerable errors. First we calculate $\gamma_I(\Delta x)$ from the intensity distributions, and then obtain $R_U(\Delta x)$ =

$\langle U(x_1)U^*(x_2)\rangle$ based on Eqs.(10) and (11). Next we perform the numerical inverse Fourier transform $FT^{-1}[R_U(\Delta x)]$, which, based on Eq.(8), is a function of spectrum coordinate f_q given by $FT^{-1}[R_U(\Delta x)] = (\lambda/z)\cos\theta_1 FT^{-1}[F(\Delta x_0)]\,|_{f_q}$. Making the numerical product of $(\cos\theta_1)^{-1}FT^{-1}[R_U(\Delta x)]$, where $(\cos\theta_1)^{-1}$ can be reciprocated from f_q with the relation given in the above context, and then performing the Fourier transform of this product, i.e., $FT\{(\cos\theta_1)^{-1}FT^{-1}[R_U(\Delta x)]\}$, we have the data of $(\lambda/z)F(\Delta x_0)$. Normalizing these data, and then making the square numeration, we obtain the data of $[F(\Delta x_0)]^2$. By the least squares fit of the data to Eq.(12) with $R_h(\Delta x_0)$ taking the form for the self-affine surface, we can extract the surface parameters of w, ξ and α. In principle, this algorithm can eliminate the effect of the inclination factor. We have practically performed the algorithm for the data processing, and the results are very close to those given in Fig.3. This indicates that the inclination factor does not significantly influence the results under the conditions we have used in the simulation in this paper, and Eq.(12) can be directly used for the least squares fit.

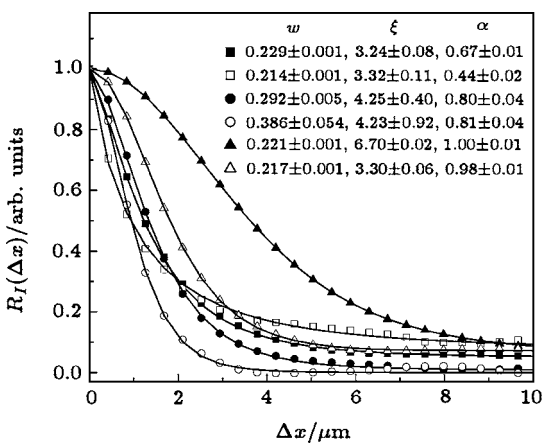


Fig.3. Curves of the speckle intensity autocorrelation functions produced by surface samples Nos.1–6 and the extracted values of surface parameters.

Additionally, we only discussed the one-dimensional case in this paper. However, by use of the scattering theory, it is not difficult to show that for isotropic two-dimensional random surface samples, the result of Eq.(12) for the normalized correlation function of the speckle intensity would be accurate enough with Δx replaced by $\rho = |\Delta r|$, where $\Delta r = (\Delta x, \Delta y)$ is the two-dimensional correlation separation and ρ is its length. This means that the results of this paper can be applied to the two-dimensional cases.

4. Conclusions

We have studied quantitatively the characteristics of the speckle structures in the deep Fresnel diffraction region by formulating a speckle autocorrelation function for the random surfaces. The method for the characterization of self-affine fractal surface is proposed. Compared with the papers in the literature in which only the lateral correlation of the random surface can be measured, this paper is advantageous in the simultaneous extraction of all the three surface parameters. This method is shown to be of satisfactory accuracy by the simulation of the speckle intensities and calculations of their autocorrelation functions. This method can be also applied directly to the practical experimental measurement of the surface parameters merely by replacing the simulations with detecting the speckle intensities.

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